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ELECTRICAL ENGINEERING

THE THEORY AND CHARACTERISTICS OF ELECTRICAL CIRCUITS AND MACHINERY

BY

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PREFACE TO THE SECOND EDITION

THE second edition contains all the material in the original text but much of it has been rewritten and a great deal of new material added.

The more important additions include sections on complex alternating waves and wave analysis, on polyphase alternating current circuits, on the construction of the characteristic curves of direct-current generators and motors, on the design of direct- and alternating-current machinery, on the Blondel diagram for the synchronous motor, on the symbolic method of analysis of the induction motor, on alternating-current commutator motors, and finally a chapter on electrical measuring instruments.

The chapter on direct-current machinery has been entirely rewritten and much enlarged, and to make it complete a short chapter outlining the design of a direct-current generator has been added.

This chapter and the other sections dealing with design are not intended to cover the work required in a course on design but only to give the student some idea of the formulæ and constants involved.

The chapter on measuring instruments has been placed at the end of the book because some of the principles involved cannot be well understood by the student until he has mastered the theory of the more important electrical machines.

It was originally intended to introduce problems at the end of each chapter to be worked out by the student and also to give a list of references to the most important articles covering the subject matter of the chapter, but there was danger of making the volume too cumbersome. A number of good books of problems have been published recently and the student will find good lists of references at the ends of the various sections in the Standard Handbook.

My thanks are due to those teachers who have offered valuable criticisms of the arrangement and method of treatment in the first edition.

CLARENCE V. CHRISTIE.

MCGILL UNIVERSITY, MONTREAL,
August, 1917.

PREFACE TO THE FIRST EDITION

THIS book has been compiled as a foundation for lecture courses for junior and senior students in Electrical Engineering.

The theory and characteristics of electrical machines are developed from the fundamental principles of electrostatics and electromagnetics. Only the more standard types have been discussed since familiarity with the principles of their operations will guide the student to a complete understanding of other machines which differ only in minor respects. This general groundwork may be extended to suit the requirements of particular classes.

C. V. C.

McGILL UNIVERSITY, MONTREAL,
October 14, 1913.

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ELECTRICAL ENGINEERING

CHAPTER I

ELECTROSTATICS

1. Electrification.—Bodies which are charged with electricity are said to be electrified. Charges are of two kinds called positive and negative. Bodies which have a positive charge are acted upon by forces tending to make them give up their charge; bodies which have a negative charge are acted upon by forces tending to convey a positive charge to them equal to their negative charge. These forces are exerted through the medium separating the charges and the medium is in a state of stress.

The body with the positive charge is at a higher potential than the body with the negative charge and the difference of potential between the two is a measure of the tendency for electricity to pass from one to the other.

2. Electrical Conductors and Insulators.—If two metallic bodies charged to different potentials are joined by a metal wire, electricity will flow from one to the other until the potential of both is the same and the transfer of electricity will take place almost instantaneously. The metal wire is therefore a good conductor of electricity; or, it offers a low resistance to the passage of electricity through it.

If the two charged bodies had been joined by a glass rod, there would have been no transfer of electricity between them, or, it would have taken place so slowly that it could only be detected by the most delicate instruments. Glass is therefore a very bad conductor; or, it offers a very high resistance to the passage of electricity. It is called a non-conductor or insulator.

As all materials conduct to a certain extent, it is not possible to divide them absolutely into conductors and insulators, but, since the resistance of a good insulator is many million times that of a good conductor, they may be so divided for practical purposes.

In the first class are silver, which is the best conductor, copper and other metals, graphite, impure water and solutions of salts. In the second class are air, which when dry is an almost perfect insulator, glass, paraffin, ebonite, porcelain, rubber, shellac, oils and the numerous insulating compounds used in electrical engineering.

3. Electrostatics and Electromagnetics.—Electrostatics comprises phenomena related to electric charges at rest and to the stresses produced in the fields surrounding them. These phenomena become of great importance where very large differences of potential must be provided for, as for example in the design and operation of all high voltage apparatus and systems.

Electromagnetics comprises phenomena related to electricity in motion, that is, to currents of electricity and the magnetic fields produced by them. Almost all the problems to be solved by the electrical engineer come under this head.

4. Laws of Electrostatics.—*First Law.*—Like charges of electricity repel one another; unlike charges attract one another.

Second Law.—The force exerted between two charges of electricity is proportional to the product of their strengths and is inversely proportional to the square of the distance between them; it also depends on the nature of the medium separating them.

This law can be expressed by the formula,

$$f = \frac{qq_1}{kr^2}, \tag{1}$$

where q and q_1 are the charges of electricity, r is the distance between them in centimeters, k is a constant depending on the medium separating the charges and is called its specific inductive capacity or dielectric constant. The unit of quantity is so chosen that the dielectric constant for air is unity; for all other substances it is greater than unity. The table on page 43 gives the dielectric constants for some of the most common dielectrics. f is the force in dynes exerted between the two charges; if the charges are of the same kind the force between them is a repulsion and f is positive.

5. Coulomb.—One electrostatic unit of quantity is that quantity which, when placed at a distance of 1 cm. in air from a similar quantity, repels it with a force of one dyne.

The practical unit of quantity is the coulomb; one coulomb is 3×10^9 electrostatic units; it is the quantity of electricity which

passes a point in a conductor when one ampere flows for one second (see Art. 63).

6. Electrostatic Field.—Any space in which electrostatic forces act is called an electrostatic field. The direction of the force at any point in the field is the direction in which a unit positive charge placed at the point tends to move and its intensity is the force in dynes exerted on the unit charge.

The electrostatic field is conveniently represented by lines of electrostatic induction or dielectric flux drawn in the direction of the force. In air the number of lines per square centimeter is equal to the force in dynes at the point and in a medium of dielectric constant k the number of lines per square centimeter is equal to k times the force. This may be stated in another way: Unit electrostatic force produces one line of dielectric flux per square centimeter in air and k lines per square centimeter in a medium of dielectric constant k.

The electrostatic force at a point is expressed in dynes and is represented by $\mathfrak{F}$; the dielectric flux density at a point is expressed in lines per square centimeter and is represented by $\mathfrak{D}$.

7. Field Surrounding a Point Charge.—At a distance r cm. in air from an isolated charge q, a unit charge is acted on by a force

$$\mathfrak{F} = \frac{q}{r^2} \text{ dynes;} \tag{2}$$

and the dielectric flux density produced at the point is

$$\mathfrak{D} = \mathfrak{F} = \frac{q}{r^2} \text{ lines per square centimeter.} \tag{3}$$

This density is produced over the surface of a sphere of radius r and therefore the total dielectric flux from the charge q is

$$\psi = \frac{q}{r^2} 4\pi r^2 = 4\pi q \text{ lines.}$$

Consider the same charge surrounded by a medium of dielectric constant k.

The force exerted on a unit charge at a distance r cm. from q is

$$\mathfrak{F} = \frac{q}{kr^2} \text{ dynes;}$$

the dielectric flux density produced is

$$\mathfrak{D} = \mathfrak{F}k = \frac{q}{r^2} \text{ lines per square centimeter,}$$

and therefore the dielectric flux from charge q is, as before,

$$\psi = 4\pi q \text{ lines.} \tag{4}$$

Fig. 1 represents the electrostatic field about a positive point charge. The lines of flux extend out radially in all directions.

8. Dielectric Flux from a Unit Charge.—The dielectric flux from a unit charge is 4π lines by equation 4. Thus if a dielectric flux ψ starts from any surface the positive charge on that surface is $q = \frac{\psi}{4\pi}$ units, and if a flux ψ ends on any surface the negative charge on that surface is $-q = \frac{\psi}{4\pi}$ units.

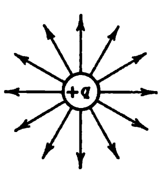


FIG. 1.—Field surrounding a point charge.

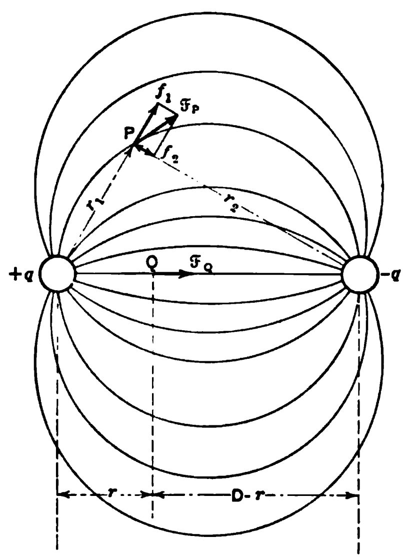


FIG. 2.—Field between two point charges.

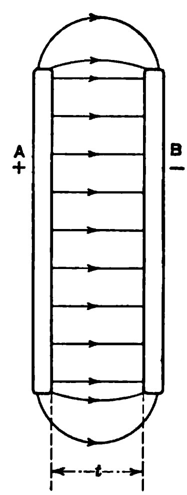


FIG. 3.—Field between two parallel plates.

A unit positive charge is always associated with each 4π lines leaving a surface and a unit negative charge with each 4π lines entering a surface.

9. Field between Two Point Charges.—The field between two point charges q and $-q$ consists of curved lines extending from the positive to the negative charge.

The direction and intensity of the force at any point such as P may be obtained by combining the forces exerted at the point by the two charges as shown in Fig. 2. If a unit positive charge is placed at the point P it is repelled by q with a force $f_1 = \frac{q}{r_1^2}$

and is attracted by $-q$ with a force $f_2 = \frac{q}{r_2^2}$. The resultant force at the point is $\mathfrak{F}_P$ and it is the vector sum of f_1 and f_2.

If D cm. is the distance between the charges, the force at a point on the line joining the charges distant r cm. from the charge q is

$$\mathfrak{F}_Q = \frac{q}{r^2} + \frac{q}{(D - r)^2} \text{ dynes.}$$

Since the medium is air the flux density at any point is numerically equal to the force $\mathfrak{F}$ at the point.

10. Field between Parallel Plates.—The field between two parallel plates A and B, Fig. 3, charged respectively with $+q$ and $-q$ units per square centimeter consists of straight lines normal to the plates except near the edges where they become slightly curved.

The dielectric flux density is uniform throughout the volume between the plates, and its value is

$$\mathfrak{D} = 4\pi q \text{ lines per square centimeter.} \tag{5}$$

If the medium between the plates is air, the electrostatic force at any point is

$$\mathfrak{F} = \mathfrak{D} = 4\pi q \text{ dynes.} \tag{6}$$

If the medium has a dielectric constant k, the force is

$$\mathfrak{F} = \frac{\mathfrak{D}}{k} = \frac{4\pi q}{k} \text{ dynes.} \tag{7}$$

11. Potential.—The difference of potential between two points is measured by the work done in carrying a unit charge from one to the other against the electrostatic force in the field; it is therefore the line integral of the force between the points.

The difference of potential between the two plates in Fig. 3 is

$$e = \int_0^t \mathfrak{F}\, dr = \mathfrak{F}t = \frac{4\pi q}{k} t, \tag{8}$$

where t cm. is the distance between the plates.

Difference of potential tends to cause electricity to flow from one point to the other and is therefore called electromotive force.

Unit difference of potential (electrostatic) exists between two points when one erg of work is done in conveying unit charge from one to the other.

The earth is usually taken as the arbitrary zero of potential, and the potentials of other bodies are given with reference to it.

If a difference of potential is produced between two points on a conductor, electricity will flow from the point of high potential until the potentials of all parts of the conductor are the same.

When, however, the difference of the potential is maintained by an electric generator a current of electricity flows continuously from one point to the other.

When a difference of potential is produced between two conductors separated by an insulating material, the material is subjected to a stress and lines of dielectric flux pass through it.

12. Volt.—The practical unit difference of potential or electromotive force is called the volt. One electrostatic unit is equal to 300 volts.

13. Induced Charges of Electricity.—When a positively charged body *A*, Fig. 4, is brought near to an insulated conductor *BC*, a negative charge is induced on the nearer side *B* and an equal positive charge on the farther side *C*. The explanation is

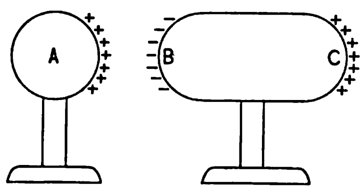


FIG. 4.—Induced charges.

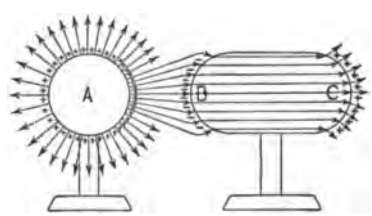


FIG. 5.—Induced charges.

that the potential of the end *B* due to the charge on *A* is higher than that of *C*, but, since *BC* is a conductor, as soon as there is a difference of potential between two points on it, electricity flows from the region of high potential at *B* to the region of low potential at *C*; thus a positive charge appears at *C* and an equal negative charge at *B* and these charges so distribute themselves over the surface of *BC* that the potential at every point on it is the same, being the sum of the potentials due to the charge on *A* and the two charges on *BC*.

The same result is illustrated in Fig. 5, which shows the dielectric flux produced.

The flux from *A* extends out radially in all directions. In the region surrounding *BC* the lines are deflected and a large number pass through the conductor *BC*, since its dielectric constant is infinite and therefore it offers an easy path. For every 4π lines

entering at B a unit negative charge appears on the surface, and for every 4π lines leaving at C a unit positive charge appears.

14. Equivalent Charges.—A uniformly distributed charge on the surface of a sphere acts as though it were concentrated at the center. If a sphere of radius R cm. has a charge Q uniformly distributed over its surface, the density of the charge is $\frac{Q}{4\pi R^2}$; and since 4π lines emanate from unit charge, the flux density at the surface of the sphere is $\frac{4\pi Q}{4\pi R^2} = \frac{Q}{R^2}$ lines per square centimeter. If the charge Q is concentrated at the center of the sphere, then, by formula (3), the flux density at a distance R cm. from the charge is $\frac{Q}{R^2}$ and therefore the uniformly distributed charge on the surface of the sphere may be represented by an equal charge concentrated at the center.

Similarly a uniformly distributed charge on the surface of a cylinder may be represented by an equal charge uniformly distributed along the axis of the cylinder.

15. Distribution of Potential in the Space Surrounding a Point Charge.—In Fig. 6 a positive charge q is placed at the point O

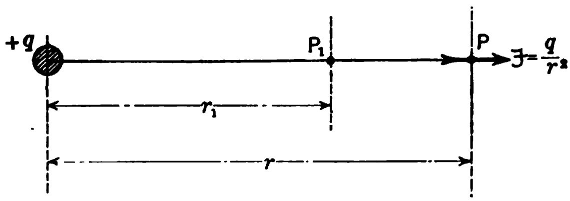


FIG. 6.—Distribution of potential about a point charge.

at an infinite distance from all other charges. The potential at a point P_1, distant r_1 cm. from O, is the work done in moving unit charge from a point of zero potential to the point P_1 against the forces in the field. The intensity of the force at a distance r cm. from O is by equation 2

$$\mathfrak{F} = \frac{q}{r^2} \text{ dynes};$$

the work done in moving a unit charge against this force through a distance dr is

$$\mathfrak{F}\,dr = q\frac{dr}{r^2} \text{ ergs};$$

and the work done in moving the charge from a point of zero potential to P_1 is

$$e_1 = \int_{r_1}^{\infty} \mathfrak{F}\, dr = \int_{r_1}^{\infty} q \frac{dr}{r^2}$$

$$= q\left[-\frac{1}{r}\right]_{r_1}^{\infty}$$

$$= \frac{q}{r_1} \text{ ergs;} \tag{9}$$

therefore, the potential at a point due to a charge of electricity is equal to the charge in electrostatic units divided by its distance in centimeters from the point.

16. Potential at a Point Due to a Number of Charges.—If there are several charges q_1, q_2, q_3, etc., in the field, the potential at any point is the sum of the potentials due to the separate charges and is

$$e = \frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \text{etc.} = \sum \frac{q}{r}, \tag{10}$$

where r_1, r_2, r_3, etc., are the distances from the various charges to the point (Fig. 7).

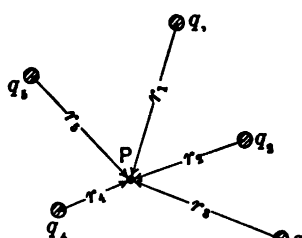


Fig. 7.—Potential due to a number of charges.

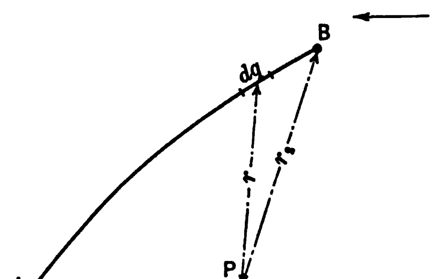


Fig. 8.—Potential due to a charged surface.

17. Potential at a Point Due to a Charged Surface.—In Fig. 8 AB is a surface with a non-uniform distribution of charge over it; if dq is a small element of charge at a distance r cm. from the point P, the potential at P due to the charge dq is

$$de = \frac{dq}{r},$$

and the potential due to the total charge on the surface is

$$e = \int \frac{dq}{r}. \tag{11}$$

18. Equipotential Surfaces.—Surfaces of which all points are at the same potential are called equipotential surfaces.

In Fig. 9, A is an isolated sphere of radius R, charged with Q units of electricity. Any spherical surface drawn about the center of A is an equipotential surface. The potential of surface (1) is $\frac{Q}{r_1}$, that of (2) is $\frac{Q}{r_2}$ and that of the sphere A is $\frac{Q}{R}$.

The difference of potential between surfaces (1) and (2) is $\frac{Q}{r_1} - \frac{Q}{r_2}$ and is the work that must be done in taking a unit charge from any point on (2) to any point on (1). It makes no difference what path the charge follows, because its path can always be resolved into two displacements, one along the equipotential surface and the other normal to it; no work is done in moving along the equipotential surface, since there is no opposing force and therefore all the work is done in the displacement along the normal.

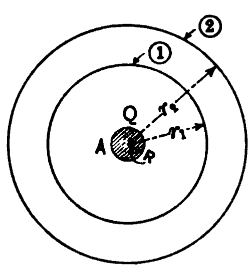


FIG. 9.—Equipotential surfaces.

The electrostatic force on an equipotential surface is normal to the surface, therefore lines of induction pass normally through equipotential surfaces.

Electric conductors not carrying current are equipotential surfaces, since if differences of potential did exist electricity would flow from the points of high potential until the potential became uniform throughout the conductor.

19. Potential Gradient.—The potential gradient at a point is the space rate of change of potential at the point measured in the direction of the electrostatic force.

The difference of potential between two points is

$$e = \int_0^D \mathfrak{F}\, dr, \tag{12}$$

where $\mathfrak{F}$ is the electrostatic force at any point and D is the distance between the points. The potential gradient at any point is

$$g = \frac{de}{dr} = \mathfrak{F} \tag{13}$$

and is equal to the electrostatic force at the point.

In practical units the potential gradient is expressed in volts per centimeter or kilovolts per centimeter.

20. Potential and Potential Gradient Near a Charged Sphere. — Determine the potential and potential gradient in the field surrounding an isolated sphere of radius R charged with Q units of electricity (Fig. 10).

The charge Q is uniformly distributed over the surface of the sphere and may be considered as an equal charge Q concentrated at the center (see Art. 14).

The potential at a point P, distant r cm. from the center of the sphere is

$$e = \frac{Q}{r}.$$

The potential gradient at P is

$$g = \frac{de}{dr} = -\frac{Q}{r^2}, \tag{14}$$

and is equal to the electrostatic force at the point.

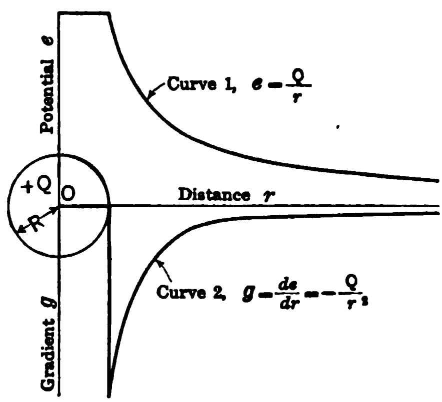


FIG. 10.—Potential and potential gradient near a charged sphere.

Curve 1, Fig. 10, shows the variation of potential with distance measured from the center of the sphere. The equation of this curve is

$$e = \frac{Q}{r}.$$

Across the sphere the potential is constant and is equal to $\frac{Q}{R}$, but near the surface it falls off very rapidly.

The potential gradient at any point is represented by the slope of the tangent to the potential curve at that point; its values are plotted in curve 2. The equation of this curve is

$$g = \frac{de}{dr} = -\frac{Q}{r^2}.$$

21. Potential and Potential Gradient between Two Charged Spheres.—Determine the potential and the potential gradient at any point between two spheres A and B (Fig. 11) of radius R cm. if the distance between their centers is D cm. and they have charges of Q and $-Q$ respectively. This last condition means that all the lines of dielectric flux which leave the sphere A fall on the sphere B.

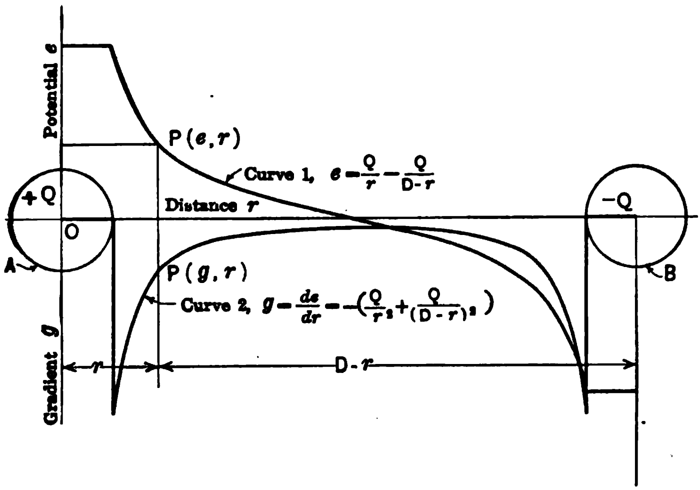


FIG. 11.—Potential and potential gradient between two charged spheres.

At a point P on the line joining the centers of the two spheres and at a distance of r cm. from the center of A the potential due to the charge Q on A is $\frac{Q}{r}$, and that due to the charge $-Q$ on B is $\frac{-Q}{D-r}$; the actual potential at P is, therefore,

$$e = \frac{Q}{r} - \frac{Q}{D-r}. \tag{15}$$

Midway between the spheres the potential is

$$\frac{Q}{D/2} - \frac{Q}{D - D/2} = O.$$

At the surface of A it is

$$\frac{Q}{R} - \frac{Q}{D-R}. \tag{16}$$

The potentials at all points between A and B are plotted in curve 1 (Fig. 11).

The potential gradient at P is

$$g = \frac{de}{dr} = -\frac{Q}{r^2} - \frac{Q}{(D-r)^2} = -\left(\frac{Q}{r^2} + \frac{Q}{(D-r)^2}\right). \quad (17)$$

Midway between the spheres the potential gradient is

$$-\left(\frac{Q}{(D/2)^2} + \frac{Q}{(D/2)^2}\right) = -\frac{8Q}{D^2}, \quad (18)$$

which is its minimum value and at the surface of either sphere it is

$$\frac{Q}{R^2} + \frac{Q}{(D-R)^2}, \quad (19)$$

its maximum value.

The potential gradient is plotted in curve 2, Fig. 11.

It has been assumed that the charges on the spheres are uniformly distributed, this is not correct as may be seen by an inspection of the field between two point charges in Fig. 2. The flux and the charges are concentrated on the sides of the spheres which are facing one another and the centers of the charges are drawn together through a very short distance. Where the distance between the spheres is large compared to the radius, the error is small.

22. Capacity.—Conductors of different sizes and shapes have different capacities for storing electricity. If two conductors of different capacities are charged with equal quantities of electricity, they will be raised to different potentials, the one of small capacity will be raised to a high potential and the one of large capacity to a low potential.

The potential to which a body is raised varies directly as the quantity of electricity stored in it and inversely as its capacity and may be expressed by the formula

$$E = \frac{Q}{C}, \quad (20)$$

where Q is the quantity of electricity, or charge,
C is the capacity of the body,
E is the potential.

The capacity therefore is

$$C = \frac{Q}{E}, \quad (21)$$

and is equal to the charge divided by the potential, or it is equal to the charge per unit potential.

A body has unit capacity (electrostatic) when one unit of electricity is required to raise its potential by unity.

A sphere with a radius of 1 cm. has unit capacity, because if a charge of one unit be given to it it will act as though it were concentrated at the center and will produce at the surface unit potential. The capacity of a sphere of radius R cm. is R electrostatic units.

23. Farad.—The practical unit of capacity is the farad. A conductor has a capacity of one farad when one coulomb of electricity is required to raise its potential by one volt.

$$1 \text{ farad} = \frac{1 \text{ coulomb}}{1 \text{ volt}} = \frac{3 \times 10^9}{1/300} = 9 \times 10^{11} \text{ electrostatic units.} \quad (22)$$

The farad is a very large unit and capacities are usually expressed in microfarads or millionths of a farad.

24. Dielectric Permeance.—The foregoing explanation of capacity is apt to be misleading. A conductor has capacity only with respect to surrounding objects, since the electrostatic energy is not stored on or in the conductor itself but in the field between the conductor and surrounding conductors.

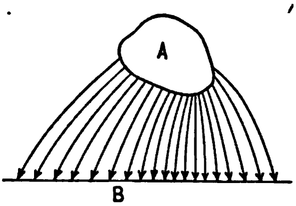


FIG. 12.—Permeance of the path between a conductor and a plane.

In Fig. 12, A is a conductor placed near a large conducting plane B. Assume a potential difference E to exist between A and B, then E measures the work in ergs that must be done in carrying a unit charge from B to A against the electrostatic forces in the field. These forces produce a dielectric flux passing from A to B.

The total dielectric flux ψ is proportional to the difference of potential E and to the permeance of the path $\mathcal{P}$.

The permeance $\mathcal{P}$ is directly proportional to the cross-sectional area of the path and to its dielectric constant and is inversely proportional to the length of the path. It is usually difficult to calculate the permeance directly, since the section of the path varies throughout its length. In such cases the dielectric flux ψ or the electric charge $Q = \frac{\psi}{4\pi}$ is assumed and the potential difference E calculated as in the preceding examples.

The permeance is then

$$\mathcal{P} = \frac{\psi}{E} = \frac{4\pi Q}{E} = 4\pi C, \qquad (23)$$

and is equal to the capacity of the system consisting of the two conductors and the dielectric between them multiplied by 4π.

25. Condenser.—An electrical condenser is an arrangement of conductors which is capable of storing a large quantity of electricity, or which has a large capacity. It is one in which a large flux is produced when a given potential difference is applied to its terminals.

The capacity of a condenser is measured by the amount of charge necessary to raise its potential by unity, and is therefore equal to the positive charge on it at any time divided by the potential difference between its terminals; this relation is expressed by the equation

$$C = \frac{Q}{E}.$$

26. Parallel Plate Condenser.—Determine the capacity of a condenser formed of two parallel plates each having an area of

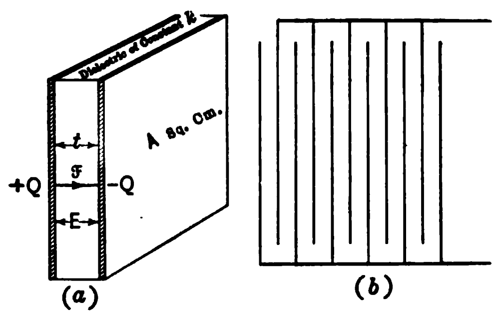


FIG. 13.—Parallel plate condensers.

A sq. cm. separated by t cm. of material of dielectric constant k Fig. 13(a).

If a difference of potential E is applied between the terminals a dielectric flux ψ is produced passing from the plate of high potential to the plate of low potential; a positive charge $Q = \frac{\psi}{4\pi}$ therefore appears on one plate and a negative charge $-Q = -\frac{\psi}{4\pi}$ on the other; the density of the charge on the plates is $q = \frac{Q}{A}$

units per square centimeter, and the dielectric flux density between the plates is $\mathfrak{D} = 4\pi q = \frac{4\pi Q}{A} = \frac{\psi}{A}$ lines per square centimeter. The thickness of the dielectric is assumed to be so small that the lines of flux travel directly across from one plate to the other.

The intensity of the electrostatic force is

$$\mathfrak{F} = \frac{\mathfrak{D}}{K} = \frac{4\pi Q}{Ak}. \tag{24}$$

The difference of potential between the plates is the work done in carrying a unit charge from one plate to the other against the force $\mathfrak{F}$; it is therefore

$$E = \mathfrak{F}t = \frac{4\pi Qt}{Ak},$$

since $\mathfrak{F}$ is constant; and the capacity of the condenser is

$$\begin{aligned} C &= \frac{Q}{E} \\ &= \frac{Q}{\frac{4\pi Qt}{Ak}} = \frac{Ak}{4\pi t} \text{ electrostatic units.} \end{aligned} \tag{25}$$

The capacity therefore varies directly with the area of the plates and with the dielectric constant of the material separating them, and inversely as the distance between them.

When the plates are separated by air the capacity is

$$C = \frac{A}{4\pi t}. \tag{26}$$

In order to increase the capacity of such a condenser a large number of plates are used joined in multiple and separated by very thin sheets of dielectric as shown in Fig. 13*b*.

27. Capacity of Concentric Cylinders.—Determine the capacity of a condenser formed of two concentric cylinders, Fig. 14, of radii R_1 and R_2 cm.

If a charge of q units per centimeter length is given to the inner cylinder, lines of dielectric flux pass out radially and produce, at a distance r cm. from the axis of the cylinder, a flux density

$$\mathfrak{D} = \frac{4\pi q}{2\pi r} = \frac{2q}{r} \text{ lines per square centimeter.}$$

If the dielectric constant of the material is k, the electrostatic force at the point is

$$\mathfrak{F} = \frac{D}{k} = \frac{2q}{kr} \text{ dynes,}$$

and the difference of potential between the two cylinders is

$$E = \int_{R_1}^{R_2} \frac{2q}{k} \frac{dr}{r} = \frac{2q}{r} \log \frac{R_2}{R_1} \text{ ergs.} \tag{27}$$

The capacity per centimeter length of the condenser is

$$C = \frac{q}{E} = \frac{q}{\frac{2q}{k} \log \frac{R_2}{R_1}} = \frac{k}{2 \log \frac{R_2}{R_1}} \text{ electrostatic units.} \tag{28}$$

This is the case of a single-conductor cable with a lead sheath, and the capacity is usually expressed in farads per mile; it is

$$C = \frac{k}{2 \log \frac{R_2}{R_1}} \times \frac{2.54 \times 12 \times 5{,}280}{9 \times 10^{11}}$$

$$= \frac{k}{2 \times 2.303 \log_{10} \frac{R_2}{R_1}} \times \frac{2.54 \times 12 \times 5{,}280}{9 \times 10^{11}}$$

$$= 3.82 \times \frac{k}{\log_{10} \frac{R_2}{R_1}} \times 10^{-6} \text{ farads per mile.} \tag{29}$$

28. Potential and Potential Gradient between Concentric Cylinders.—Assume that the potential of the inner cylinder in Fig. 14 is E and that of the outer one is zero.

The charge per centimeter length is

$$q = CE = \frac{kE}{2 \log \frac{R_2}{R_1}} \text{ by equation (28).}$$

The potential at radius r is

$$e = \int_r^{R_2} \mathfrak{F}\, dr = \int_r^{R_2} \frac{2q}{kr}\, dr = \int_r^{R_2} \frac{E}{\log \frac{R_2}{R_1}} \frac{dr}{r}$$

$$= \frac{E}{\log \frac{R_2}{R_1}} \left[\log r\right]_r^{R_2} = E \frac{\log \frac{R_2}{r}}{\log \frac{R_2}{R_1}}. \tag{30}$$

The potential gradient at radius r is

$$g = \frac{de}{dr} = \frac{2q}{kr} = \frac{E}{r \log \frac{R_2}{R_1}}; \tag{31}$$

it has its maximum value at the surface of the inner cylinder.

$$g_{\max.} = \frac{E}{R_1 \log \frac{R_2}{R_1}}. \tag{32}$$

Curve 1, Fig. 14, shows the variation of the gradient from one conductor to the other.

The area under the curve represents the difference of potential, E, between the conductors.

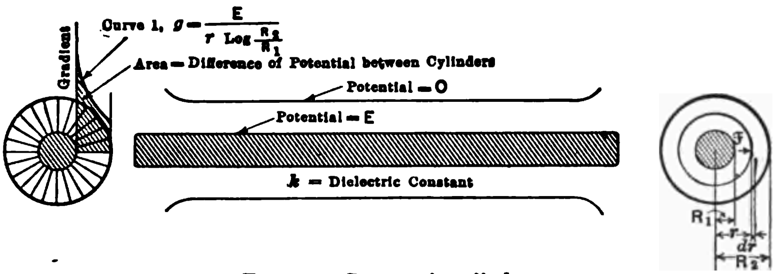


FIG. 14.—Concentric cylinders.

The ends of the outer cylinder are shown turned out at a radius not less than the distance between the conductors. This prevents concentration of flux at the ends which might cause a breakdown if the difference of potential were very great.

29. Capacity of Parallel Conductors.—Determine the capacity of two parallel cylindrical conductors A and B, Fig. 15(b) of radius R cm. suspended in air at a distance of D cm. between centers.

When a difference of potential E is applied between them a dielectric flux ψ passes across from A to B and a positive charge $Q = \frac{\psi}{4\pi}$ appears on A and an equal negative charge $-Q = -\frac{\psi}{4\pi}$ on B.

If q is the charge per centimeter length on A, the electrostatic force at any point Q, on the line joining the centers, distant r cm. from A and $D - r$ cm. from B is the resultant of the forces exerted by the charges on A and B acting independently.

From each centimeter length of A, $4\pi q$ lines of dielectric flux extend out normally and produce at the point Q a flux density

$$\mathfrak{D}_A = \frac{4\pi q}{2\pi r} = \frac{2q}{r} \text{ lines per square centimeter.}$$

The electrostatic force at the point Q is

$$\mathfrak{F}_A = \mathfrak{D}_A = \frac{2q}{r} \text{ dynes}$$

and acts from A to B.

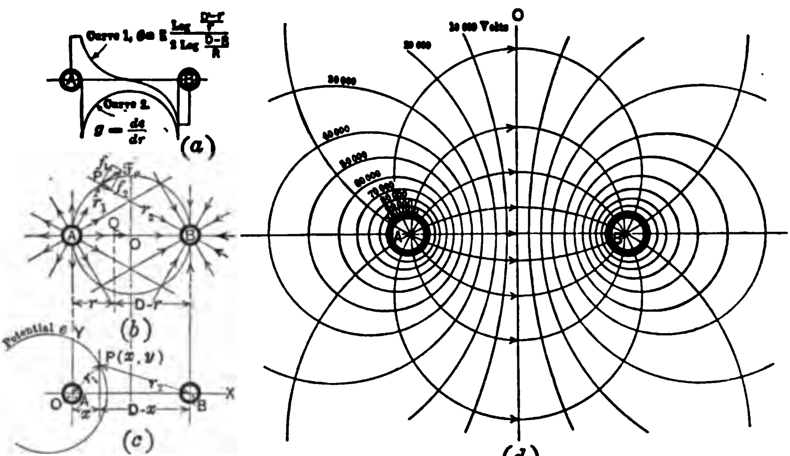


FIG. 15.—Electrostatic field between two parallel cylindrical conductors.

The charge $-q$ on B produces at Q a force

$$\mathfrak{F}_B = \frac{2q}{D - r} \text{ dynes}$$

in the same direction as $\mathfrak{F}_A$.

The resultant force at Q is

$$\mathfrak{F} = \mathfrak{F}_A + \mathfrak{F}_B = \frac{2q}{r} + \frac{2q}{D - r} \text{ dynes.} \tag{33}$$

The work done in moving a unit positive charge from B to A is equal to the difference of potential between the two conductors; it is

$$E = \int_R^{D-R} \left(\frac{2q}{r} + \frac{2q}{D - r}\right) dr$$

$$= 2q\Big[\log r - \log (D - r)\Big]_R^{D-R}$$

$$= 2q\left[\log \frac{D - R}{R} - \log \frac{R}{D - R}\right] = 4q \log \frac{D - R}{R}. \tag{34}$$

The capacity of the two conductors per centimeter length is

$$C = \frac{q}{E} = \frac{q}{4q \log \dfrac{D-R}{R}} = \frac{1}{4 \log \dfrac{D-R}{R}} \text{ electrostatic units.} \tag{35}$$

The capacity per mile of a line consisting of two conductors is

$$\begin{aligned} C &= \frac{2.54 \times 12 \times 5{,}280}{4 \times 2.303 \log_{10} \dfrac{D-R}{R} \times 9 \times 10^{11}} \\ &= \frac{19.4 \times 10^{-9}}{\log_{10} \dfrac{D-R}{R}} \text{ farads.} \end{aligned} \tag{36}$$

It is sometimes useful to separate the capacity of the line into the capacities of the two conductors forming it. The potential of the point O midway between the conductors is zero and the potential of conductor A is

$$E_A = \frac{E}{2} = 2q \log \frac{D-R}{R}$$

and, therefore, the capacity of A per centimeter length between the conductor and the neutral or point of zero potential is

$$C_A = \frac{q}{E_A} = \frac{q}{2q \log \dfrac{D-R}{R}} = \frac{1}{2 \log \dfrac{D-R}{R}} \text{ electrostatic units} \tag{37}$$

and the capacity of each conductor in farads per mile is

$$C = \frac{38.8 \times 10^{-9}}{\log_{10} \dfrac{D-R}{R}} \tag{38}$$

30. Potential and Potential Gradient between Parallel Conductors.—The potential at the point Q, Fig. 15(b), is the work done in moving a unit positive charge from the neutral point O to the point Q; it is

$$\begin{aligned} e &= \int_r^{D/2} \left(\frac{2q}{r} + \frac{2q}{D-r} \right) dr \\ &= 2q \Big[\log r - \log (D-r) \Big]_r^{D/2} \\ &= 2q \left[\log \frac{D}{2r} - \log \frac{D}{2(D-r)} \right] \\ &= 2q \log \frac{D-r}{r} \end{aligned}$$

but
$$q = CE = \frac{E}{4 \log \frac{D-R}{R}} \text{ from equation (35),}$$

and, therefore,

$$e = 2 \times \frac{E}{4 \log \frac{D-R}{R}} \times \log \frac{D-r}{r} = E \frac{\log \frac{D-r}{r}}{2 \log \frac{D-R}{R}} \qquad (39)$$

This equation is plotted in curve 1, Fig. 15(a).

The potential of conductor A is $\frac{E}{2}$ and that of B is $-\frac{E}{2}$.

The potential gradient, at any point Q, which is equal to the intensity of the electrostatic field at the point, is given by the equation

$$g = \frac{de}{dr} = \frac{2q}{r} + \frac{2q}{D-r} = 2q\left\{\frac{D}{r(D-r)}\right\}$$

$$= \frac{E}{2 \log \frac{D-R}{R}}\left\{\frac{D}{r(D-r)}\right\} \qquad (40)$$

or expressed in terms of the potential of the conductor A it is

$$g = \frac{E_A}{\log \frac{D-R}{R}}\left\{\frac{D}{r(D-r)}\right\}. \qquad (41)$$

For small values of r and R this may be written

$$g = \frac{E_A}{r \log \frac{D}{R}}. \qquad (42)$$

If E and E_A are expressed in volts the potential gradient g is expressed in volts per centimeter.

Equation 40 is plotted in curve 2, Fig. 15(a).

If E is taken as 200,000 volts, D as 200 cm. and R as 1 cm., the maximum gradient, which occurs at the surface of the conductor is

$$g_{\text{max.}} = \frac{E_A}{\log \frac{D-R}{R}}\left\{\frac{D}{R(D-R)}\right\} \qquad (43)$$

$$= \frac{100{,}000}{\log 199}\left\{\frac{200}{199}\right\} = 19{,}000 \text{ volts per centimeter and the}$$

minimum gradient, which occurs at the point midway between the conductors, is

$$g_{\text{min.}} = \frac{E_A}{\log \frac{D-R}{R}} \left\{ \frac{D}{\frac{D}{2} \times \frac{D}{2}} \right\} = \frac{4E_A}{D \log \frac{D-R}{R}} \tag{44}$$

$$= \frac{100{,}000}{\log 199} \times \frac{1}{50} = 3{,}800 \text{ volts per centimeter.}$$

If the two conductors had been suspended in a medium of dielectric constant k and the same difference of potential applied between them the capacity and the charge per centimeter would have been increased in the ratio k but the potential and potential gradient would remain as before.

31. Dielectric Field and Equipotential Surfaces between Parallel Conductors.—The field between two parallel conductors is shown in Fig. 15(d). It is obtained by superposing the fields due to the conductors A and B separately. At the point P in Fig. 15(b) the two forces acting are $f_1 = \frac{2q}{r_1}$ due to the charge on A and $f_2 = \frac{2q}{r_2}$ due to the charge on B. These two forces may be combined vectorially to give the resultant force $\mathfrak{F}_P$, which is tangent to the line of force passing through P. The lines of force are excentric circles cutting the conductors normally and passing through points slightly displaced from their centers.

A number of equipotential surfaces are also shown in Fig. 15(d); they are excentric cylinders surrounding the conductors with their centers on the line joining the conductor centers.

The equations of these surfaces may be found as follows:

In Fig. 15(c) the point P has coördinates r_1 and r_2 referred to the centers of the two conductors, and coördinates x and y referred to the rectangular axes. It is required to determine the equation of the equipotential surface passing through P.

The potential at P is

$$e = 2q \int_{r_1}^{D/2} \frac{dr}{r} - 2q \int_{r_2}^{D/2} \frac{dr}{r}$$

$$= 2q \left[\log \frac{D/2}{r_1} - \log \frac{D/2}{r_2} \right] = 2q \log \frac{r_2}{r_1}$$

$$= 2CE \log \frac{r_2}{r_1} \tag{45}$$

and
$$\log \frac{r_2}{r_1} = \frac{e}{2CE}$$
or
$$\frac{r_2}{r_1} = \epsilon^{\frac{e}{2CE}} = k = \text{a constant for the surface.}$$

Expressing r_1 and r_2 in terms of x and y,

$$r_2 = kr_1 \text{ or } r_2^2 = k^2 r_1^2$$
$$(D - x)^2 + y^2 = k^2 (x^2 + y^2)$$

or, simplifying,

$$(k^2 - 1)\, x^2 + 2Dx + (k^2 - 1)\, y^2 = D^2$$
$$x^2 + \frac{2D}{k^2 - 1}\, x + y^2 = \frac{D^2}{k^2 - 1}$$
$$x^2 + \frac{2D}{k^2 - 1}\, x + \frac{D^2}{(k^2 - 1)^2} + y^2 = \frac{D^2}{k^2 - 1} + \frac{D^2}{(k^2-1)^2} = \frac{k^2 D^2}{(k^2-1)^2}$$
$$\left(x + \frac{D}{k^2 - 1}\right)^2 + y^2 = \left(\frac{kD}{k^2 - 1}\right)^2 \qquad (46)$$

This is the equation of a circle with its center at the point

$$\left(- \frac{D}{k^2 - 1}, 0\right) \text{ and of radius } \left(\frac{kD}{k^2 - 1}\right). \qquad (47)$$

Thus the equipotential surfaces are cylinders (the traces in the xy plane are circles) and may be determined by assuming any potential e and substituting the value $k = \epsilon^{\frac{e}{2CE}}$ in the equations above.

32. Capacity of a Single Wire to Earth.—Determine the capacity of a single wire of radius R cm., suspended at a height H cm. above the earth. If the wire is raised to a potential E above the earth potential, lines of dielectric flux will pass from the wire to earth as shown in Fig. 16. If ψ is the flux from each centimeter length of A, then $q = \frac{\psi}{4\pi}$ is the charge on each centimeter of A, and a corresponding negative charge $-q$ appears on the earth, but it is not evenly distributed being most concentrated directly beneath the wire.

The flux passing from the wire A to the earth would be unchanged if the charge on the earth were collected on a second wire B placed at a distance H cm. below the earth. Its potential would be $-E$.

The difference of potential between A and B is $2E$ and from equation (34)

$$2E = 4q \log \frac{2H - R}{R},$$

thus the capacity of the single wire with respect to the earth per centimeter length is

$$C = \frac{q}{E} = \frac{q}{2q \log \frac{2H - R}{R}} = \frac{1}{2 \log \frac{2H - R}{R}} \text{ e.s. units,} \qquad (48)$$

and the capacity in farads per mile is

$$C = \frac{38.8 \times 10^{-9}}{\log_{10} \frac{2H - R}{R}}. \qquad (49)$$

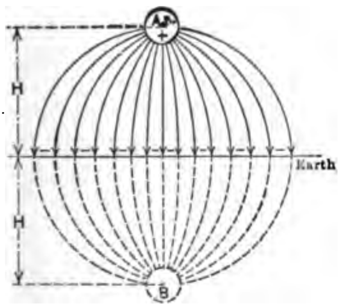


FIG. 16.—Capacity of a wire to earth.

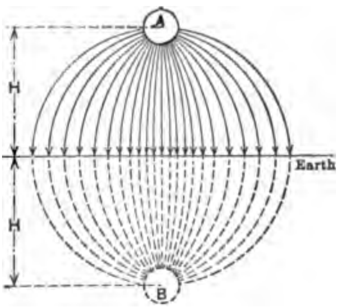


FIG. 17.—Capacity of a sphere to earth.

33. Capacity of a Sphere to Earth.—In Fig. 17, A is a sphere of radius R cm. at a height H cm. above the earth. If A is raised to a potential E above earth potential, a dielectric flux ψ is produced passing from the sphere to the earth as shown and a positive charge $Q = \frac{\psi}{4\pi}$ appears on A and an equal negative charge on the earth.

Without changing the distribution of flux in any way the negative charge on the earth may be assumed to be collected on a sphere B similar to A but placed at a distance H cm. below the surface. The distribution of potential between two such spheres was worked out in Art. 21. The potential midway between them is zero, the potential at the surface of A is

$$E = \frac{Q}{R} - \frac{Q}{2H - R} \text{ by equation (16),}$$

and therefore the capacity of the sphere is

$$C = \frac{Q}{E} = \frac{Q}{\frac{Q}{R} - \frac{Q}{2H - R}} = \frac{1}{\frac{1}{R} - \frac{1}{2H - R}} \text{ electrostatic units.} \quad (50)$$

When H is very large compared to R the capacity is

$$C = \frac{1}{1/R} = R \text{ as in Art. 22.}$$

34. Condensers in Multiple.—If a number of condensers of capacities C_1, C_2 and C_3 are connected in multiple, as shown in Fig. 18, and a difference of potential E is applied to the terminals, each condenser receives a charge proportional to its capacity,

$$Q_1 = C_1E,$$
$$Q_2 = C_2E,$$
$$Q_3 = C_3E,$$

and the total charge on the system is

$$Q = Q_1 + Q_2 + Q_3 = E\,(C_1 + C_2 + C_3).$$

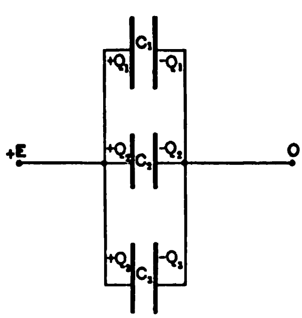


Fig. 18.—Capacities in multiple.

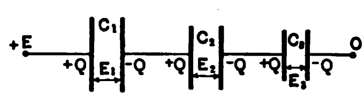


Fig. 19.—Capacities in series.

The capacity of the system is

$$C = \frac{Q}{E} = C_1 + C_2 + C_3, \quad (51)$$

and therefore the capacity of a number of condensers connected in multiple is equal to the sum of their separate capacities.

35. Condensers in Series.—When a number of condensers of capacities C_1, C_2 and C_3 are connected in series, as in Fig. 19, and a difference of potential E is applied to the terminals of the system, a charge Q appears on each condenser and the potential E

divides up among the condensers in inverse proportion to their capacities.

The drop of potential across condenser (1) is

$$E_1 = \frac{Q}{C_1},$$

that across (2) is

$$E_2 = \frac{Q}{C_2},$$

and that across (3) is

$$E_3 = \frac{Q}{C_3};$$

but

$$\begin{aligned} E &= E_1 + E_2 + E_3 \\ &= \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \\ &= Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right), \end{aligned}$$

and, therefore, the capacity of the system consisting of three condensers in series is

$$C = \frac{Q}{E} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}. \tag{52}$$

When two equal condensers of capacity C are connected in series, their combined capacity is

$$C_1 = \frac{1}{\frac{1}{C} + \frac{1}{C}} = \frac{C}{2}, \tag{53}$$

and is equal to one-half of the capacity of either condenser alone.

38. Energy Stored in a Condenser.—When a condenser is being charged, work is done in raising the charge through the difference of potential between the terminals, and this amount of energy is stored in the electrostatic field of the condenser.

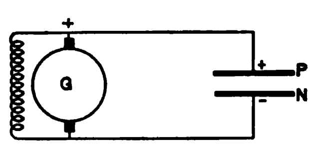


FIG. 20.

In Fig. 20, PN is a condenser of capacity C formed of two parallel plates separated by t cm. of a dielectric of constant k. When a potential difference e is produced between the plates by the generator G, electricity flows from N to P until the charge on P is

$$q = Ce;$$

if the potential e is increased by de the charge q is increased by $dq = C\,de$ and the work done in raising the charge dq through the difference of potential e is

$$dw = e\,dq = Ce\,de.$$

The total work done in charging the condenser with a quantity of electricity Q, or to a difference of potential E, is

$$\begin{aligned} W &= \int dw = \int_0^E Ce\,de \\ &= C\left[\frac{e^2}{2}\right]_0^E \\ &= C\frac{E^2}{2} \text{ ergs.} \end{aligned} \tag{54}$$

Thus the work done in charging a condenser, or the energy stored in the electrostatic field of the condenser, is equal to one-half of the capacity multiplied by the square of the difference of potential between the terminals.

Equation (54) may be expressed in two other forms by substituting for E its value $\frac{Q}{C}$;

$$W = \frac{1}{2}QE, \tag{55}$$

$$W = \frac{1}{2}\frac{Q^2}{C}. \tag{56}$$

If the area of the plate P is A sq. cm., then the dielectric flux density between the plates is

$$\mathfrak{D} = \frac{4\pi Q}{A} \text{ lines per square centimeter,}$$

and the electrostatic force is

$$\mathfrak{F} = \frac{\mathfrak{D}}{k} \text{ dynes.}$$

The potential difference between the plates is

$$E = \mathfrak{F}t = \frac{\mathfrak{D}t}{k},$$

and the capacity of the condenser by equation (25) is

$$C = \frac{Ak}{4\pi t}.$$

Substituting these values for E and C in equation (54) gives a fourth expression for the energy stored in the field, namely

$$W = \frac{1}{2} \times \frac{Ak}{4\pi t} \times \left(\frac{\mathfrak{D}t}{k}\right)^2$$

$$= At \times \frac{\mathfrak{D}^2}{8\pi k} \text{ ergs.} \tag{57}$$

Since the volume of the field is At c.c. and the flux density is uniform, the energy stored per cubic centimeter of the field is

$$w = \frac{\mathfrak{D}^2}{8\pi k} \text{ ergs,} \tag{58}$$

or

$$w = \frac{\mathfrak{F}^2 k}{8\pi} \text{ ergs.} \tag{59}$$

Thus, the energy stored per cubic centimeter in an electrostatic field is equal to the square of the dielectric flux density multiplied by $\frac{1}{8\pi k}$, or is equal to the square of the intensity of the electrostatic force multiplied by $\frac{k}{8\pi}$.

From equation (54) a very useful definition of capacity may be obtained,

$$C = \frac{2W}{E^2}, \tag{60}$$

or the capacity of a condenser is equal to twice the energy stored in its field divided by the square of the difference of potential across its terminals, or the capacity is equal to twice the energy stored when the difference of potential is unity.

37. Stresses in an Electrostatic Field.—The energy stored in an electrostatic field is

$$W = C\frac{E^2}{2} \text{ ergs;}$$

and the energy stored per cubic centimeter is

$$w = \frac{\mathfrak{D}^2}{8\pi k} \text{ ergs.}$$

These two equations represent the potential energy of the field. Stresses exist throughout the field tending to reduce the potential energy to a minimum; first, there is a tension along the lines of induction tending to shorten them and to draw the bounding

surfaces of the field together and so reduce the volume to zero; second, there is a pressure at right angles to the lines tending to spread them apart and so reduce the density in the field. Since the system is in equilibrium these two stresses are of equal magnitude.

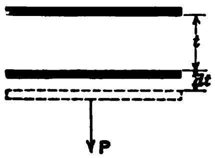


FIG. 21.

To obtain an expression for the stress per square centimeter on the bounding surfaces, consider the parallel plate condenser in Fig. 21. The energy stored in the field is by equation (57)

$$W = At \times \frac{\mathfrak{D}^2}{8\pi k} \text{ ergs.}$$

If a force of P dynes is applied to one of the plates and the distance between the plates is increased by amount dt, the work done is $P\,dt$ ergs. The charges on the plates are assumed to remain constant and therefore the flux density remains constant, but the volume of the field is increased by the amount $A\,dt$ c.c., and the energy stored in it is increased by $A\,dt \times \frac{\mathfrak{D}^2}{8\pi k}$, but the increase in the stored energy is equal to the work done by the force P and, therefore,

$$P\,dt = A\,dt \times \frac{\mathfrak{D}^2}{8\pi k}$$

and

$$P = A \times \frac{\mathfrak{D}^2}{8\pi k} \text{ dynes.} \tag{61}$$

This is the pull exerted by the field on each plate of the condenser tending to draw them together.

The pull per square centimeter is

$$p = \frac{P}{A} = \frac{\mathfrak{D}^2}{8\pi k} \text{ dynes,} \tag{62}$$

thus, the pull per square centimeter on any charged surface is equal to the square of the induction density at the point divided by $8\pi k$.

This is the value of the tension along the lines of force tending to shorten them, and also the value of the pressure at right angles to the lines tending to spread them apart.

38. Force Exerted on a Dielectric by an Electrostatic Field.—Fig. 22 shows two metallic plates of area A sq. cm. separated

by a dielectric of constant k and thickness t cm. Determine the pull required to remove the dielectric from the condenser against the opposing force due to the field.

(*a*) Assume, first, that the condenser has a charge Q and is not connected to any source of potential.

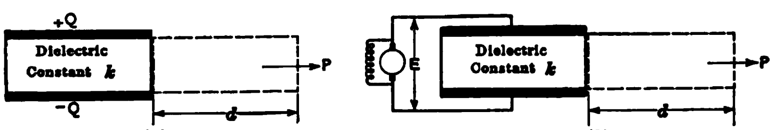
The energy stored in the condenser is

$$W_1 = \frac{1}{2}\frac{Q^2}{C_1} = \frac{1}{2}\frac{Q^2}{\frac{Ak}{4\pi t}} = \frac{2\pi t Q^2}{Ak};$$

when the dielectric is removed the stored energy is

$$W_2 = \frac{1}{2}\frac{Q^2}{C_2} = \frac{1}{2}\frac{Q^2}{\frac{A}{4\pi t}} = \frac{2\pi t Q^2}{A}$$

where C_1 is the capacity before and C_2 is the capacity after the dielectric is removed.



(*a*) Fig. 22. (*b*)

The change of stored energy is

$$W_2 - W_1 = \frac{2\pi t Q^2}{A}\left(1 - \frac{1}{k}\right),$$

and this is equal to the average pull P multiplied by the distance through which it is exerted $= Pd$.

Therefore, the average pull is

$$P = \frac{W_2 - W_1}{d} = \frac{2\pi t Q^2}{Ad}\left(1 - \frac{1}{k}\right) \text{ dynes, Fig. 22}(a) \qquad (63)$$

but $Q = C_1E_1 = \dfrac{AkE_1}{4\pi t}$, where E_1 is the initial difference of potential between the plates; and, therefore,

$$P = \frac{2\pi t}{Ad}\frac{A^2k^2E_1^2}{16\pi^2t^2}\left(1 - \frac{1}{k}\right) = \frac{Ak(k-1)E_1^2}{8\pi t d} \text{ dynes.} \qquad (64)$$

The final potential difference between the plates is

$$E_2 = \frac{Q}{C_2} = kE_1.$$

(*b*) Assume, second that the difference of potential E_1 is maintained constant by a generator, Fig. 22(*b*), and find the pull required to remove the dielectric.

Initial stored energy is

$$W_1 = C_1 \frac{E_1^2}{2} = \frac{Ak}{4\pi t}\frac{E_1^2}{2};$$

the final stored energy is

$$W_2 = C_2 \frac{E_1^2}{2} = \frac{A}{4\pi t}\frac{E_1^2}{2}.$$

The loss of stored energy is

$$W_1 - W_2 = \frac{AE_1^2}{8\pi t}(k-1).$$

This amount of energy is given back to the generator supplying the charge and as before the average pull required is

$$P^1 = \frac{W_1 - W_2}{d} = \frac{AE_1^2(k-1)}{8\pi td}. \tag{65}$$

The ratio of the pulls required in the two cases (*a*) and (*b*) is

$$\frac{P}{P^1} = \frac{\dfrac{Ak(k-1)E_1^2}{8\pi td}}{\dfrac{A(k-1)E_1^2}{8\pi td}} = k.$$

39. Effects of Introducing Dielectrics of Various Specific Inductive Capacities into a Uniform Field.—1. Fig. 23 shows a parallel plate condenser with air as dielectric.

t = distance between plates in centimeters.
A = area of each plate in square centimeters.
E = difference of potential between plates.
ψ = dielectric flux.
$\mathfrak{D} = \dfrac{\psi}{A}$ = dielectric flux density.
$\mathfrak{F} = \mathfrak{D}$ = electrostatic force in the field.

Since the force is constant throughout the field, therefore,

$$E = \mathfrak{F}t$$

and the electrostatic force or the stress in the air is

$$\mathfrak{F} = \frac{E}{t}.$$

If E is expressed in volts, the potential gradient or the stress may be expressed as

$$g = \frac{E}{t} \text{ volts per centimeter.}$$

Assuming that $E = 25{,}000$ volts and $t = 1$ cm. the gradient or stress is $g = \frac{E}{t} = \frac{25{,}000}{1} = 25{,}000$ volts per centimeter. Air has a dielectric strength of 31,000 volts per centimeter and it will therefore not break down in this case.

2. In Fig. 24 a sheet of glass of thickness $0.3t$ and dielectric constant $k = 6$ is introduced into the field as shown and the difference of potential is the same as before.

FIG. 23.

FIG. 24.

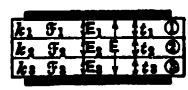


FIG. 25.

The dielectric flux density $\mathfrak{D}$ is constant throughout the field; the electrostatic force in the air is

$$\mathfrak{F}_A = \mathfrak{D};$$

the drop of potential across the air portion of the field is

$$E_1 = 0.7t \times \mathfrak{F}_A = 0.7t \times \mathfrak{D};$$

the electrostatic force in the glass is

$$\mathfrak{F}_G = \frac{\mathfrak{D}}{k} = \frac{\mathfrak{D}}{6};$$

The drop of potential across the glass is

$$E_2 = 0.3t \times \mathfrak{F}_G = 0.3t \times \frac{\mathfrak{D}}{6} = 0.05t \times \mathfrak{D};$$

the difference of potential between the plates is

$$\begin{aligned} E &= E_1 + E_2 \\ &= 0.7t \times \mathfrak{D} + 0.05t \times \mathfrak{D} \\ &= 0.75t\mathfrak{D}, \end{aligned}$$

and the dielectric flux density is

$$\mathfrak{D} = \frac{E}{0.75t} = \frac{4}{3}\frac{E}{t};$$

thus the stress in the air is

$$\mathfrak{F}_A = \mathfrak{D} = \frac{4}{3}\frac{E}{t},$$

and is greater than it was in the first case.

Again, assuming $E = 25{,}000$ volts and $t = 1$ cm., the gradient in the air is $g = \frac{4}{3}\frac{E}{t} = \frac{4}{3} \times 25{,}000 = 33{,}000$ volts and the air will break down.

In this case by introducing a dielectric of high specific inductive capacity and much greater dielectric strength than the air, the air is made to break down. It now becomes a conductor and the full stress of 25,000 volts comes on the glass. The potential gradient in the glass becomes $g = \frac{25,000}{0.3} = 83,000$ volts per centimeter.

3. Fig. 25 shows the same pair of plates with three sheets of dielectric introduced between them, of thickness t_1, t_2 and t_3 and dielectric constants k_1, k_2 and k_3 respectively.

$\mathfrak{D}$ = dielectric flux, which is constant throughout the field.

$$\mathfrak{F}_1 = \frac{\mathfrak{D}}{k_1} = \text{stress in layer (1)},$$

$$\mathfrak{F}_2 = \frac{\mathfrak{D}}{k_2} = \text{stress in layer (2)},$$

$$\mathfrak{F}_3 = \frac{\mathfrak{D}}{k_3} = \text{stress in layer (3)},$$

$$E_1 = \mathfrak{F}_1 t_1 = \frac{\mathfrak{D}}{k_1} t_1 = \text{drop of potential across (1)},$$

$$E_2 = \mathfrak{F}_2 t_2 = \frac{\mathfrak{D}}{k_2} t_2 = \text{drop of potential across (2)},$$

$$E_3 = \mathfrak{F}_3 t_3 = \frac{\mathfrak{D}}{k_3} t_3 = \text{drop of potential across (3)};$$

the difference of potential between the plates is

$$E = E_1 + E_2 + E_3 = \mathfrak{D}\left(\frac{t_1}{k_1} + \frac{t_2}{k_2} + \frac{t_3}{k_3}\right).$$

and the dielectric density is

$$\mathfrak{D} = \frac{E}{\frac{t_1}{k_1} + \frac{t_2}{k_2} + \frac{t_3}{k_3}},$$

and the total flux is

$$\psi = \mathfrak{D}A = \frac{E}{\frac{t_1}{k_1} + \frac{t_2}{k_2} + \frac{t_3}{k_3}} A = E\mathcal{P}$$

where

$$\mathcal{P} = \frac{\psi}{E} = \frac{A}{\frac{t_1}{k_1} + \frac{t_2}{k_2} + \frac{t_3}{k_3}} \text{ equation (23)},$$

is the permeance of the path between the plates.

The drop of potential across(1) is

$$E_1 = \frac{\mathfrak{D}t_1}{k_1} = \frac{E}{\frac{k_1}{t_1}\left(\frac{t_1}{k_1} + \frac{t_2}{k_2} + \frac{t_3}{k_3}\right)}; \tag{66}$$

and the potential gradient is

$$g_1 = \frac{E_1}{t_1} = \frac{E}{k_1\left(\frac{t_1}{k_1} + \frac{t_2}{k_2} + \frac{t_3}{k_3}\right)} \tag{67}$$

The potential gradients in (2) and (3) may be expressed by similar equations.

From the last two examples it is seen that a uniform field can be made non-uniform by introducing dielectrics of various specific inductive capacities, and that the stresses in the various dielectrics vary inversely as their specific inductive capacities.

4. In Fig. 26 a block of dielectric constant k is introduced into the uniform field as shown.

The stress in the air is

$$\mathfrak{F}_A = \frac{E}{t},$$

and is the same as before the dielectric was introduced.

The stress in the dielectric is also

$$\mathfrak{F}_D = \mathfrak{F}_A = \frac{E}{t},$$

but the flux density in the dielectric is

$$\mathfrak{D} = \mathfrak{F}_D k = \frac{E}{t} k = \mathfrak{F}_A k.$$

FIG. 26.

FIG. 27.

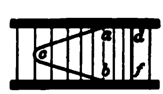


FIG. 28.

The permeance of the path is increased and an increased flux is produced passing between the plates, but since the increased flux is all confined to the dielectric there is no increase in the stress in the air.

5. In Fig. 27, the same block of dielectric is shown with a groove cut in it. The air in the groove is very highly stressed

3

and as the difference of potential between the plates is increased, this air will be the first to break down.

6. Fig. 28 is another example of the same thing. The stress on the air in the pockets *a* and *b* is high. As the difference of potential is increased this air will break down and become conducting and there will be a discharge over the surface *acb* at a potential difference much lower than that required to break over the shorter surface *df*, where the stress is uniform.

7. In Fig. 29, *A* is a small piece of material with a high dielectric constant such as a drop of oil. It offers a local path of high

FIG. 29.

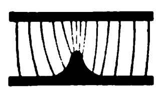


FIG. 30.

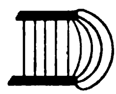


FIG. 31.

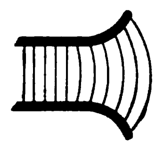


FIG. 32.

permeability, and a greater flux density is produced in it than in other parts of the field, and therefore the stress in the air at its surface will be greater than the average stress throughout the field.

8. In Fig. 30 the drop of oil is replaced by a knob on the conductor forming one boundary of the field. Since the dielectric constant of the conducting knob is infinity the stress in the air at its surface will be greater than in the case of the oil.

9. Fig. 31 shows the field at the edge of the condenser in case (1). In this region the dielectric flux density is not uniform, and just at the edges of the plates it is greater than in the main body of the field. The electrostatic stress is also greater than the average value.

This condition may be corrected by turning out the edges of the plates or electrodes at a radius not less than the distance between them as shown in Fig. 32.

40. Effect of Introducing Conductors into Electrostatic Fields. —An insulated conductor may be introduced into an electrostatic field without changing the flux distribution if it is placed entirely on an equipotential surface. Fig. 33(*b*) shows such a case. The conductor takes the potential of the surface on which it lies.

In Fig. 33(*c*) the conductor does not lie along one of the original equipotential surfaces and the distribution of flux is changed as

shown. The conductor is an equipotential surface and the lines of force enter it normally. Its potential is fixed by the relative capacities between it and the two plates forming the boundaries of the field. If it is placed symmetrically between the plates its potential is the mean of the potentials of the plates.

In Fig. 15 which represents the field between two parallel cylindrical conductors an insulated conducting cylinder may be

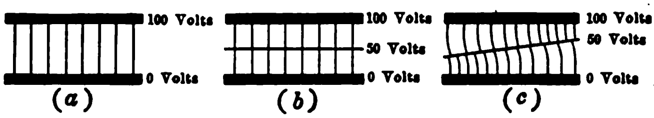


FIG. 33.

placed on any one of the equipotential surfaces without changing the field; and if this cylinder is then connected to a source of potential of the same value, the original conductor may be removed without changing the field external to the cylinder.

41. Graded Insulation for Cables.—Fig. 34(a) shows a single-conductor cable insulated with a dielectric of constant $k_1 = 6$;

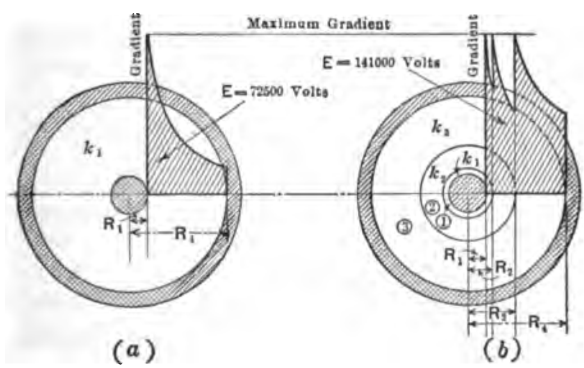


FIG. 34.—Graded insulation for cables.

the radius of the conductor is $R_1 = 0.4$ cm. and the inside radius of the sheath is $R_4 = 2.4$ cm. If the sheath is grounded and the conductor is raised to a potential E, the potential at any point in the dielectric at a distance r cm. from the center of the conductor is

$$e = E \frac{\log \frac{R_4}{r}}{\log \frac{R_4}{R_1}} \text{ by equation (30)}$$

and the potential gradient or stress at the point is

$$g = \frac{de}{dr} = \frac{E}{r \log \frac{R_4}{R_1}} \text{ volts per centimeter by equation (31).}$$

The gradient is plotted in the figure and the hatched area under the curve represents the difference of potential E.

The maximum gradient or stress occurs at the surface of the conductor where $r = R_1$; it is

$$g_{\text{max.}} = \frac{E}{R_1 \log \frac{R_4}{R_1}} \tag{68}$$

If the maximum allowable stress is assumed to be 100,000 volts per centimeter, then the maximum difference of potential at which the cable can be operated safely is

$$E = g_{\text{max.}} \times R_1 \log R_2 \frac{R_4}{R_1}$$

$$= 100{,}000 \times 0.4 \log \frac{2.4}{0.4} = 72{,}500 \text{ volts.} \tag{69}$$

In this case the stress in the outer layers of the dielectric is far below the maximum allowable stress and the material is not used to the best advantage.

The capacity of this cable per centimeter length, equation (28) is

$$C = \frac{k_1}{2 \log \frac{R_4}{R_1}} = \frac{6}{2 \log \frac{2.4}{0.4}} = 1.585 \text{ electrostatic units.}$$

Fig. 34(*b*) shows the same cable insulated with three layers of materials of dielectric constants $k_1 = 6$, $k_2 = 4$ and $k_3 = 2$; the outside radii of the three layers are $R_2 = 0.6$, $R_3 = 1.2$ and $R_4 = 2.4$ cm.

If q is the charge per centimeter length, the dielectric flux density at radius r is

$$\mathfrak{D} = \frac{2q}{r} \text{ lines per square centimeter}$$

and the electrostatic force or stress in the medium is

$$\mathfrak{F} = \frac{2q}{kr} \text{ dynes,}$$

where k has different values in the three dielectrics.

In order to make the stresses in the various parts of the insulation equal it would be necessary to place next to the conductor a material of high dielectric constant k_1 and to decrease this constant gradually in succeeding layers in inverse proportion to the distance from the center of the conductor. This would be a very expensive process and it is not necessary since good results can be obtained by using three or four layers of dielectric.

The stress in the outer layer is

$$\mathfrak{F}_3 = \frac{2q}{k_3 r}, \tag{70}$$

and the drop of potential across it is

$$E_3 = \int_{R_3}^{R_4} \frac{2q}{k_3 r}\, dr = \frac{2q}{k_3} \log \frac{R_4}{R_3}; \tag{71}$$

the stress in the second layer is

$$\mathfrak{F}_2 = \frac{2q}{k_2 r}, \tag{72}$$

and the drop of potential across it is

$$E_2 = \int_{R_2}^{R_3} \frac{2q}{k_2 r}\, dr = \frac{2q}{k_2} \log \frac{R_3}{R_2}; \tag{73}$$

the stress in the inner layer is

$$\mathfrak{F}_1 = \frac{2q}{k_1 r}, \tag{74}$$

and the drop of potential across it is

$$E_1 = \int_{R_1}^{R_2} \frac{2q}{k_1 r}\, dr = \frac{2q}{k_1} \log \frac{R_2}{R_1}. \tag{75}$$

The difference of potential between the conductor and the sheath is

$$E = E_1 + E_2 + E_3 = \frac{2q}{k_1} \log \frac{R_2}{R_1} + \frac{2q}{k_2} \log \frac{R_3}{R_2} + \frac{2q}{k_3} \log \frac{R_4}{R_3}. \tag{76}$$

The capacity of the cable per centimeter length is

$$C = \frac{q}{E} = \frac{1}{2\left(\frac{1}{k_1} \log \frac{R_2}{R_1} + \frac{1}{k_2} \log \frac{R_3}{R_2} + \frac{1}{k_3} \log \frac{R_4}{R_3}\right)} \tag{77}$$

$$= \frac{1}{2\left(\frac{1}{6} \log \frac{0.6}{0.4} + \frac{1}{4} \log \frac{1.2}{0.6} + \frac{1}{2} \log \frac{2.4}{1.2}\right)} = 0.85 \text{ electrostatic units.}$$

The maximum stress or gradient in (1) is

$$\mathfrak{G}_{\max.} = \frac{2q}{k_1R_1} = \frac{2CE}{k_1R_1} \text{ electrostatic units,}$$

or

$$g_{\max.} = \frac{2CE}{k_1R_1} \text{ volts per centimeter;} \tag{78}$$

the maximum gradient in (2) is

$$g_{\max} = \frac{2CE}{k_2R_2}; \tag{79}$$

and the maximum gradient in (3) is

$$g_{\max.} = \frac{2CE}{k_3R_3}. \tag{80}$$

If the three materials have the same dielectric strength, the thickness should be so chosen that the three maximum stresses are equal. The required condition is that $k_1R_1 = k_2R_2 = k_3R_3$. In the present case the three materials are assumed to have the same maximum allowable stress of 100,000 volts per centimeter.

If the allowable maximum stress in layer (2) had been only 50,000 volts per centimeter or one-half of that in (1), R_2 should have been chosen so that $k_2R_2 = 2k_1R_1$ and would have been 1.2 instead of 0.6.

The maximum difference of potential which can be applied to this cable can be found by substituting numerical values in any one of the equations for maximum gradient.

From equation (78)

$$g_{\max.} = \frac{2CE}{k_1R_1}$$

and thus

$$E = g_{\max.} \times \frac{k_1R_1}{2C} = 100{,}000 \times \frac{6 \times 0.4}{2 \times 0.85} = 141{,}000 \text{ volts.}$$

The gradient from conductor to sheath is shown by the discontinuous curve in Fig. 34(*b*), and the area under the curve represents the safe operating voltage of the cable.

In this case by grading the insulation on the cable without changing the outside diameter the safe operating voltage has been increased by 95 per cent. and at the same time the capacity has been decreased about 45 per cent.

42. Air Films in Generator Slot Insulation.—Fig. 35 shows a section of one slot of a three-phase, 11,000-volt generator with a chain winding. The insulation on the coils in addition to the insulation between turns consists of $t_1 = 0.2$ cm. of molded mica with a dielectric constant $k_1 = 4$, $t_2 = 0.4$ cm. of varnished cloth of dielectric constant $k_2 = 5$, and an outer wrapping of fiber of thickness $t_3 = 0.1$ cm. and dielectric constant $k_3 = 2$. A film of air of thickness $t_4 = 0.05$ cm. is assumed to be included in the slot. Determine the maximum stress in the air.

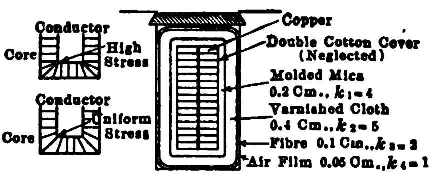


FIG. 35.—Slot insulation for an 11,000 volt generator.

The maximum voltage to neutral, *i.e.*, between any coil and the core, is found at the terminal coil of any one of the phases and its value is $\frac{11,000}{\sqrt{3}} \times \sqrt{2} = 8,960$ volts (see Art. 123). The side of the coil and the side of the slot may be considered as forming the plates of a condenser with four layers of insulation and the stress in the air may be found from equation (67),

$$g_{air} = \frac{E}{k_4\left(\frac{t_1}{k_1} + \frac{t_2}{k_2} + \frac{t_3}{k_3} + \frac{t_4}{k_4}\right)} = \frac{8,960}{1\left(\frac{0.2}{4} + \frac{0.4}{5} + \frac{0.1}{2} + \frac{0.05}{1}\right)}$$

$$= \frac{8,960}{0.23} = 39,000 \text{ volts per centimeter.}$$

This is well above the critical stress in air (31,000 volts per centimeter) and the air will therefore break down and ozone and nitric acid will form, which will attack the insulation. If the air is continually renewed by the expanding and contracting of the coil due to alternate heating and cooling, the corrosive action may go on until the insulation is broken down and the coil becomes grounded. The corners of the conductors should be rounded off to prevent local concentration of flux and increased stress as shown in Fig. 35.

43. Condenser Bushing.—The leads from transformers, in some cases must be insulated for very high voltages and it is difficult to make the terminals of reasonable size. This can be accomplished only by making each part of the insulating material take care of its proper proportion of the total stress. One method of obtaining this result is illustrated in Fig. 36, which is a cross-section of a condenser bushing.

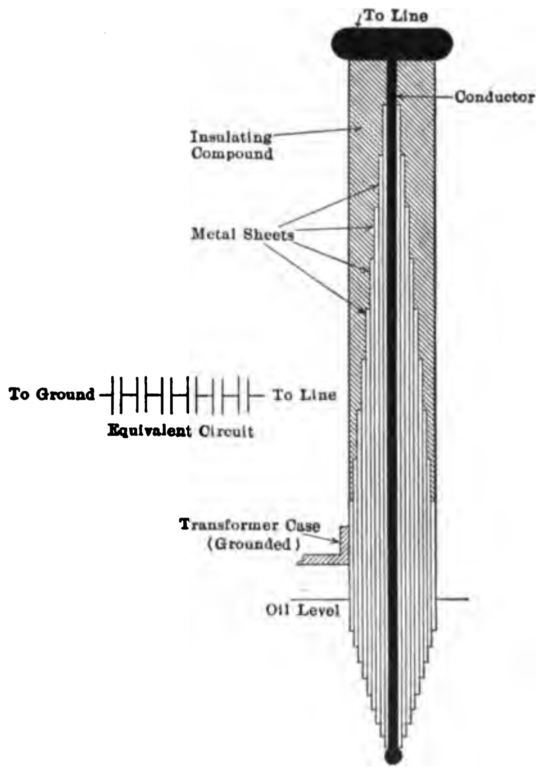


FIG. 36.—Condenser bushing.

The conductor is a hollow tube of sufficient radius to reduce the stress at its surface to a safe value and it is placed as far away from the other terminal as possible. The tube is wrapped with layers of insulating material separated by their sheets of metal. Adjacent metal sheets with the dielectric between them form condensers which should all have approximately the same capacity, but since, due to the increase in diameter, the outer plates

tend to have greater areas than the inner ones they must be made shorter. The lengths of succeeding plates are reduced by equal amounts in order to get as long a surface leakage path as possible and if necessary the stresses can be equalized by varying the thickness of the dielectric. The whole bushing is covered with a cylinder filled with an insulating compound.

44. Dielectric Strength.—The dielectric strength of an insulating material is measured by the difference of potential required to puncture it, but this is a quantity which is dependent to a large extent on the conditions under which the stress is applied. The test is usually made by applying an alternating voltage to the specimen, which is placed between two electrodes. Frequencies of from 40 to 60 cycles per second are generally used and the shape of the voltage wave is approximately sinusoidal. In this case the ratio of the maximum voltage, which is responsible for the puncture, to the effective voltage, which is indicated by the voltmeter, is $\sqrt{2}$ to 1.

The dielectric strength is variously expressed in volts per mil, volts per inch, volts per millimeter, or volts per centimeter.

The most important conditions affecting the test are: (1) thickness of the specimen; (2) shape of the electrodes; (3) medium in which the test is made; (4) temperature; (5) pressure; (6) time of application of the voltage; (7) frequency of the impressed voltage; (8) wave shape of the impressed voltage.

1. Fig. 37 shows the variation of apparent dielectric strength with thickness.

When an alternating voltage is impressed, the changes in the direction of the stress cause a loss in the dielectric resulting in a rise of temperature, which decreases the dielectric strength. A thin specimen can more easily radiate this heat than a thicker one and so its temperature does not rise to the same extent. Further, it is more difficult to make thick sheets of dielectric as homogeneous and free from flaws as thin ones. Both of these conditions tend to make the thin specimen show greater strength under test.

When a considerable thickness of dielectric is required it is advisable to build it up of a number of thin sheets held by an insulating varnish as this makes the product more flexible and it is not likely that a flaw will extend through more than one sheet.

2. If the electrodes are not properly shaped, excessive stresses

may occur at certain points and cause a breakdown at a lower voltage than normal. For example, transformer oil which will stand 225 to 250 volts per mil when tested between half-inch discs will only stand about 200 volts per mil when tested between half-inch spheres.

3. If a sheet of material is tested between spherical electrodes under oil it will show a lower dielectric strength than it would if tested in air, since the air will break down first and so give the effect of flat electrodes.

4. In general an increase of temperature results in a decrease in dielectric strength unless the increase of temperature reduces the amount of moisture in the dielectric. It is a complex phenomenon.

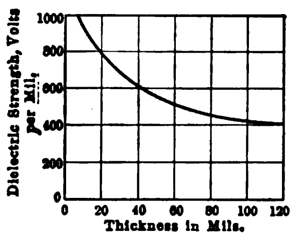


FIG. 37.—Typical curve of disruptive strength *vs.* thickness.

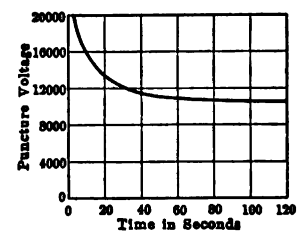


FIG. 38.—Typical curve of puncture voltage *vs.* time.

5. The dielectric strength of air and oil increase directly with the pressure.

6. Fig. 38 shows the variation of dielectric strength with time of application of the voltage.

A given material will withstand a much higher voltage for a short time than it will for an extended period. For this reason, in making high-voltage tests on machines the stress must be applied for a specified time.

7. Increase of frequency results in an increase of loss and a rise in temperature, which decreases the dielectric strength.

8. The breakdown is the result of the maximum value of the impressed voltage. If the wave is peaked the ratio of maximum to effective value will be greater than with a sine wave and the material will break down with an effective voltage lower than normal.

In the following table are given approximate values of the dielectric constants and dielectric strengths for materials in common use in electrical engineering.

TABLE OF DIELECTRIC CONSTANTS AND DIELECTRIC STRENGTHS

Material	Dielectric constant k	Dielectric strength, volts per mil	Remarks
Air	1.0	79	31,000 volts per cm.
Asphalt	2.7	30	60 mils thick
Cellulose acetate	....	300–1,800	
Cloth, oiled	....	750–1,000	10 mils thick
Cloth, varnished	3.5–5.5	500–1,300	5–16 mils thick
Cotton impregnated	....	86	7 mils thick
Cotton not impregnated	....	21	7 mils thick
Ebonite	1.9–3.5	1,700–3,750	Up to 20 mils
Fiber	2.0	150–250	
Fullerboard	7.5 at 100°C.	....	Boiled in transformer oil.
Fullerboard varnished	2.9 at 25°C.	300	
Glass	5.5–10.0	150–300	
Gutta percha	3.0–5.0		
Mica	2.5–6.0	1,000–2,500	
Molded mica or micanite	2.5–6.0	900	
Paraffin wax	1.9–2.3	300	
Porcelain	4.0–6.0	300 220–240	100 mils thick 500 mils thick
Rubber	2.0–3.0	300–500	
Rubber compounds	3.0–4.0	300–500	
Shellac	2.75		
Slate	6.0–7.0	5–10	
Transformer oil	2.5	225–250 200	Between 0.5-in. discs 0.2 in. apart. Between 0.5-in. spheres 0.15-in. apart.
Vacuum	0.9994		

45. Breakdown.—When a discharge takes place between two electrodes in a gaseous or liquid dielectric, the natural circulation of the material heals the break and the insulating qualities are not impaired. If the discharge in oil is very heavy some of the oil becomes carbonized and the carbon particles tend to line up in the field of greatest stress and reduce the dielectric strength.

Particles of moisture in oil due to their high dielectric constant seek the most intense parts of the field and cause an increase of stress and tend to cause breakdown.

One-tenth of 1 per cent. of water in transformer oil will reduce its dielectric strength to 20 per cent. of its original value.

With porcelain or glass when breakdown occurs the material is ruptured and its insulating value destroyed.

In the case of cotton or silk fabrics the overstress causes local breakdown accompanied by carbonization and a weak spot develops which gradually extends until complete rupture occurs.

46. Dielectric Losses.—If a constant voltage is impressed on the terminals of a condenser (Fig. 13) a dielectric field is set up and the dielectric material is in a state of stress. So long as the voltage is maintained constant there is not consumption of energy due to this cause. When, however, an alternating voltage is impressed the stresses in the dielectric alternate in direction. If the dielectric were perfectly homogeneous all the energy stored in the field during the increase of voltage would be returned as it decreased again and no energy would be consumed. This occurs in the case of gaseous and of some liquid dielectrics. In solid dielectrics some energy is consumed in reversing the stress in the material and appears as heat causing a rise in the temperature of the dielectric. The loss of energy is proportional to the square of the impressed voltage, that is, to the square of the dielectric stress and is called the dielectric hysteresis loss; it, however, differs from magnetic hysteresis in that it causes a slight lag of flux in time behind the voltage. This is noticeable only at high frequencies.

A further energy loss occurs in dielectrics due to the leakage of current between the electrodes. This current is directly proportional to the impressed voltage and is inversely proportional to the resistance of the dielectric. The resulting loss is proportional to the square of the current and therefore to the square of the voltage. It occurs with both direct and alternating voltages.

The insulation resistance of dielectrics or insulating materials is a very complex quantity and varies through a wide range with temperature and other conditions. Fig. 68 shows the variation of the resistance of slot insulation with temperature. The presence of minute quantities of moisture in the dielectric reduces the insulation resistance to very low values.

There is no direct relation between the insulation resistance of a material and its dielectric strength. For instance, dry air has a very high insulation resistance but very low dielectric strength.

Ordinarily the dielectric losses are small, but in the case of high-voltage cables where the stresses are very high they may be large enough to cause a dangerous rise in temperature which in

turn will decrease the insulation resistance and further increase the loss.

47. Surface Leakage.—When dielectric surfaces are clean and dry, very little surface leakage takes place but if dust or moisture is present the leakage of current may become serious.

The flashing-over of clean dielectric surfaces is due to the breakdown of one dielectric, usually air, at the surface of a stronger dielectric. In Fig. 28 leakage occurs over the long surface *acb* due to the breaking down of the air in pockets at *a* and *b*; this air becomes conducting. The shorter surface *df* does not exhibit the phenomenon to nearly so marked an extent.

In the ordinary pin-type insulator, discharges over the surface take place due to the breakdown of the air in the pockets between the petticoats combined with the presence of dust and moisture.

48. Corona.—When the electrostatic stress or potential gradient at any point in air exceeds about 30,000 volts per centimeter a brush discharge takes place and the air becomes conducting and luminous. The discharge does not necessarily extend from the positive to the negative electrode but exists only in the region where the dielectric strength of the air has been exceeded.

In the case of two parallel wires suspended in air the discharge or corona first appears at any rough points on the wires and finally forms a luminous envelope about them which increases in diameter as the voltage is raised.

If the increase in the effective diameter of the wire due to the presence of the corona results in a decrease of the stress a flash-over between the wires will not result; but if the increase in diameter so decreases the distance between the conductors that the stress at the surface of the conducting envelope is not reduced below the breakdown stress of the air, the diameter of the envelope increases until a flash takes place from one wire to the other.

Corona is accompanied by a loss of energy proportional to the square of the voltage rise above the critical voltage at which the discharge begins.

CHAPTER II

MAGNETISM AND ELECTROMAGNETICS

49. Magnetization.—When bodies are magnetized magnetic forces act at every point throughout their volume and lines of magnetic induction pass through them. There are two kinds of magnetic poles just as there are two kinds of electric charges; as a positive electric charge appears where a dielectric flux leaves a surface, so a positive magnetic pole appears where a magnetic flux leaves a surface. The positive magnetic pole is called a north pole. Similarly a negative magnetic pole or south pole appears where a magnetic flux enters a surface.

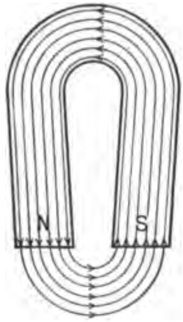


FIG. 39.—Magnet.

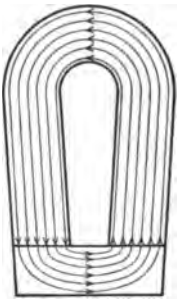


FIG. 40.—Magnet with armature on.

Thus a body which is magnetized has a north pole at one part of its surface and an equal south pole at another part unless the magnetic path forms a closed circuit as in Fig. 40.

Fig. 39 represents a horseshoe magnet. The lines of magnetic induction pass through it in the direction shown, leaving the surface at *N* and entering it again at *S*. Thus *N* is a positive magnetic pole or a north pole and *S* is a negative magnetic pole or a south pole.

Fig. 40 represents the same magnet with its armature on. The lines of magnetic induction pass in the same direction as before, but the circuit is closed and the poles do nor appear until a gap is made by removing the armature.

50. Laws of Magnetism.—*First Law.*—Like magnetic poles repel one another; unlike magnetic poles attract one another.

Second Law.—The force exerted between two magnetic poles is proportional to the product of their strengths and is inversely proportional to the square of the distance between them. This law can be expressed by the formula

$$f = \frac{mm_1}{r^2} \tag{81}$$

where m and m_1 are the pole strengths, r is the distance between them in centimeters, and f is the force exerted between them in dynes. If m and m_1 are like poles the force is a repulsion and f is positive.

The unit of pole strength is defined as follows: A magnetic pole has unit strength if, when placed at a distance of 1 cm. from a similar pole, it repels it with a force of one dyne.

The force exerted on a unit pole at a distance of r cm. from a pole of strength m is

$$f = \frac{m}{r^2} \text{ dynes.} \tag{82}$$

51. Magnetic Field.—The space surrounding a magnetic pole or a current of electricity in which magnetic forces act is called a magnetic field. The direction of the force at any point in the field is the direction in which a unit north pole placed at the point would tend to move and its intensity is the force in dynes exerted on the unit pole.

The magnetic field is represented by lines of magnetic induction or magnetic flux drawn in the direction of the force.

Unit magnetic force or unit magnetizing force produces one line of magnetic flux per square centimeter in air and μ lines per square centimeter in a magnetic material of permeability μ.

The magnetic force at a point is expressed in dynes and is represented by $\mathcal{H}$; the magnetic flux density at a point is expressed in lines per square centimeter and is represented by $\mathcal{B}$.

Fig. 41 shows the magnetic fields produced in certain cases. The lines of induction are all closed lines and extend from a north to a south pole in air and from a south to a north pole inside the magnetic material or the generator of m.m.f.

52. Magnetic Flux.—The total number of lines of magnetic induction passing through a given section is called the magnetic flux through the section and is represented by Φ.

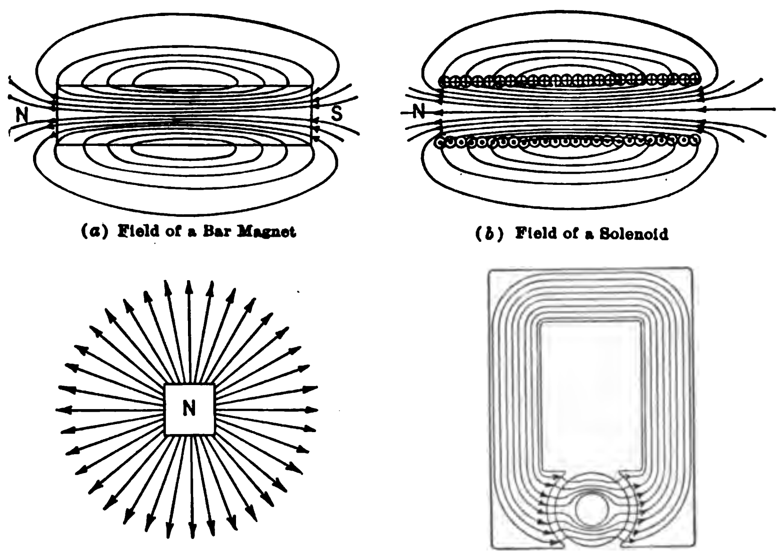
The unit of magnetic flux, which is one line, is called the maxwell.

53. Flux from Unit Pole.—At every point on a sphere of 1 cm. radius, surrounding a unit pole as center, a similar unit pole is repelled with a force of one dyne. There must therefore be one line of induction per square centimeter passing through the surface, and since the surface is 4π sq. cm. the total flux from the unit pole is

$$\Phi = 4\pi \text{ lines.}$$

The flux from a pole of strength m is

$$\Phi = 4\pi m \text{ lines.}$$



(a) Field of a Bar Magnet (b) Field of a Solenoid

(c) Field near an Isolated North Pole (d) Field of a Dynamo

FIG. 41.—Magnetic fields.

Thus a unit north pole is associated with each 4π lines leaving a surface and a unit south pole with each 4π lines entering a surface.

54. Magnetic Potential.—The magnetic potential of an isolated magnetic pole is the work done in carrying a unit north pole from an infinite distance to the point against the forces in the magnetic field.

Fig. 42 shows a north magnetic pole of strength m. Its field

extends out radially in all directions and the magnetic force at a distance of r cm. from m is

$$\mathcal{H} = \frac{m}{r^2} \text{ dynes.}$$

The magnetic potential of the point P at a distance of r_1 cm. from m is the work done in carrying a unit north pole from an infinite distance to the point against the force of repulsion of m.

The work done is

$$W = \int_{r_1}^{\infty} \mathcal{H}\, dr = \int_{r_1}^{\infty} m\frac{dr}{r^2} = \frac{m}{r_1} \text{ ergs.}$$

Therefore the magnetic potential of a point at a distance of r_1 cm. from an isolated magnetic pole of strength m is

$$M = \frac{m}{r_1}. \tag{83}$$

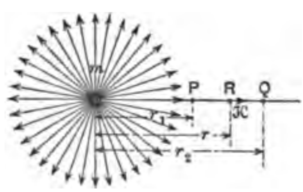


FIG. 42.—Magnetic potential.

The difference of magnetic potential between the points P and Q in Fig. 42 is

$$M = \int_{r_1}^{r_2} \mathcal{H}\, dr = \int_{r_1}^{r_2} m\frac{dr}{r^2} = \frac{m}{r_1} - \frac{m}{r_2}.$$

Thus the difference of magnetic potential between two points is the line integral of the magnetic force between them.

55. Magnetomotive Force.—The difference of magnetic potential or the line integral of the magnetic force between two points is called the magnetomotive force (m.m.f.) between the points. It causes magnetic flux to pass from one to the other.

Unit m.m.f. will produce one line of magnetic flux through a $(\text{cm.})^3$ of air and μ lines through a $(\text{cm.})^3$ of a magnetic material of permeability μ. It is called the gilbert.

Since m.m.f. is the line integral of the magnetic force or

$$M = \int \mathcal{H}\, dr,$$

therefore the magnetic force is the space rate of change of m.m.f. or

$$\mathcal{H} = \frac{dM}{dr}, \tag{84}$$

and thus magnetic force is the m.m.f. per centimeter.

56. Permeability.—Permeability is the ratio of the magnetic conductivity of a substance to the magnetic conductivity of air and is represented by μ.

Lines of magnetic flux pass through air or any other substance except iron, nickel or cobalt, as easily as they do through a vacuum. The permeability of such substances is for all practical purposes the same and is taken as unity. Iron and its compounds and to a lesser degree nickel and cobalt are found to allow magnetic lines to pass through them much more easily than empty space; that is, a given m.m.f. will produce a much larger flux through a volume of iron than it will through an equal volume of air. The permeability of the iron is therefore greater than that of the air and is expressed by some number greater than unity.

The permeability of magnetic materials is not constant but varies with the induction density as shown in Art. 72.

57. Magnetic Reluctance.—The reluctance of a magnetic circuit may be defined as the resistance offered by the circuit to the passage of magnetic flux through it and is represented by $\mathcal{R}$.

If a m.m.f. M is applied to a path of length l cm., of uniform section A sq. cm. and of permeability μ, the m.m.f. per centimeter will be $\frac{M}{l}$ and this will produce through each square centimeter a flux or flux density $\mathcal{B} = \frac{M}{l}\mu$ lines and the total flux through the path will be

$$\phi = \mathcal{B}A = \frac{M}{l}\mu A = \frac{M}{\frac{l}{A\mu}} = \frac{M}{\mathcal{R}}.$$

Thus the reluctance of a path of uniform section is

$$\mathcal{R} = \frac{l}{A\mu}, \tag{85}$$

and is directly proportional to its length and inversely proportional to its sectional area and to the permeability of the material forming it.

The equation connecting the m.m.f. acting on a path, the re-

luctance of the path and the flux through the path can be written in three ways:

1. $$\Phi = \frac{M}{\mathcal{R}}, \tag{86}$$

the flux is equal to the m.m.f. divided by the reluctance;

2. $$M = \Phi\mathcal{R} \tag{87}$$

the m.m.f. is equal to the flux multiplied by the reluctance;

3. $$\mathcal{R} = \frac{M}{\Phi}, \tag{88}$$

the reluctance is equal to the m.m.f. divided by the flux.

Assuming the flux to be unity in the last equation the reluctance of the path may be defined as the m.m.f. required to produce unit flux through it.

58. Permeance.—The permeance of a magnetic path is the reciprocal of its reluctance and is represented by $\mathcal{P}$; thus

$$\mathcal{P} = \frac{1}{\mathcal{R}} = \frac{\Phi}{M}, \tag{89}$$

and assuming that the m.m.f. acting is unity, the permeance may be defined as the flux produced through the path by unit m.m.f.

The permeance of a path of uniform section is

$$\mathcal{P} = \frac{A\mu}{l} \tag{90}$$

and is directly proportional to the sectional area and to the permeability, and is inversely proportional to the length of the path.

59. Electromagnetics.—The region surrounding a conductor carrying a current of electricity is a magnetic field. A current of electricity therefore represents a m.m.f. If the conductor is isolated from other magnetic forces the lines of force will form circles around it.

Maxwell's Corkscrew Rule.—The direction of the current and that of the resulting magnetic force are related to one another as the forward travel and the twist of an ordinary corkscrew. This rule is illustrated in Fig. 43.

The symbol ⊕ represents a current flowing down and ⊙ a current flowing up.

Faraday discovered that a current is induced in a closed coil of wire when a magnet is brought near it. The same effect is noticeable if a coil of wire carrying current is moved to or from the closed coil, or if the second coil is fixed in position and the current in it is varied. The induced current only exists while the magnet or inducing coil is moving with respect to the fixed coil or while the current in the inducing coil is varying.

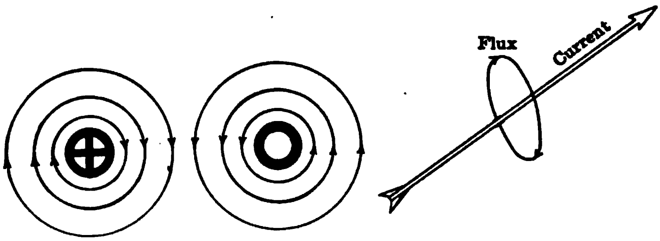


FIG. 43.—Magnetic flux produced by an electric current.

The induced current is due to the fact that a difference of potential or electromotive force is produced in the circuit by changing the number of lines of magnetic flux threading through it or by causing lines of magnetic flux to cut across it.

60. Laws of Induction.—*First Law.*—A change in the number of lines which pass through a closed circuit induces a current around the circuit in such a direction as to oppose the change in the flux threading the circuit.

Second Law.—The electromotive force induced around a closed circuit is equal to the rate of change of the flux which passes through the circuit; or the electromotive force induced in a conductor is equal to the rate at which it cuts across lines of magnetic flux.

61. Unit of Electromotive Force.—The absolute unit of electromotive force (e.m.f.) is the e.m.f. induced in a coil of one turn when the flux threading the coil is changing at the rate of one line per second; or it is the e.m.f. induced in a conductor when it is cutting one line per second. It is called the abvolt.

The practical unit is the e.m.f. produced by cutting 10^8 lines per second and is called the volt. E.m.f. is commonly called voltage.

To change from absolute units of e.m.f. to volts divide by 10^8.

If a coil of wire has n turns and the flux through it is changing at the rate $\frac{d\phi}{dt}$ lines per second, the e.m.f. induced in the coil is

$$e = -n\frac{d\phi}{dt} \text{ absolute units.} \tag{81}$$

The negative sign is used because when the flux is decreasing the induced e.m.f. is in the positive direction, that is, it tends to prevent the decrease of the flux.

62. Force Exerted by a Magnetic Field on an Electric Circuit.—Every part of an electric circuit situated in a magnetic field is acted upon by a force at right angles both to the direction of the current and to the lines of force, and the circuit as a whole is acted upon by forces tending to move it into the position where it will include the greatest possible flux.

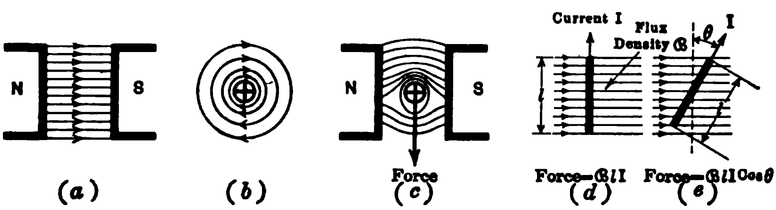


FIG. 44.—Force on an electric conductor in a magnetic field.

In Fig. 44, (a) represents a uniform magnetic field between two unlike poles and (b) represents the field surrounding a conductor carrying current. If the conductor is placed in the uniform field the resultant distribution will be as shown in (c). The intensity of the field above the conductor will be greater than that below and a force f will act on the conductor at right angles to it and to the lines of flux, tending to push it out of the field. This force is directly proportional to the intensity of the field or the flux density, to the length of the conductor in the field and to the strength of the current, or

$$f = \mathcal{B}lI \text{ dynes,} \tag{92}$$

where $\mathcal{B}$ is the flux density in lines per square centimeter,

l is the length of the conductor in centimeters,

I is the strength of the current in absolute units.

It has been assumed that the conductor lies at right angles to the lines of force as shown at (d); if it is inclined at an angle θ to this direction as at (e) then the force is reduced to

$$f = \mathcal{B}lI \cos\theta. \tag{93}$$

In Fig. 45(*a*), *AB* is a section through a coil of wire carrying current. *A* is acted upon by a force f_A tending to move it down and *B* is acted upon by a smaller force f_B, tending to move it up. The coil will move down until *A* and *B* are in fields of equal strength and the flux threading the coil is maximum. The flux produced by the current in the coil is in the same direction as the main field. (*b*) shows the same coil reversed and in this case it will be forced out of the field. (*c*) shows the coil near the position of maximum inclosure of flux but with the flux produced by the current opposing the main flux. The coil is forced around to the position *A'B'*.

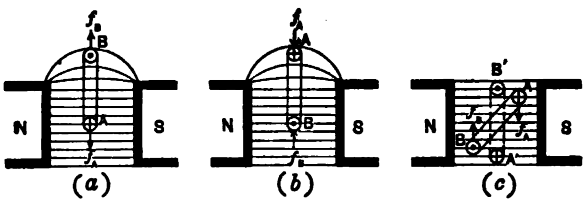


FIG. 45.—Force exerted by a magnetic field on a coil of wire.

63. Unit Current.—If a conductor carrying one absolute unit of current is placed in a magnetic field of unit strength at right angles to the lines of force, each centimeter of its length will be acted upon by a force of one dyne. This absolute unit of current is called the absampere.

The practical unit is one-tenth of the absolute unit and is called the ampere.

64. Transformation of Mechanical Energy to Electrical Energy.—In Fig. 46, if the conductor is moved through a distance dx at right angles to the flux against the force f, the work done is

$$dw = f\,dx = \mathfrak{B}lI\,dx = \mathfrak{B}l\,dx\,I,$$

but $\mathfrak{B}l\,dx$ is the flux cut in moving through the distance dx and is $= d\phi$, therefore

$$dw = I\,d\phi, \tag{94}$$

and the work done in moving a current across a magnetic field is equal to the product of the current and the flux cut.

In moving completely across the pole face the work done is

$$W = \int I\,d\phi = I\Phi,$$

where Φ is the flux from the pole.

If I is expressed in absolute units and Φ in maxwells, W is in ergs.

If the motion through the distance dx takes place in time dt sec., the work done is

$$dw = I\,d\phi = I\,dt\frac{d\phi}{dt} = eI\,dt, \tag{95}$$

where $e = \dfrac{d\phi}{dt}$ is the e.m.f. generated in the conductor and $I\,dt = dq$ is the quantity of electricity raised through the difference of potential e. Therefore, the mechanical work supplied to move the conductor through the distance dx against the force f is used up in doing the electrical work of raising a quantity of

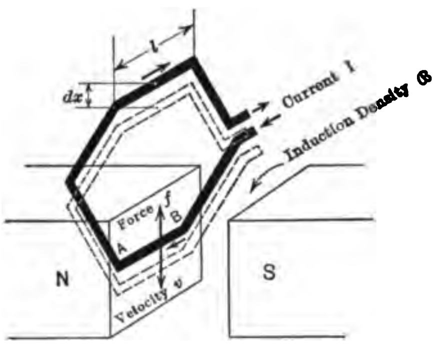


FIG. 46.—Transformation of mechanical energy to electrical energy.

electricity dq through a difference of potential e, or in driving a current I against an e.m.f. e for a time dt. Thus mechanical energy is transformed into electrical energy. This is what takes place in an electric generator.

If electric power is supplied to drive the current I against the e.m.f. $e = \dfrac{d\phi}{dt}$, for a time dt energy is supplied

$$dw = e\,I\,dt = \frac{d\phi}{dt}\,I\,dt = I\,d\phi;$$

the conductor exerts a force $f = \mathcal{B}l\,I$ dynes through a distance dx and does mechanical work,

$$f\,dx = \mathcal{B}lI\,dx = I\,d\phi.$$

This is the action of an electric motor.

65. Electric Power and Energy.—When an electric circuit carrying a current I absamp. incloses a flux ϕ, which is not pro-

duced by the current, the potential electrical energy of the system is

$$w = \phi I \text{ ergs.} \tag{96}$$

This amount of energy must have been expended in bringing the electric circuit into its position.

Power is the rate of flow or the rate of transformation of energy. The mechanical power required to move the conductor AB across the field and thus to vary the flux ϕ is

$$P = \frac{dw}{dt} = I\frac{d\phi}{dt} = \mathfrak{B} l I \frac{dx}{dt} = fv \text{ dyne-cm. per second,}$$

where $v = \frac{dx}{dt}$ is the velocity of the conductor normal to the field.

The electric power generated during the motion is

$$P = \frac{dw}{dt} = I\frac{d\phi}{dt} = eI \text{ ergs per second,}$$

where $e = \frac{d\phi}{dt}$ is the e.m.f. generated in the conductor by cutting the flux.

The electric power in a circuit is the product of the current and the e.m.f. in the circuit.

The practical unit of power is the watt. It is the power in a circuit carrying one ampere when the e.m.f. across it is one volt.

$$P_{\text{Watts}} = e_{\text{Volts}} \times I_{\text{Amperes}}. \tag{97}$$

The kilowatt, which is 1,000 watts, is more commonly used where the amounts of power are large. One horsepower is equivalent to 746 watts.

The electric energy transformed in a circuit is the product of the power and the time. The practical units of electric energy are the watt-second or joule, the watt-hour and the kilowatt-hour.

The absolute unit of electric energy is the erg.

$$\begin{aligned} 1 \text{ watt-sec.} &= 1 \text{ volt} \times 1 \text{ amp.} \times 1 \text{ sec.} \\ &= 1 \text{ abvolt} \times 10^{8} \times 1 \text{ absamp.} \times 10^{-1} \times 1 \text{ sec.} \\ &= 10^{7} \text{ ergs.} \end{aligned}$$

66. Intensity of Magnetic Fields Produced by Electric Currents.— The following cases are of special importance: (A) At the center of a circular loop of wire carrying a current I absolute units (Fig. 47).

If $\mathcal{H}$ is the field intensity at O, the center of the loop, a magnetic pole of strength m placed at this point will be acted upon by a force

$$f = m\mathcal{H} \text{ dynes}$$

in a direction perpendicular to the plane of the coil.

The pole will produce at the wire a field of intensity

$$\mathcal{H}_1 = \frac{m}{r^2} \text{ dynes,}$$

and a flux density

$$\mathcal{B} = \frac{m}{r^2} \text{ lines per square centimeter,}$$

where r cm. is the radius of the loop.

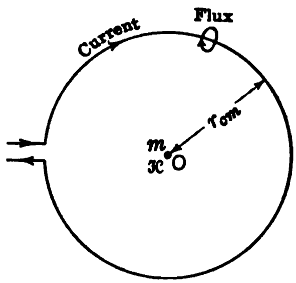


FIG. 47.—Intensity of the magnetic field at the center of a circular coil of wire.

This field will act on the wire with a force

$$f_1 = \mathcal{B}lI \text{ dynes}$$

in a direction perpendicular to the plane of the coil, where $l = 2\pi r$ is the length of the wire in centimeters.

Substituting the values of $\mathcal{B}$ and l gives

$$f_1 = \frac{m}{r^2} \times 2\pi r I = m\frac{2\pi I}{r} \text{ dynes,}$$

but the forces f and f_1 are equal and therefore

$$m\mathcal{H} = m\frac{2\pi I}{r},$$

and

$$\mathcal{H} = \frac{2\pi I}{r} \text{ dynes.} \tag{98}$$

The flux density at the center of the loop is

$$\mathcal{B} = \mathcal{H} = \frac{2\pi I}{r} \text{ lines per square centimeter.}$$

If I is expressed in amperes the field intensity at the center of the loop is

$$\mathcal{H} = \frac{0.2\pi I}{r} \text{ dynes.} \tag{99}$$

(*B*) At a distance of r cm. from a long straight wire carrying a current of I absolute units (Fig. 48).

If $\mathcal{H}$ is the intensity of the field, the work done in moving a unit magnetic pole around the wire at a distance r cm. from it against the force $\mathcal{H}$ is

$$w = 2\pi r\mathcal{H} \text{ ergs.}$$

The work done is equal to the product of the current and the flux cut and, therefore,

$$2\pi r\mathcal{H} = 4\pi I,$$

and

$$\mathcal{H} = \frac{2I}{r} \text{ dynes;} \tag{100}$$

the intensity of the field varies directly as the strength of the current and inversely as the distance from the wire.

The flux density at distance r is

$$\mathcal{B} = \mathcal{H} = \frac{2I}{r} \text{ lines per square centimeter.} \tag{101}$$

If I is in amperes the field intensity is

$$\mathcal{H} = \frac{0.2I}{r} \text{ dynes,} \tag{102}$$

and the flux density is

$$\mathcal{B} = \frac{0.2I}{r} \text{ lines per square centimeter.} \tag{103}$$

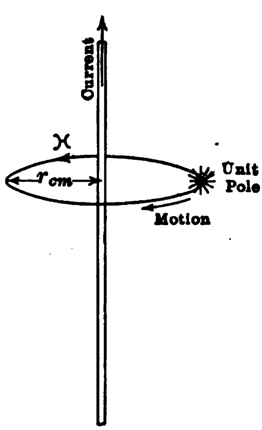


FIG. 48.—Intensity of the magnetic field near a long straight wire.

Unit current would produce a flux density of 2 lines per square centimeter at a distance of 1 cm. from a straight wire and it would produce this flux through a distance of 2π cm., or it would produce a flux density of 4π lines per square centimeter through a distance of 1 cm. in air. Thus one absolute unit of current represents a m.m.f. of 4π gilberts and one ampere represents a m.m.f. of 0.4π gilberts.

(*C*) Between two parallel wires A and B, Fig. 49, at a distance of D cm. apart and carrying equal currents I but in opposite directions.

The field intensity or magnetic force at point P distant r cm. from A and $D - r$ cm. from B is the resultant of the magnetic forces due to the currents in A and B. Since these forces act in the same direction at all points between the wires they can be added directly; the field intensity is by equation 100

$$\mathcal{H} = \frac{2I}{r} + \frac{2I}{D - r} \text{ dynes,} \tag{104}$$

and the flux density at P is

$$\mathcal{B} = \mathcal{H} = \frac{2I}{r} + \frac{2I}{D - r} \text{ lines per square centimeter.} \tag{105}$$

√(*D*) At any point on the axis of a short coil of radius r cm. (Fig. 50).

n = number of turns in the coil,
I = current in the coil in absolute units.

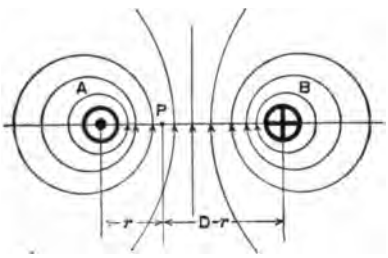


FIG. 49.—Magnetic field between two parallel wires.

Take any point P on the axis at a distance x cm. from the plane of the coil and let $\mathcal{H}$ be the field intensity there. If a pole of strength m is placed at P it will be acted on by a force of $m\mathcal{H}$ dynes perpendicular to the plane of the coil. The force exerted on the coil by the pole is equal and opposite to the force exerted on the pole by the coil.

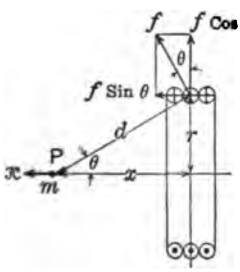


FIG. 50.—Intensity of the magnetic field on the axis of a short coil.

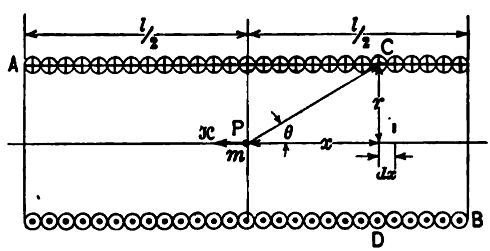


FIG. 51.—Magnetic field in a solenoid.

The field intensity at the wire due to the pole m at P is $\frac{m}{d^2}$ dynes and the flux density is $\frac{m}{d^2}$ lines per square centimeter, where d cm. is the distance from the point to the wire; the length of wire is $2\pi rn$ cm. and therefore the force exerted on it is

$$f = \frac{m}{d^2} \times 2\,\pi rnI \textit{ dynes.}$$

This force acts at right angles to the lines of flux and may be resolved into two components, $f \cos \theta$ in the plane of the coil and $f \sin \theta$ perpen-

dicular to the plane of the coil. The component $f \cos \theta$ taken around the loop is zero and therefore the component $f \sin \theta$ is equal in magnitude to the force $m\mathcal{H}$.

The field intensity or magnetic force at P is, therefore,

$$\mathcal{H} = \frac{f \sin \theta}{m} = \frac{2\pi rnI}{d^2} \sin \theta$$

$$= \frac{2\pi rnI}{r^2 + x^2} \times \frac{r}{\sqrt{r^2 + x^2}} = \frac{2\pi r^2 nI}{(r^2 + x^2)^{3/2}} \text{ dynes,} \tag{106}$$

since

$$d = \sqrt{r^2 + x^2} \text{ and } \sin \theta = \frac{r}{\sqrt{r^2 + x^2}}.$$

(*E*) On the axis of a long solenoid. In Fig. 51 AB is a solenoid of length l cm. and radius r cm. If n is the number of turns in the solenoid, the number of turns in the section CD of width dx is $\frac{n}{l}\,dx$.

The field intensity at P due to the section CD is, by equation (106),

$$d\mathcal{H} = \frac{2\pi r^2 I}{(r^2 + x^2)^{3/2}} \times \frac{n}{l}\,dx,$$

where I is the current in the solenoid.

The field intensity due to the complete solenoid is

$$\mathcal{H} = \int_{-\frac{l}{2}}^{\frac{l}{2}} \frac{2\pi r^2 I}{(r^2 + x^2)^{3/2}} \times \frac{n}{l}\,dx.$$

To integrate this let angle $CPT = \theta$, then

$$x = r \cot \theta,$$

$$(r^2 + x^2)^{3/2} = r^3(1 + \cot^2 \theta)^{3/2} = r^3 \operatorname{cosec}^3 \theta = \frac{r^3}{\sin^3 \theta},$$

$$dx = -r \operatorname{cosec}^2 \theta \, d\theta = -\frac{r\,d\theta}{\sin^2 \theta};$$

if the solenoid is assumed to be very long the limits may be taken as 0 and π, and, therefore,

$$\mathcal{H} = \frac{2\pi r^2 nI}{l} \int_\pi^0 \frac{\sin^3 \theta}{r^3} \times \left(-\frac{r\,d\theta}{\sin^2 \theta}\right)$$

$$= -\frac{2\pi nI}{l} \int_\pi^0 \sin \theta \, d\theta$$

$$= \frac{2\pi nI}{l} [\cos \theta]_\pi^0$$

$$= \frac{4\pi nI}{l} \text{ dynes.} \tag{107}$$

If the current is expressed in amperes

$$\mathcal{H} = \frac{0.4\pi nI}{l} \text{ dynes,} \tag{108}$$

and the field intensity on the axis of a long solenoid is proportional to the product of amperes and turns or ampere-turns and is inversely proportional to the length of the solenoid.

The field intensity throughout the volume enclosed by the solenoid is practically uniform except near the ends and can be expressed by equation 108.

67. Magnetomotive Force of a Solenoid.—The m.m.f. of a solenoid is the line integral of the magnetic forces along any closed path through it and is measured by the work done in carrying a unit magnetic pole around the closed path (Fig. 52).

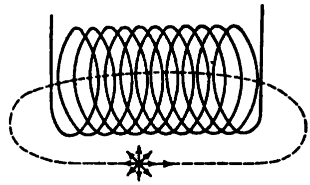


FIG. 52.—Magnetomotive force of a solenoid.

The work done is equal to the product of the current and the flux cut, and thus

$$\text{m.m.f.} = 4\pi nI, \text{ where } I \text{ is in absolute units,}$$

or

$$\text{m.m.f.} = 0.4\pi nI, \text{ where } I \text{ is in amperes.}$$

The m.m.f. is proportional to the ampere-turns of the coil. It does not make any difference whether it is a small current in a large number of turns or a large current in a small number of turns.

The m.m.f. of one ampere in one turn or one ampere-turn is 0.4π gilberts.

In studying the characteristics of electrical machinery it is more convenient to use the ampere-turn as the unit of m.m.f. Thus the m.m.f. of a field coil of n turns carrying a current I amp. is specified as nI ampere-turns instead of $0.4\pi nI$ gilberts.

68. Examples.—1. The solenoid in Fig. 53 has nI ampere-turns and is wound on a ring of non-magnetic material. The m.m.f. of the solenoid is $M = 0.4\pi nI$ gilberts.

If l cm. is the mean length of the path and A sq. cm. is the sectional area of the path, the reluctance is

$$\mathfrak{R} = \frac{l}{A},$$

and the flux inside the ring is

$$\Phi = \frac{M}{R} = \frac{0.4\pi nI}{l/A} \text{ lines.}$$

The flux density in the ring is

$$\mathfrak{B} = \frac{\Phi}{A} \text{ lines per square centimeter,}$$

and the magnetizing force is

$$\mathcal{H} = \mathcal{B} = \frac{\Phi}{A} = \frac{0.4\pi nI}{l}$$

and is the m.m.f. per centimeter.

If the solenoid is wound on an iron ring of permeability μ, the reluctance is reduced and becomes

$$\mathcal{R}_1 = \frac{l}{A\mu},$$

the flux is increased to

$$\Phi_1 = \frac{0.4\,\pi nI}{\dfrac{l}{A\mu}}$$

and the flux density or induction density is increased to

$$\mathcal{B}_1 = \frac{\Phi_1}{A};$$

the magnetizing force remains the same as before,

$$\mathcal{H} = \frac{\mathcal{B}_1}{\mu} = \frac{\Phi_1}{A\mu} = \frac{0.4\pi nI}{l}.$$

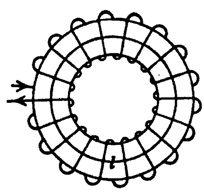


FIG. 53.—Ring solenoid.

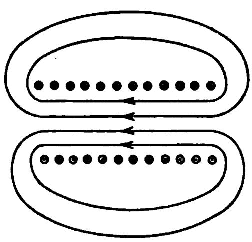


FIG. 54.—Solenoid in air.

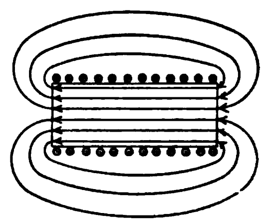


FIG. 55.—Solenoid wound on an iron bar.

2. Fig. 54 represents a solenoid of n turns carrying a current of I amp.; the m.m.f. is $M = 0.4\pi nI$ gilberts. If l cm. is the length of the solenoid and A sq. cm. is its sectional area, the reluctance of the path through it is

$$\mathcal{R}_1 = \frac{l}{A}.$$

The m.m.f. required to drive the flux Φ through this reluctance is

$$M_1 = \Phi\frac{l}{A}\text{ gilberts.}$$

But the flux going out at one end has to pass, around through the air and in at the other end as shown. The reluctance of this return path is difficult to calculate and is not of great practical importance; its length is greater than l but its sectional area is very much greater than A and its reluctance is small compared to $\mathcal{R}_1$. Thus in the case of long solenoids of small section the reluctance of the return path may be neglected and the assumption may be made that the whole m.m.f. M is utilized in driving the flux through the reluctance $\mathcal{R}_1$. Thus

$$\Phi = \frac{M}{\mathcal{R}_1} = \frac{0.4\,\pi nI}{l/A},$$

the induction density in the solenoid is

$$\mathcal{B} = \frac{\Phi}{A} = \frac{0.4\pi nI}{l/\cancel{X}}$$

and the magnetizing force at any point inside the solenoid is

$$\mathcal{H} = \mathcal{B} = \frac{0.4\,\pi nI}{l/\cancel{X}} \text{ dynes, as in equation (108).}$$

If an iron bar of length l and permeability μ is placed in the solenoid the reluctance of the path through the solenoid is reduced in the ratio $\frac{1}{\mu}$ and the reluctance of the return path is no longer negligible in comparison with it and its value must be calculated (Fig. 55).

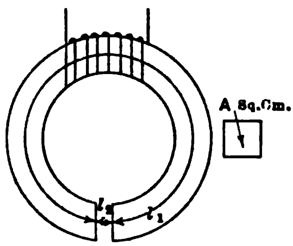


FIG. 56.—Ring with an air gap.

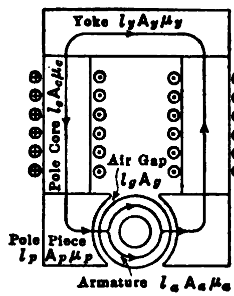


FIG. 57.—Magnetic circuit of a dynamo.

3. In Fig. 56 a solenoid of nI ampere-turns is wound on an iron ring of permeability μ and a section of length l_2 is cut from the iron. If l_1 is the mean length of the path through the iron and its sectional area is A sq. cm. the reluctance of the path through the iron is

$$\mathcal{R}_1 = \frac{l_1}{A\mu},$$

the reluctance of the path through the air is

$$\mathcal{R}_2 = \frac{l_2}{A}$$

and the total reluctance of the path is

$$\mathcal{R} = \mathcal{R}_1 + \mathcal{R}_2 = \frac{l_1}{A\mu} + \frac{l_2}{A}.$$

The flux through the path is

$$\Phi = \frac{M}{\mathcal{R}} = \frac{0.4\pi nI}{\frac{l_1}{A\mu} + \frac{l_2}{A}} \tag{109}$$

The flux density in the iron is $\mathcal{B} = \frac{\Phi}{A}$ and is the same as in the air.

The magnetizing force or m.m.f. per centimeter in the iron is

$$\mathcal{H}_1 = \frac{\mathcal{B}}{\mu} = \frac{\Phi}{A\mu},$$

and in the air it is

$$\mathcal{H}_2 = \mathcal{B} = \frac{\Phi}{A}.$$

The m.m.f. consumed in the iron is

$$M_1 = \mathcal{H}_1 l_1 = \frac{\Phi}{A\mu} l_1$$

and the m.m.f. consumed in the air is

$$M_2 = \mathcal{H}_2 l_2 = \frac{\Phi}{A} l_2.$$

4. Fig. 53 represents the magnetic circuit of a bipolar dynamo. It consists of a number of parts of different materials as follows:

One yoke y of section A_y, length l_y, and permeability μ_y,
Two pole cores c,c of section A_c, length l_c, and permeability μ_c,
Two pole pieces p,p of section A_p, length l_p, and permeability μ_p,
Two air gaps g,g of section A_g, length l_g, and permeability $\mu_g = 1$,
One armature a of section A_a, length l_a, and permeability μ_a.

The reluctance of these parts are, respectively,

$$\mathcal{R}_y = \frac{l_y}{A_y\mu_y}, \quad \mathcal{R}_c = \frac{2l_c}{A_c\mu_c}, \quad \mathcal{R}_p = \frac{2l_p}{A_p\mu_p}, \quad \mathcal{R}_g = \frac{2l_g}{A_g}, \quad \mathcal{R}_a = \frac{l_a}{A_a\mu_a};$$

and the reluctance of the whole circuit is

$$\mathcal{R} = \mathcal{R}_y + \mathcal{R}_c + \mathcal{R}_p + \mathcal{R}_g + \mathcal{R}_a.$$

The m.m.f. M is provided by field coils placed on the pole cores as shown, and the flux through the circuit is

$$\Phi = \frac{M}{\mathcal{R}} = \frac{M}{\mathcal{R}_y + \mathcal{R}_c + \mathcal{R}_p + \mathcal{R}_g + \mathcal{R}_a},$$

and is equal to the m.m.f. divided by the total reluctance; and the m.m.f. is

$$\begin{aligned} M &= \Phi\mathcal{R} = \Phi\mathcal{R}_y + \Phi\mathcal{R}_c + \Phi\mathcal{R}_p + \Phi\mathcal{R}_g + \Phi\mathcal{R}_a \\ &= M_y + M_c + M_p + M_g + M_a, \end{aligned}$$

where M_y is the part of the total field m.m.f. required to drive the flux through the yoke, etc.

The m.m.f. M_g required to drive the flux across the air gaps is sometimes as much as 80 per cent. of the total m.m.f.

5. Determine the reluctance of the ring in Fig. 58 made up of three parts of lengths l_1, l_2 and l_3 cm. respectively and sectional areas A_1, A_2 and A_3 sq. cm. and permeabilities μ_1, μ_2 and μ_3. The m.m.f. of the solenoid is M.

The reluctance of section (1) is $\mathcal{R}_1 = \dfrac{l_1}{A_1\mu_1}$;

the reluctance of section (2) is $\mathcal{R}_2 = \dfrac{l_2}{A_2\mu_2}$;

the reluctance of section (3) is $\mathcal{R}_3 = \dfrac{l_3}{A_3\mu_3}$;

the flux through section (1) is $\Phi_1 = \dfrac{M}{\mathcal{R}_1}$;

the flux through section (2) is $\Phi_2 = \dfrac{M}{\mathcal{R}_2}$;

the flux through section (3) is $\Phi_3 = \dfrac{M}{\mathcal{R}_3}$;

the total flux through the ring is

$$\begin{aligned}\Phi &= \Phi_1 + \Phi_2 + \Phi_3 \\ &= \frac{M}{\mathcal{R}_1} + \frac{M}{\mathcal{R}_2} + \frac{M}{\mathcal{R}_3} \\ &= M\left(\frac{1}{\mathcal{R}_1} + \frac{1}{\mathcal{R}_2} + \frac{1}{\mathcal{R}_3}\right). \qquad (110)\end{aligned}$$

But the flux is equal to the m.m.f. divided by the reluctance of the ring, or

$$\Phi = \frac{M}{\mathcal{R}},$$

and therefore

$$\mathcal{R} = \frac{M}{\Phi} = \frac{1}{\dfrac{1}{\mathcal{R}_1} + \dfrac{1}{\mathcal{R}_2} + \dfrac{1}{\mathcal{R}_3}}. \qquad (111)$$

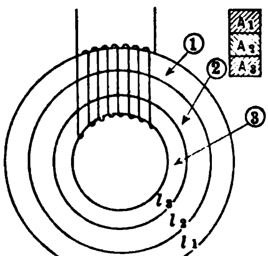


FIG. 58.—Parallel magnetic paths.

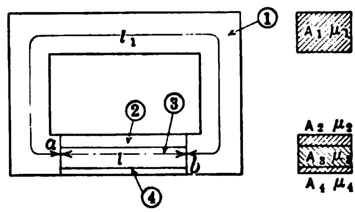


FIG. 59.—Series-parallel magnetic circuit.

6. Fig. 59 shows a ring of iron with a piece set in made up of three parts of different permeabilities. The lengths, sections and permeabilities are indicated and the reluctances of the various parts are $\mathcal{R}_1 = \dfrac{l_1}{A_1\mu_1}$, $\mathcal{R}_2 = \dfrac{l}{A_2\mu_2}$, $\mathcal{R}_3 = \dfrac{l}{A_3\mu_3}$, and $\mathcal{R}_4 = \dfrac{l}{A_4\mu_4}$. Determine the reluctance of the circuit.

Let M = the m.m.f. applied to the ring,
Φ = flux through the ring,
M_1 = m.m.f. consumed in section (1),
M_2 = m.m.f. consumed in section ab,

then $M_1 = \Phi\mathcal{R}_1$, and $M_2 = \Phi\mathcal{R}_{ab}$.

The reluctance of the section ab consisting of three paths in multiple must be found. The flux Φ passing through it divides into three parts which are inversely proportional to the reluctances of the paths,

$$\Phi_2 = \frac{M_2}{\mathcal{R}_2}, \quad \Phi_3 = \frac{M_2}{\mathcal{R}_3}, \quad \Phi_4 = \frac{M_2}{\mathcal{R}_4},$$

therefore,

$$\Phi = \Phi_2 + \Phi_3 + \Phi_4 = M_2\left(\frac{1}{\mathcal{R}_2} + \frac{1}{\mathcal{R}_3} + \frac{1}{\mathcal{R}_4}\right) = \frac{M_2}{\mathcal{R}_{ab}},$$

and

$$\mathcal{R}_{ab} = \frac{1}{\dfrac{1}{\mathcal{R}_2} + \dfrac{1}{\mathcal{R}_3} + \dfrac{1}{\mathcal{R}_4}}.$$

The reluctance of the whole circuit is

$$\mathcal{R} = \frac{M}{\Phi} = \frac{M_1 + M_2}{\Phi} = \mathcal{R}_1 + \mathcal{R}_{ab}$$

$$= \mathcal{R}_1 + \frac{1}{\frac{1}{\mathcal{R}_2} + \frac{1}{\mathcal{R}_3} + \frac{1}{\mathcal{R}_4}}.$$

69. Energy Stored in the Magnetic Field.—When a current i c.g.s. units flows in the solenoid, Fig. 60, wound on an iron ring, a flux is produced,

$$\Phi = \frac{4\pi n i}{l} A\mu,$$

where n is the number of turns in the solenoid,
A is the sectional area of the ring,
l is the mean length of the ring and
μ is its permeability and is treated as though it were a constant.

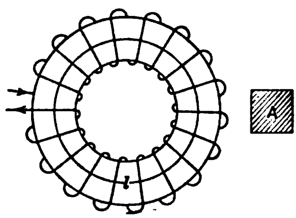


FIG. 60.—Solenoid on an iron ring.

When the current increases by a small amount di the flux increases by an amount $d\phi = \frac{4\pi n A\mu}{l} di$, and the work done is equal to the product of the current and the increase of the flux,

$$dw = ni\, d\phi = \frac{4\pi n^2 A\mu}{l} i\, di \text{ ergs.}$$

The work done while the current is building up to its full value I is

$$W = \int dw = \frac{4\pi n^2 A\mu}{l} \int_0^I i\, di$$

$$= \frac{4\pi n^2 A\mu}{l} \frac{I^2}{2} \text{ ergs} \tag{112}$$

$$= \mathcal{L} \frac{I^2}{2} \text{ ergs,} \tag{113}$$

where $\mathcal{L} = \frac{4\pi n^2 A\mu}{l}$ is a constant, if μ is constant, and is called the inductance of the circuit.

This amount of energy is stored in the magnetic field of the solenoid and may be expressed in other forms.

The energy stored is

$$W = \frac{4\pi n^2 A\mu}{l}\frac{I^2}{2} \text{ ergs},$$

but the magnetic force in the solenoid is

$$\mathcal{H} = \frac{4\pi n I}{l},$$

therefore,

$$W = \frac{\mu \mathcal{H}^2}{8\pi} Al,$$

or since the induction density is

$$\mathcal{B} = \mu \mathcal{H},$$

the energy may be expressed as

$$W = \frac{\mathcal{B}^2}{8\pi\mu} Al.$$

The product Al represents the volume of the magnetic field and therefore the energy stored in the field per unit volume is

$$w = \frac{W}{Al} = \frac{\mu \mathcal{H}^2}{8\pi}. \qquad (114)$$

or

$$w = \frac{\mathcal{B}^2}{8\pi\mu}. \qquad (115)$$

or

$$w = \frac{\mathcal{H}\mathcal{B}}{8\pi} \text{ ergs}. \qquad (116)$$

When the magnetic field is produced in air the field intensity $\mathcal{H}$ is equal to the flux density $\mathcal{B}$, and the energy stored per cubic centimeter is

$$w = \frac{\mathcal{H}^2}{8\pi} \text{ ergs}, \qquad (117)$$

or

$$w = \frac{\mathcal{B}^2}{8\pi} \text{ ergs}. \qquad (118)$$

70. Stress in the Magnetic Field.—The force between two magnetic poles is not exerted at a distance but is transmitted through the medium separating them and the medium is stressed.

There is a tension along the lines of force or induction tending to shorten them and draw the boundary surfaces of the field together and so decrease the magnetic energy stored in the field; there is also a pressure at right angles to the lines tending to spread them apart, and so decrease the flux density in the field and, therefore, also the energy stored. Since the medium is in equilibrium these two forces are equal in magnitude.

The pull on the bounding surfaces is usually expressed in dynes per square centimeter and may be found as follows:

The energy stored in a magnetic field in air was found to be $\frac{\mathcal{B}^2}{8\pi}$ ergs in Art. 69

Fig. 61 represents a horseshoe magnet with its armature removed a distance x. If $\mathcal{B}$ is the flux density in the field between the magnet poles and the armature and A sq. cm. is the area of the two pole faces, the volume of the field is Ax c.c. and the energy stored in it is

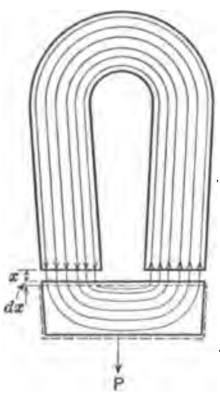


Fig. 61.—Pull of a magnet.

$$W = \frac{\mathcal{B}^2}{8\pi} Ax \text{ ergs.}$$

If a pull P is exerted on the armature and it is moved through a distance dx, the work done is $P\,dx$ and this is equal to the increase in the energy stored in the field; therefore,

$$P\,dx = \frac{\mathcal{B}^2}{8\pi} A\,dx$$

and

$$P = \frac{\mathcal{B}^2 A}{8\pi} \text{ dynes;} \tag{119}$$

this is the pull of the magnet on the armature.

The pull per square centimeter is

$$p = \frac{P}{A} = \frac{\mathcal{B}^2}{8\pi} \text{ dynes,} \tag{120}$$

and this is the tension along the lines of induction in the field and is also equal in magnitude to the pressure at right angles to the lines.

71. Force between Parallel Wires Carrying Current.—Parallel wires carrying currents I_1 and I_2 absamp. will either be attracted

or repelled depending on whether the currents are in the same or opposite directions. Fig. 62 shows the fields in the two cases. When the currents are in the same direction the lines of force combine to form lines surrounding the two wires and these lines tend to shorten, or the circuit tends to change so that the reluctance is a minimum and the flux a maximum. When the currents are in opposite directions the lines of force pass between the wires and they tend to spread out resulting in a repulsion between the wires. In this case again the forces tend to decrease the reluctance of the circuit by increasing its section.

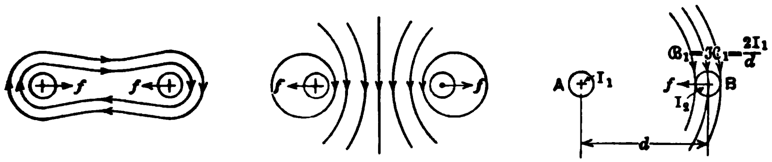


FIG. 62.—Force between two parallel wires carrying current.

To obtain an expression for the force between the two wires refer to Fig. 62. A and B are the two wires carrying currents I_1 and I_2 respectively and d cm. is the distance between their centers.

The field intensity and flux density at wire B produced by the current I_1 in A is

$$\mathfrak{B}_1 = \frac{2I_1}{d} \text{ lines per square centimeter.}$$

This field acts on the wire B carrying current I_2 with a force

$$f = \mathfrak{B}_1 I_2 = \frac{2I_1I_2}{d} \text{ dynes per centimeter length.} \tag{121}$$

This is the force of repulsion between the two wires.

72. Magnetic Characteristics.—If an iron ring, Fig. 60, which has been completely demagnetized, is gradually magnetized, by increasing the current in the exciting coil from zero, the induction density in the ring increases with the magnetizing force as shown in curve 1, Fig. 63. With feeble magnetizing forces the gradient of the curve is small and the permeability is low; as the force increases the curve becomes very steep and nearly straight and the permeability increases rapidly and becomes very large; as the force is further increased the curve rounds off and the gradient becomes small again and a large increase in magnetizing force is required to produce any considerable increase in density, the permeability decreases to a low value and the material is

said to be saturated. Curve 1 is called a "magnetization curve" or "saturation curve," or $\mathcal{B}$–$\mathcal{H}$ curve of the material.

Curve 2, Fig. 63, shows the relation between the permeability $\mu = \frac{\mathcal{B}}{\mathcal{H}}$ and the induction density $\mathcal{B}$; when the density is low

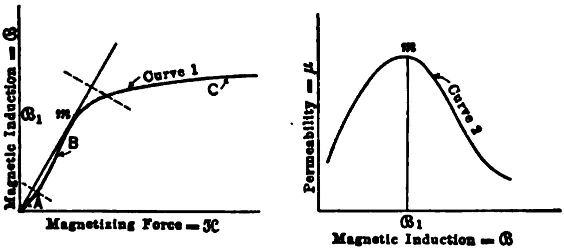


FIG. 63.—Magnetic characteristics.

the permeability is low; as the density is increased the permeability increases until it reaches a maximum at the point m where the tangent from the origin touches curve 1; above this point the permeability decreases again.

In Figs. 64 and 65 are shown magnetization curves for materials used in electrical machine design. These curves are plotted with

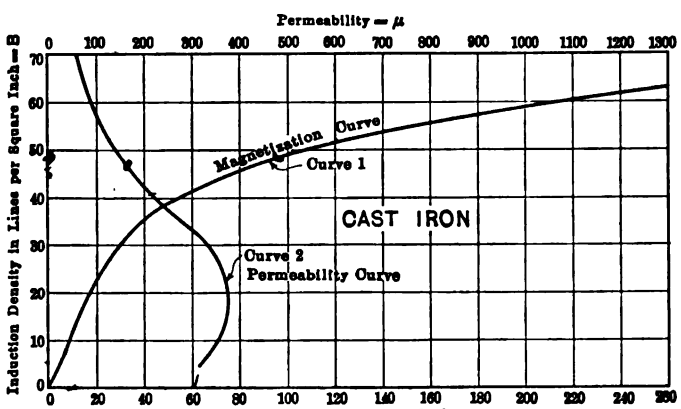


FIG. 64.—Magnetic characteristics of cast iron.

induction density B expressed in lines per square inch on a base of ampere-turns per inch instead of magnetizing force $\mathcal{H}$. This results in a change of scales only since B lines per square inch $= (2.54)^2\ \mathcal{B}$, and T ampere-turns per inch corresponds to a magnetizing force $\mathcal{H} = \frac{0.4\pi T}{2.54}$.

Permeability curves for cast iron and cast steel are also shown. For cast iron the maximum value of μ occurs at a density of about 4,000 lines per square centimeter or 25,000 lines per square inch and for steel at about 6,500 lines per square centimeter or 40,000 lines per square inch. Densities below these points are not of very great importance.

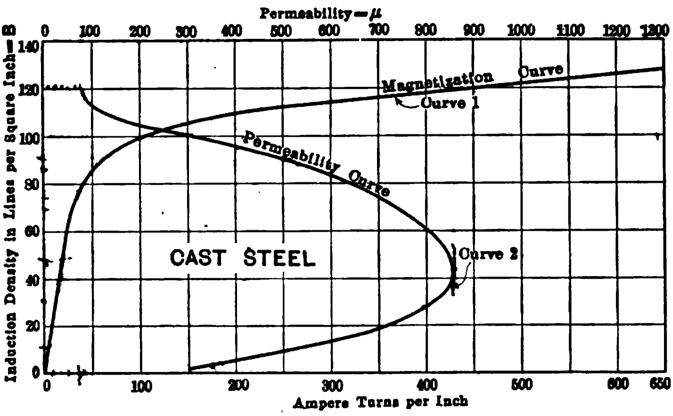


FIG. 65.—Magnetic characteristics of cast steel.

73. Hysteresis.—If the exciting current i or magnetizing force $\mathcal{H}$, of the solenoid in Art. 69, is increased the flux density $\mathfrak{B}$ increases, following the curve OA in Fig. 66 until it reaches its maximum value $\mathfrak{B}_m$ at A. If $\mathcal{H}$ is now gradually reduced to zero, $\mathfrak{B}$ will not come back on AO but will follow the curve AR and when $\mathcal{H}$ is zero $\mathfrak{B}$ will have the value $\mathfrak{B}_r$ represented by OR. This is called the residual magnetism; it is not a fixed quantity but depends on the degree of magnetization of the material. To remove the residual magnetism $\mathcal{H}$ is reversed and increased to the point C. $\mathfrak{B}$ is now zero. The magnetizing force represented by OC is called the coercive force. As $\mathcal{H}$ is still increased $\mathfrak{B}$ increases to its maximum value $\mathfrak{B}_m$ again at the point A' with the same magnetizing force as at A. Again reducing $\mathcal{H}$, $\mathfrak{B}$ follows the curve $A_1R_1C_1$ to A and closes the loop. This closed curve is called a hysteresis loop.

The area of the hysteresis loop represents the amount of energy consumed in carrying the material through the cycle of magnetism.

From Art. 64 the work done in increasing the flux threading the solenoid by an amount $d\phi$ is

$$dw = \frac{ni}{10}\, d\phi \text{ ergs,}$$

Where n is the number of turns on the coil and i is the current in amperes when the flux threading the coil is ϕ. If the ring has a constant section of A sq. cm. and a length of l cm., then, since $\mathcal{H} = \dfrac{4\pi ni}{10l}$ and $d\phi = A\,d\mathcal{B}$, the work done is

$$dw = \frac{Al}{4\pi}\, \mathcal{H}\, d\mathcal{B},$$

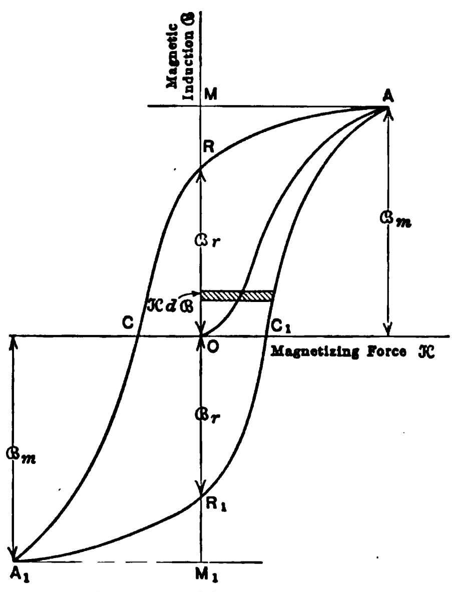


FIG. 66.—Hysteresis loop.

and the work done during a complete cycle is

$$W = \int dw = \frac{Al}{4\pi} \int_{\mathcal{B}_m}^{\mathcal{B}_m} \mathcal{H}\, d\mathcal{B} \text{ ergs.}$$

Al is the volume of the ring in cubic centimeters and therefore the loss in ergs per cycle per cubic centimeter of iron is

$$w_h = \frac{1}{4\pi} \int_{\mathcal{B}_m}^{\mathcal{B}_m} \mathcal{H}\, d\mathcal{B}. \qquad (122)$$

Referring to Fig. 66 and starting from the point C_1, the energy per cubic centimeter supplied to increase the induction density

from O to $\mathfrak{B}_m$ is $\frac{1}{4\pi}$ (area OC_1AM) ergs. While $\mathfrak{B}$ decreases from $\mathfrak{B}_m$ to $\mathfrak{B}_r$, energy is given back $= \frac{1}{4\pi}$ (area AMR). The energy required to remove the residual magnetism is $\frac{1}{4\pi}$ (area ROC). Similarly below the axis the energy consumed is

$$\frac{1}{4\pi} \text{ (area } OCA_1M_1 - \text{area } A_1M_1R_1 + \text{area } OR_1C_1\text{) ergs.}$$

Thus the hysteresis loss per cubic centimeter per cycle is $\frac{1}{4\pi}$ (area of loop) ergs.

The area of the loop increases faster than the maximum density. Steinmetz gives the following equation for the loss per cubic centimeter per cycle in terms of the maximum density:

$$\omega_h = \eta \mathfrak{B}_m^{1.6} \text{ ergs,} \tag{123}$$

where η is called the hysteretic constant of the material.

The hysteresis loss in a volume of V c.c. of iron at a frequency of f cycles per second is

$$W_h = \eta \mathfrak{B}_m^{1.6} f V 10^{-7} \text{ watts.} \tag{124}$$

The following table gives ordinary values for the hysteretic constant for commercial materials.

Good dynamo sheet steel	0.002
Fair	0.003
Silicon steel	0.00076–0.004
Cast iron	0.011–0.016
Cast iron	0.011–0.016
Cast steel	0.003–0.012
Good commercial iron after assembly	0.0027

Iron can be carried through a cycle of magnetism or hysteresis cycle (1) by applying an alternating m.m.f. and producing an alternating flux as in the transformer; (2) by producing a revolving flux which cuts stationary iron as in the stator of an induction motor; and (3) by revolving iron in a stationary magnetic field as in direct-current generators and motors.

The energy expended in hysteresis heats the iron and causes a loss in efficiency.

74. Magnetic Materials.—The most important magnetic materials used in electrical design are cast iron, cast steel, ordinary sheet steel and silicon sheet steel.

Cast iron has low permeability and large hysteresis loss. It is used for parts of the magnetic circuit where the induction density is low and is in a constant direction. It is cheap and can be made into castings of complex form.

Cast steel has a much higher permeability and smaller hysteresis loss. It replaces cast iron where greater strength or greater permeability is required or when the appearance would be improved by reducing the section.

Sheet steel is used for parts carrying alternating fluxes of all densities. It is necessary to use thin sheets in such places to reduce the loss due to eddy currents set up in the iron as it cuts across the flux (see Art. 184). To further reduce the eddy-current loss the iron should have as high an electrical resistance as possible.

Silicon steel contains a small percentage of silicon which has the effect of reducing the hysteresis loss and increasing the electrical resistance and so reducing the eddy-current loss. Much higher flux densities can thus be used and the weight and cost of machines reduced. Silicon steel is used principally for transformers, where very high efficiency is required. It is quite expensive.

75. Effect of Chemical Composition and Physical Treatment on Hysteresis Loss.—Hysteresis loss is dependent on:

1. Chemical composition of the iron.
2. Heat treatment.
3. Mechanical treatment.

1. The general effect of impurities is to decrease the permeability and increase the iron loss but there are exceptions.

Carbon decreases the permeability and lowers the saturation point; it increases the residual magnetism and the coercive force. These effects are greater in hardened steel than in soft iron. Such materials are valuable for permanent magnets but are not used in rotating machinery.

Silicon in certain percentages has a very beneficial effect; 2.5 to 4 per cent. of silicon alloyed with the iron increases the permeability and decreases the hysteresis loss to a marked degree. It also increases the electrical resistance.

T. D. Yensen has obtained samples of iron, alloyed with certain percentages of silicon and melted *in vacuo*, which exhibit magnetic qualities far superior to any commercial materials. These

materials are only in the experimental stage. The table below shows a comparison between various steels.

Aluminum has an effect similar to that of silicon but the improvement is not so great.

2. Annealing the material after manufacture has a very important effect. In certain cases it has reduced the loss in sheet iron to half.

It is found that the hysteresis loss in some steels increases with continued heating during use. This is known as aging and may have very serious results.

Silicon steel is non-aging.

3. Punching, hammering and bending increase the hysteresis loss in sheet steel.

The following table shows a comparison between Yensen's experimental steels and some commercial materials.

Material	Maximum permeability	Density for maximum permeability in lines per square centimeter	Hysteresis loss in ergs per cubic centimeter per cycle		Electrical resistance in microhms per centimeter cube
			$\mathcal{B}_{max.} =$ 10,000	$\mathcal{B}_{max.} =$ 15,000	
Experimental steel with 0.15 per cent. silicon	66,500	6,500	286	916	11.8
Experimental steel with 3.40 per cent. silicon	63,300	6,500	280	1,025	48.5
Standard transformer steel	3,850	7,000	3,320	5,910	11.0
4 per cent. silicon steel	3,160	4,000	2,260	3,030	51.0

76. Theories of Magnetism.—The magnetization of iron is accompanied by molecular changes and energy is consumed. Ewing was the first to formulate a theory to explain the resulting phenomena. He imagined iron to be made up of magnetic molecules. In unmagnetized material these molecules are arranged in groups, in which the elementary magnetic poles neutralize one another. When a weak magnetizing force or m.m.f. is applied, the less stable groups are broken up and their component molecules turned with their magnetic axes in the direction of the impressed m.m.f., making the material as a whole slightly magnetic. This stage is represented by the lowest section, *A*, of the magnetization curve, Fig. 63. If the magnetizing force is removed, the molecules return to their original groupings and no magnetism remains. When, however, an increasingly powerful

m.m.f. is applied, the more stable groups break up and a large proportion of the molecules turn in the direction of the external field and form new stable groupings; the $\mathcal{B}$–$\mathcal{H}$ curve rises sharply and the permeability is large. This is section B of the curve. If at this point the m.m.f. is reduced to zero, only a part of the molecules resume their old groupings, the majority remaining in their new positions. In this way the phenomenon of residual magnetism is accounted for.

With an ever increasing external m.m.f. more and more of the molecules line up and the material approaches saturation. Beyond this point any increase in m.m.f. only produces an increase in flux density comparable with that produced in a non-magnetic material (section C).

The later electron theory of magnetism replaces Ewing's magnetized molecules by electromagnets. It is assumed that each molecule of iron is the seat of an electric current or whirl of electrons and therefore exerts a small m.m.f. An external m.m.f. tends to draw these into line with itself as discussed above and thus increases the resultant m.m.f. of the system and consequently the flux and induction density. With this assumption the conception of permeability is unnecessary as increasing permeability is replaced by increasing m.m.f. which has the same effect.

77. Lifting Magnets.—Design a direct-current electromagnet capable of lifting a block of cast iron weighing 2,000 lb.

The pull of a magnet is given, by equation (119),

$$P = \frac{\mathcal{B}^2 A}{8\pi} \text{ dynes}$$

where $\mathcal{B}$ is the flux density in the air gap in lines per square centimeter and A is the area of the pole surfaces in square centimeters.

$$1 \text{ lb.} = 444{,}000 \text{ dynes}$$

and therefore

$$P = \frac{\mathcal{B}^2 A}{8\pi} = 444{,}000 \times 2{,}000 = 888 \times 10^6 \text{ dynes.}$$

Assuming an air-gap density of 6,000 lines per square centimeters (39,000 lines per square inch) the required pole area is

$$A = \frac{888 \times 10^6 \times 8\pi}{(6{,}000)^2} = 616 \text{ sq. cm.} = 95.5 \text{ sq. in.}$$

If the magnet is made in the shape shown in Fig. 67(a) the

area of each leg is $\frac{95.5}{2} = 48$ sq. in. The remaining dimensions may be chosen to make the magnet of a suitable shape but space must be left for the winding.

The number of ampere-turns required for the exciting winding may be found as follows:

Assume that the contact between the magnet and the mass to be raised is not perfect and that an air gap of ¼-in. is left. The ampere-turns required for the gap = $0.3132 \times 39{,}500 \times 0.25 \times 2 = 6{,}170$. (Art. 223). Length of the path through the steel magnet = 36 in.; the ampere-turns per inch required for a density of 39,500 lines is 17 (Fig. 65) and the total ampere-turns = $17 \times 32 = 544$. Length of the path in the cast iron

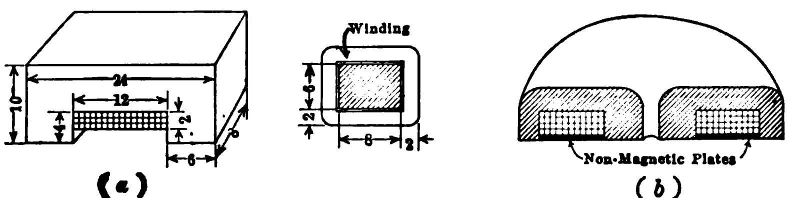


FIG. 67.—Lifting magnets.

may be assumed to be 24 in. and the section 60 sq. in.; the flux density in the cast iron is $\frac{39{,}500 \times 48}{60} = 31{,}600$; the ampere-turns per inch = 32 (Fig. 64) and the total ampere-turns for the cast iron = $32 \times 24 = 768$.

The exciting ampere-turns for the magnet = $6{,}170 + 544 + 768 = 7{,}482$.

In order that the coil may not become overheated it is necessary to limit the power loss to a value which can be radiated from the external cylindrical surface of the coil without too great a temperature rise. For a magnet in continuous service a loss of about 0.7 watts can be taken care of by each square inch of the radiating surface. The length of the coil is 12 in. and the periphery is 44 in. and thus the radiating surface is $44 \times 12 = 528$ sq. in. The allowable loss is therefore $528 \times 0.7 = 370$ watts. If the voltage is 220 volts, as is usually the case, the current is $I = \frac{370}{220} = 1.7$ amp. and the resistance of the coil is $R = \frac{220}{1.7} = 130$ ohms.

The number of turns $= T = \dfrac{\text{ampere-turns}}{\text{current}} = \dfrac{7{,}480}{1.7} = 4{,}400,$ the length of the mean turn is 36 in. = 3 ft. and the length of wire is $4{,}400 \times 3 = 13{,}200$ ft.

The resistance of a wire is give by equation (131).

$$R = \rho \frac{\text{length in feet}}{\text{section in circ. mils}}$$

The specific resistance ρ may be taken as 12 for a temperature of 60°C. and substituting in the equation above the section of the conductor in circular mils may be obtained:

$$130 = 12 \times \frac{13{,}200}{\text{circ. mils}}.$$

and

$$\text{circ. mils} = \frac{12 \times 13{,}200}{130} = 1{,}220.$$

Referring to the wire table on page 85, it is seen that No. 19 wire has a section of 1,288 circ. mils and it would therefore be used.

Lifting magnets are usually made ring-shaped as shown in Fig. 67(*b*) and a large factor of safety is required on account of the varied nature of the material to be handled.

CHAPTER III

ELECTRIC CIRCUITS

78. Ohm's Law.—When an e.m.f. is applied to the terminals of a conductor, a current is produced which is directly proportional to the e.m.f. and is inversely proportional to the resistance of the conductor;

$$I = \frac{E}{R} \text{ amp.} \tag{125}$$

where I is the current in amperes,
E is the e.m.f. in volts,
R is the resistance in ohms.

This is Ohm's Law.

A conductor has a resistance of one ohm, when an e.m.f. of one volt is required to drive a current of one ampere through it.

When, therefore a current I flows through a resistance R, e.m.f. is consumed;

$$E = IR \text{ volts.} \tag{126}$$

79. Joule's Law.—Whenever a current flows through a resistance, electric energy is transformed into heat energy. The power or the rate at which energy is transformed in the circuit is equal to the product of the current and the e.m.f. consumed in driving the current through the resistance of the circuit.

$$P = EI \text{ watts,}$$

but

$$E = IR \text{ and, therefore,}$$

$$P = I^2R \text{ watts;} \tag{127}$$

thus, the power consumed in the circuit is equal to the square of the current multiplied by the resistance. This is Joule's Law.

The power consumed in the resistance of circuits represents a loss of power except in such cases as the incandescent lamp, where it is utilized in producing light, or the electric heater, where the heat developed is applied to a useful purpose.

80. Heat Units.—When a current of I amp. flows through a resistance of R ohms for a time t sec. heat is developed

$$H = I^2Rt \text{ watt-sec. of joules.} \tag{128}$$

The practical heat units are the British thermal unit and the calorie.

The British thermal unit, B.t.u., is the heat energy required to raise 1 lb. of water 1°F.

$$1 \text{ B.t.u.} = 1{,}055 \text{ watt-sec.} = 1.055 \text{ kw.-sec.} \tag{129}$$

The calorie is the heat energy required to raise 1 gram of water 1°C.

$$1 \text{ calorie} = 4.2 \text{ watt-sec.} = 0.0042 \text{ kw.-sec.} \tag{130}$$

81. Examples.—1. If electric energy costs 1½ cts. per kilowatt-hour, what will it cost to raise the temperature of 100 gal. of water 60°F.?

One gallon of water weighs 8.4 lb.

The energy required to raise 100 gal. = 840 lb. of water 60°F. is

$$840 \times 1{,}055 \times 60 \text{ watt-sec.} = \frac{840 \times 1{,}055 \times 60}{1{,}000 \times 3{,}600} \text{ kw.-hr.}$$

and the cost is

$$\frac{840 \times 1{,}055 \times 60}{1{,}000 \times 3{,}600} \times 0.015 = \$0.221 = 22.1 \text{ cts.}$$

2. If 2 tons (2,000 lb.) of coal are required per month to heat a house, what would it cost to supply the same amount of heat electrically at 1½ cts. per kilowatt-hour?

Assume 1 lb. of coal to give 10,000 B.t.u.

$$\text{Heat energy in 2 tons of coal} = 2 \times 2{,}000 \times 10{,}000 \text{ B.t.u.}$$

$$\text{The equivalent kilowatt-hours} = \frac{2 \times 2{,}000 \times 10{,}000 \times 1.055}{3{,}600}.$$

$$\text{and the cost} = \frac{2 \times 2{,}000 \times 10{,}000 \times 1.055}{3{,}600} \times 0.015 = \$175.83.$$

82. Resistance.—The resistance of a conductor varies directly as its length and inversely as its sectional area; it also depends on the material of which the conductor is made;

$$R = \rho \frac{l}{A}, \tag{131}$$

where

l is the length of the conductor,
A is the sectional area,
ρ is the specific resistance or resistivity of the material.

The specific resistance or resistivity may be expressed in ohms per centimeter cube or per inch cube, which is the volume resistivity; or in ohms per meter-gram, *i.e.*, the resistance of a uniform round wire 1 meter long weighing 1 gram, this is called the mass resistivity.

In engineering problems wires are usually specified by gage numbers, their lengths are given in feet and their sectional areas in circular mils. A circular mil is the area of a circle one mil or one-thousandth of an inch in diameter. The specific resistance is then the resistance of a wire one foot long and one circular mil in section.

83. Conductance.—The reciprocal of the resistance of a conductor is called its conductance and is represented by the letter G, where

$$G = \frac{1}{R}.$$

The reciprocal of the specific resistance or resistivity of a material is called its conductivity and is represented by the Greek letter γ, where

$$\gamma = \frac{1}{\rho}.$$

Conductivity should be expressed in per cent. of the conductivity of the International Annealed Copper Standard taken as 100 per cent. This standard is expressed in terms of mass resistivity as 0.15328 ohms (meter-gram), at the standard temperature 20°C. This is equivalent to 10.371 ohms per circ. mil foot at 20°C.

The conductivity of metals is usually decreased by the presence of impurities. Drawing or other cold working makes metals harder, stronger and slightly more dense and decreases their conductivity. The conductivity of copper wires ranges from 96 to 98 per cent. Aluminum wire has a conductivity of 60 or 61 per cent.

84. Effect of Temperature on Resistance.—The resistance of all pure metals increases with increase of temperature. The variation can be expressed by the formula

$$R_t = R_0(1 + \alpha t), \tag{132}$$

where R_o is the resistance at a chosen standard temperature, R_t is the resistance at a temperature $t°$ higher and α is the temperature coefficient of resistance. It is the increase in resist-

ance per degree rise in temperature expressed as a fraction of the resistance at the standard temperature. If the Centigrade scale is used and R_0 is the resistance at 0°C., the value of α for copper of 100 per cent. conductivity is 0.00427. This value may be considered as constant over the range of temperature from 0° to 100°C. With the Fahrenheit scale and R_0 taken as the resistance at 32°F., the value of α is $\frac{0.00427}{1.8} = 0.00237$.

If the temperature t_1°C. is taken as standard the formula may be written

$$R_t = R_{t_1}\{1 + \alpha\,(t - t_1)\}, \tag{133}$$

where α is not the same as before but is smaller because the increase of resistance is expressed as a fraction of the resistance at t_1°C. which is greater than the resistance at 0°C.

When t_1 is taken as the standard temperature 20°C. the value of α is

$$\alpha_{20} = \alpha_0 \frac{R_0}{R_{20}} = \frac{\alpha_0 R_0}{R_0\,(1 + \alpha_0 \times 20)} = \frac{1}{\frac{1}{\alpha_0} + 20}$$

$$= \frac{1}{234.5 + 20} = 0.00393.$$

Similarly, starting from this standard temperature coefficient at 20°C., the value of α at any temperature t is found as

$$\alpha_t = \alpha_{20} \frac{R_{20}}{R_t} = \frac{\alpha_{20}\,R_{20}}{R_{20}\,\{1 + \alpha_{20}\,(t - 20)\}} = \frac{1}{\frac{1}{\alpha_{20}} + t - 20}$$

$$= \frac{1}{\frac{1}{0.00393} + t - 20} = \frac{1}{234.5 + t}. \tag{134}$$

This result is correct if the conductivity is 100 per cent. When it is less than 100 per cent. the value of α must be varied proportionally. For copper of 95 per cent. conductivity

$$\alpha_t = \frac{1}{\frac{1}{0.95 \times 0.00393} + t - 20}. \tag{135}$$

If the resistance at t_1°C. is known, but the corresponding value of α_{t_1} is not known, the resistance at any other temperature t may be found from

$$R_t = R_{t_1}\left(\frac{1 + 0.00427t}{1 + 0.00427t_1}\right) = R_{t_1}\left(\frac{234.5 + t}{234.5 + t_1}\right), \text{ for 100 per cent. conductivity} \tag{136}$$

and

$$R_t = R_{t_1}\left(\frac{1 + 0.00427\gamma t}{1 + 0.00427\gamma t_1}\right) = R_{t_1}\left(\frac{234.5 + \gamma t}{234.5 + \gamma t_1}\right), \qquad (137)$$

where γ is the conductivity expressed as a decimal fraction.

If the resistance R_{t_1} at a known temperature t_1 is given and also the resistance R_t at an unknown temperature t, this temperature t may be found as follows:

$$\frac{R_t}{R_{t_1}} = \frac{R_0(1 + 0.00427t)}{R_0(1 + 0.00427t_1)} = \frac{234.5 + t}{234.5 + t_1},$$

or

$$\frac{R_t - R_{t_1}}{R_t} = \frac{t - t_1}{234.5 + t_1},$$

and

$$t - t_1 = \frac{R_t - R_{t_1}}{R_t}(234.5 + t_1). \qquad (138)$$

If the conductivity is γ,

$$t - t_1 = \frac{R_t - R_{t_1}}{R_t}\left(\frac{234.5}{\gamma} + t\right). \qquad (139)$$

85. Properties of Conductors.—The properties of the most important electrical conductors are given in the table below.

Material	Temperature coefficient	Density, grams per (cm.)^3	Resistance, ohms per (cm.)^3 at 0°C.	
Aluminum	0.00390	2.70	2.64	$\times 10^{-6}$
Copper	0.00393	8.89	1.59	
Lead	0.00387	11.36	18.40–19.60	
Magnesium	0.00381	1.69–1.75	4.10–5.00	
Mercury	0.00072	13.55	94.07	
Platinum	0.00367	21.20–21.70	9.00–15.50	
Silver	0.00377	10.40–10.60	1.50–1.70	
Tungsten	0.00500	19.30	5.50	

The temperature coefficients of all pure metals are positive and are quite large. Certain alloys such as manganin have very small temperature coefficients and are used in the design of electrical instruments where the resistance must remain constant over the ordinary range of temperature.

Carbon unlike the metals has a negative temperature coefficient and it is not constant.

The resistance of insulating materials decreases very rapidly with increase of temperature, but the variation is not regular and

cannot be expressed by a simple equation. Fig. 68 shows the variation of the resistance of slot insulation with temperature. The resistance is expressed in megohms or millions of ohms.

86. Resistance of Conductors.—The resistance of a circular mil foot of copper of 100 per cent. conductivity at 20°C. is 10.371 ohms; therefore, the resistance of a copper wire at 20°C. is, by equation (131),

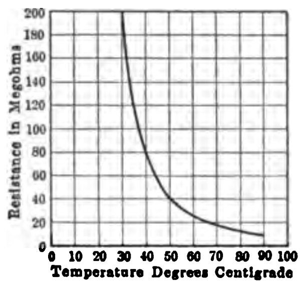


Fig. 68.—Variation of resistance of slot insulation with temperature.

$$R = 10.371 \frac{\text{length in feet}}{\text{section in circular mils.}}$$

$$= 10.371 \frac{l'}{\text{circ. mils.}} \quad (140)$$

In the case of rectangular conductors such as busbars, the section is expressed in square mils and the value of ρ is then the resistance of a square wire, one mil on each side and one foot long.

The following table gives the specific resistance of copper at various temperatures and the corresponding values of the temperature coefficient.

Temperature in degrees C.	Resistance of one circular mil foot in ohms	Resistance of one square mil foot in ohms	Temperature coefficient α
0	9.560	7.52	0.00427
15	10.160	7.98	0.00401
20	10.371	8.13	0.00393
25	10.550	8.30	0.00385
50	11.590	9.11	0.00351
75	12.620	9.85	0.00323

In this table and in wire tables generally the conductivity is assumed as 100 per cent. If the conductivity of a given wire is less than this the specific resistance must be changed accordingly; thus, the resistance of one circular mil foot of copper of 98 per cent. conductivity at 25°C. is $\frac{10.55}{0.98} = 10.76$ ohms; the resistance of one circular mil foot of aluminium of 60 per cent. conductivity at 25°C. is $\frac{10.55}{0.60} = 17.58$ ohms.

TABLE OF COPPER WIRE OF 100 PER CENT. CONDUCTIVITY

American Wire gage (B. & S.) No.	Diameter in mils at 20°C.	Section at 20°C.		Resistance, ohms per 1,000 ft. at 20°C.	Current capacity, amperes	
		Circular mils	Square inches		Rubber-covered	Other insulation
0000	460.00	211,600.0	0.1662	0.04901	225	325
000	409.60	167,800.0	0.1318	0.06180	175	275
00	364.80	133,100.0	0.1045	0.07793	150	225
0	324.90	105,500.0	0.08289	0.09827	125	200
1	289.30	83,690.0	0.06573	0.1239	100	150
2	257.60	66,370.0	0.05213	0.1563	90	125
3	229.40	52,640.0	0.04134	0.1970	80	100
4	204.30	41,740.0	0.03278	0.2485	70	90
5	181.90	33,100.0	0.02600	0.3133	55	80
6	162.00	26,250.0	0.02062	0.3951	50	70
7	144.30	20,820.0	0.01635	0.4982		
8	128.50	16,510.0	0.01297	0.6282	35	50
9	114.40	13,090.0	0.01028	0.7921		
10	101.90	10,380.0	0.008155	0.9989	25	30
11	90.74	8,234.0	0.006467	1.260		
12	80.81	6,530.0	0.005129	1.588	20	25
13	71.96	5,178.0	0.004067	2.003		
14	64.08	4,107.0	0.003225	2.525	15	20
15	57.07	3,257.0	0.002558	3.184		
16	50.82	2,583.0	0.002028	4.016	6	10
17	45.26	2,048.0	0.001609	5.064		
18	40.30	1,624.0	0.001276	6.385	3	5
19	35.89	1,288.0	0.001012	8.051		
20	31.96	1,022.0	0.0008023	10.15		
21	28.46	810.1	0.0006363	12.80		
22	25.35	642.4	0.0005046	16.14		
23	22.57	509.5	0.0004002	20.36		
24	20.10	404.0	0.0003173	25.67		
25	17.90	320.4	0.0002517	32.37		
26	15.94	254.1	0.0001996	40.81		
27	14.20	201.5	0.0001583	51.47		
28	12.64	159.8	0.0001255	64.90		
29	11.26	126.7	0.0000995	81.83		
30	10.03	100.5	0.0000789	103.2		

87. Drop of Voltage and Loss of Power in a Distributing Circuit.—The distributing circuit in Fig. 69 delivers 20 kw. at 220 volts to a receiver circuit 1,000 ft. distant; if the size of the conductors is No. 1 B. & S., determine the voltage required at the generating end of the line and the power lost in the line.

E_g is the generator voltage,
E is the receiver voltage = 220 volts,
I is the load current,
r is the resistance of each conductor.

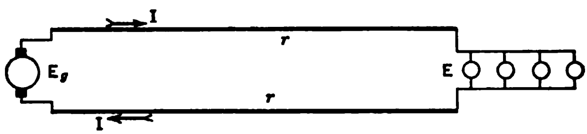


FIG. 69.—Distributing circuit.

The power delivered is

$$EI = 20{,}000 \text{ watts};$$

the current is therefore

$$I = \frac{20{,}000}{220} = 90.9 \text{ amp.};$$

the resistance of each conductor at 25°C. is

$$r = 0.126 \text{ ohms};$$

the voltage drop in each conductor is

$$e_1 = Ir = 90.9 \times 0.126 = 11.48 \text{ volts};$$

the generator voltage is therefore

$$E_g = E + 2Ir = 220 + 22.96 = 242.96 \text{ volts};$$

the drop of voltage in the circuit is

$$2e_1 = 2Ir = 22.96 \text{ volts}$$
$$= \frac{22.96}{242.96} \times 100 \text{ per cent.} = 9.45 \text{ per cent.}$$

The loss of power in the circuit is

$$2I^2r = 2 \times 90.9^2 \times 0.1261 = 2{,}100 \text{ watts};$$

the power delivered by the generator is

$$E_gI = 242.96 \times 90.9 = 22{,}100 \text{ watts};$$

therefore the power loss is

$$\frac{2{,}100}{22{,}100} \times 100 \text{ per cent.} = 9.45 \text{ per cent.};$$

and the efficiency of the transmission is 90.55 per cent.

If the drop of voltage had been limited to 10 volts what size of wire would have been required?

The drop in voltage is

$$2Ir = 2 \times 90.9 \times 10.55 \frac{1{,}000}{\text{circ. mils}} = 10 \text{ volts};$$

therefore the required section in circular mils is

$$A = 2 \times 90.9 \times 10.55 \frac{1{,}000}{10} = 192{,}000 \text{ circ. mils.}$$

88. Current-carrying Capacity of Wires.—The energy consumed in the resistance of a wire raises the temperature of the wire until the point is reached where the heat radiated and conducted from the wire is equal to the heat generated in it. When the wire is bare the heat will escape easily into the air, but when it is covered with insulating material the heat cannot escape so easily and for a given current density the temperature rise will be greater. This increase in temperature decreases the resistance of the insulating material and so decreases its insulating properties; in extreme cases the insulation may be charred and rendered useless. The last two columns of the table in Art. 86 give the values of current which can be carried safely by different sizes of wire. With rubber insulation the allowable current is about 25 per cent. less than with weatherproof insulation because the rubber is more easily affected by heat. When the insulated wires are inclosed in conduits the current-carrying capacity is less than that given in the table.

89. Examples.—**1.** If the resistance of a copper conductor at 25°C. is 10 ohms, determine its resistance at 65°C.

$$R_{65} = R_{25}\{1 + \alpha_{25}(65 - 25)\} = 10\{1 + 0.00385(65 - 25)\} = 11.55 \text{ ohms.}$$

2. Determine the resistance of a copper wire of 97 per cent. conductivity, 100,000 circ. mils in section and 50 ft. in length at 50°C.

$$\text{The resistance is } \frac{11.59}{0.97} \frac{50}{100{,}000} = 0.00597 \text{ ohms.}$$

3. Determine the resistance of an aluminum bar 0.75 in. by 0.375 in. by 100 ft. at 25°C., if the conductivity is 60 per cent.

The resistance is

$$R = \frac{8.3}{0.60} \frac{100}{750 \times 375} = 0.00492 \text{ ohms.}$$

4. If the resistance of the shunt-field winding of a generator is 30 ohms at a temperature of 25°C. and after running under load is found to be 31.5 ohms, determine the average temperature of the winding.

If t is the average temperature of the winding when hot, equation (133) gives

$$R_t = R_{25}\{1 + 0.00385(t - 25)\},$$

or substituting

$$31.5 = 30\{1 + 0.00385(t - 25)\},$$

the rise in temperature is

$$t - 25 = \frac{\frac{31.5}{30} - 1}{0.00385} = 13°\text{C}.$$

and the average temperature of the winding is

$$t = 25 + 13 = 38°\text{C}.$$

This method is used in measuring the temperature rise in the field and armature windings of electrical machines.

90. Kirchoff's Laws.—Two laws enunciated by Kirchoff are of great value in solving problems dealing with continuous-current circuits.

First Law.—The algebraic sum of all currents flowing toward or away from any junction is zero.

Second Law.—The algebraic sum of the e.m.fs. acting in a closed circuit is equal to the algebraic sum of the products of the current and resistance in the several parts of the circuit.

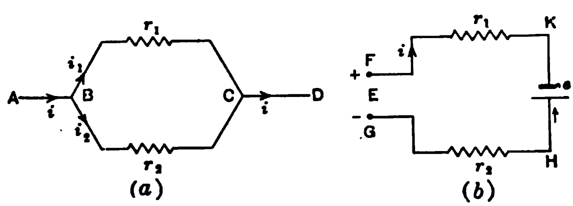


Fig. 70.—Kirchoff's laws.

The first law is illustrated by the circuit $ABCD$ in Fig. 70(a) The current i divides at B into two parts i_1 and i_2., i is numerically equal to the sum of i_1 and i_2 and therefore the algebraic sum of all currents at the junction B is

$$i - i_1 - i_2 = 0.$$

The second law is illustrated by the circuit $FGHK$ in Fig. 70(b). Between the points F and G an e.m.f. E is applied which drives a current i around the circuit in a clockwise direction against the e.m.f. e of the battery between H and K. r_1 and r_2 are the resistances of the circuit.

The e.m.f. acting in the direction of the current is $E - e$ and is consumed in driving the current i through the resistances r_1 and r_2 and by Ohm's law

$$E - e = ir_1 + ir_2.$$

91. Examples.—(*A*) Fig. 71 shows a three-wire system supplied by two similar 110-volt generators. The resistances in the various parts of the circuit are indicated. Find the currents in the three lines, assuming the direction of flow to be as indicated by the arrows.

From circuit *abgf*, $110 = i_1(r_1 + r_3 + R_1) + i_2r_4$ or $110 = 8.3i_1 + 0.4i_2$. (1)
From circuit *fgcd*, $110 = i_3(r_2 + r_5 + R_2) - i_2r_4$ or $110 = 10.3i_3 - 0.4i_2$. (2)
From circuit *abcd*, $220 = i_1(r_1 + r_3 + R_1) + i_3(R_2 + r_5 + r_2)$
or $220 = 8.3i_1 + 10.3i_3$. (3)
At the point g, $i_1 = i_2 + i_3$ (4)
Substituting (4) in (1) $110 = 8.7i_2 + 8.3i_3$ (5)
Solving (2) and (5), $i_2 = 2.4$ amp., $i_3 = 10.9$ amp. and $i_1 = i_2 + i_3 = 13.3$ amp.

(*B*) Find the currents in the various branches of the circuit in Fig. 72.

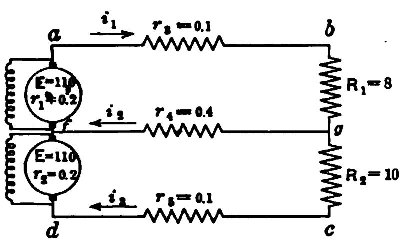


FIG. 71.

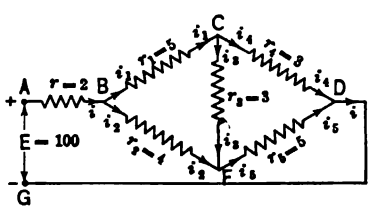


FIG. 72.

Equating the voltages around the various circuits to zero.

(*ABCDG*)	$2i + 5i_1 + 8i_4 = 100.$	(1)
(*ABFDG*)	$2i + 4i_2 + 5i_5 = 100.$	(2)
(*BCF*)	$5i_1 + 3i_3 - 4i_2 = 0.$	(3)
(*FCD*)	$3i_4 - 5i_5 - 3i_3 = 0.$	(4)
(*BCDF*)	$5i_1 + 3i_4 - 5i_5 - 4i_2 = 0.$	(5)

Equating the currents at the junction points to zero.

(*B*)	$i - i_1 - i_2 = 0$, or $i = i_1 + i_2.$	(6)
(*C*)	$i_1 - i_3 - i_4 = 0$, or $i_4 = i_1 - i_3.$	(7)
(*D*)	$i - i_4 - i_5 = 0$, or $i = i_4 + i_5.$	(8)
(*F*)	$i_2 + i_3 - i_5 = 0$, or $i_5 = i_2 + i_3.$	(9)

Substituting (7) and (9) in (4),

$$3(i_1 - i_3) - 5(i_2 + i_3) - 3i_3 = 0, \text{ or } 3i_1 - 5i_2 - 11i_3 = 0. \quad (10)$$

Substituting (7) and (9) in (5),

$$5i_1 + 3(i_1 - i_3) - 5(i_2 + i_3) - 4i_2 = 0, \text{ or } 8i_1 - 9i_2 - 8i_3 = 0. \quad (11)$$

Solving (3), (10) and (11) simultaneously,

$$i_1 = 6.77 \text{ amp.}, \; i_2 = 8.66 \text{ amp. and } i_3 = -1.76 \text{ amp.}$$

From (6), $i = 15.43$; from (7), $i_4 = 8.53$; from (9), $i_5 = 6.90$.

92. Resistances in Series.—If a voltage E is applied across a circuit consisting of a number of resistances R_1, R_2 and R_3 connected in series (Fig. 73) the total resistance of the circuit will be equal to the sum of the resistances of the different parts.

The drop of voltage across the resistance R_1 is

$$E_1 = IR_1$$

where I is the current in the circuit.

Similarly

$$E_2 = IR_2,$$

and

$$E_3 = IR_3,$$

but

$$E = E_1 + E_2 + E_3$$
$$= I(R_1 + R_2 + R_3),$$

and the resistance of the whole circuit is

$$R = \frac{E}{I} = R_1 + R_2 + R_3. \tag{141}$$

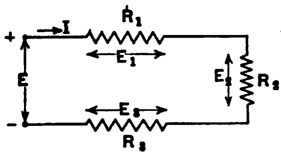


FIG. 73.—Resistances in series.

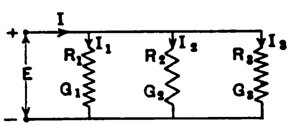


FIG. 74.—Resistances in parallel.

93. Resistances in Parallel.—Determine the resistance of a circuit consisting of a number of resistances R_1, R_2 and R_3 (Fig. 74) connected in parallel.

If an e.m.f E is applied to the circuit a current I will flow which will divide up into branch currents I_1, I_2 and I_3. By Ohm's law

$$I_1 = \frac{E}{R_1},$$
$$I_2 = \frac{E}{R_2},$$
$$I_3 = \frac{E}{R_3},$$

and

$$I = \frac{E}{R},$$

where R is the resistance of the whole circuit,

but

$$I = I_1 + I_2 + I_3$$
$$= E\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right),$$

therefore,

$$R = \frac{E}{E\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)}$$
$$= \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}. \quad (142)$$

If the conductances G, G_1, G_2 and G_3 are used instead of the terms $\frac{1}{R}$, $\frac{1}{R_1}$, $\frac{1}{R_2}$, and $\frac{1}{R_3}$ the result can be written

$$R = \frac{1}{G} = \frac{1}{G_1 + G_2 + G_3},$$

or

$$G = G_1 + G_2 + G_3; \quad (143)$$

that is, the conductance of a circuit consisting of a number of parallel branches is equal to the sum of the conductances of the branches.

94. Potentiometer.—The potentiometer shown diagrammatically in Fig. 75 may be used to obtain a variable voltage from a constant voltage supply in a continuous-current system. By changing the position of the point B any voltage E_{AB} from zero to the value of the constant supply voltage E may be applied to the load resistance R_1.

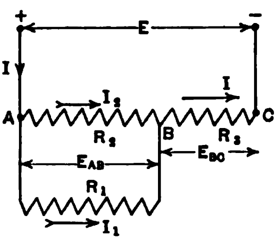


FIG. 75.—Potentiometer.

The resistance of the potentiometer from A to B is R_2 and from B to C is R_3. The various currents are indicated in the figure.

The resistance of the circuit between A and B is

$$R_{AB} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}};$$

the resistance between A and C is

$$R_{AC} = R_{AB} + R_{BC} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} + R_3 = \frac{R_1R_2}{R_1 + R_2} + R_3,$$

and the current from the mains is

$$I = \frac{E}{R_{AC}} = \frac{E}{\frac{R_1R_2}{R_1 + R_2} + R_3};$$

the drop of potential from B to C is

$$E_{BC} = IR_3 = \frac{ER_3}{\frac{R_1R_2}{R_1 + R_2} + R_3};$$

and the drop across the load circuit is

$$E_{AB} = E - E_{BC} = E - \frac{ER_3}{\frac{R_1R_2}{R_1 + R_2} + R_3} = E\frac{R_1R_2}{R_1R_2 + R_2R_3 + R_3R_1}$$

There is a continuous waste of power in the resistances R_2 and R_3 and the efficiency of the arrangement is always less than the ratio of the variable voltage E_{AB} to the constant voltage E.

95. Inductance.—When a current of electricity flows in a circuit, lines of magnetic induction interlink with the circuit; if the current remains constant the flux threading the circuit is constant, but when the current varies the flux varies proportionally and in doing so generates in the circuit an e.m.f. proportional to the rate of change of the interlinkages of flux and turns or to the product of the turns and the rate of change of the flux. This e.m.f. has a value

$$e = -n\frac{d\phi}{dt} \text{ c.g.s. units,} \tag{144}$$

where n is the number of turns in the circuit and $\frac{d\phi}{dt}$ is the rate of change of the flux interlinking with the turns. The negative sign is used because the e.m.f. produced opposes the change in the current and, therefore, the change in the flux.

The inductance of a circuit is defined as the rate of change of the flux interlinkages with the current in the circuit and is represented by $\mathfrak{L}$, thus,

$$\mathfrak{L} = n\frac{d\phi}{di} \text{ c.g.s. units.}$$

This quantity is not a constant except in circuits which do not contain any magnetic materials and in which therefore the flux varies directly with the current.

In such circuits the inductance may be defined as the number

of interlinkages of flux and turns for unit current in the circuit, or,

$$\mathfrak{L} = \frac{n\phi}{i} \text{ c.g.s. units,} \tag{145}$$

where i = current in c.g.s. units,
ϕ = flux produced by current i,
$n\phi$ = interlinkages for current i.

Since $\mathfrak{L} = n\frac{d\phi}{di} = n\frac{d\phi}{dt}\Big/\frac{di}{dt}$, the e.m.f. generated in a circuit due to its inductance is

$$e = -n\frac{d\phi}{dt} = -\mathfrak{L}\frac{di}{dt} \text{ c.g.s. units,} \tag{146}$$

It is equal to the product of the inductance of the circuit and the rate of change of the current. When e is expressed in volts and i in amperes the inductance is in henrys and is represented by L to distinguish it from the inductance $\mathfrak{L}$ expressed in c.g.s. units. Equation (146) may then be written

$$e = -L\frac{di}{dt} \text{ volts.} \tag{147}$$

The henry is defined as the inductance of a circuit in which a counter e.m.f. of one volt is generated when the current is changing at the rate of one ampere per second.

The relation between the two units of inductance is found as follows:

$$\begin{aligned} e &= -\mathfrak{L}\frac{di}{dt} \text{ c.g.s. units,} \\ &= -\mathfrak{L}\frac{di}{dt}10^{-8} \text{ volts, } \mathfrak{L} \text{ and } i \text{ in c.g.s. units,} \\ &= -\mathfrak{L}\frac{di}{dt}10^{-9} \text{ volts, } \mathfrak{L} \text{ in c.g.s. units, } i \text{ in amperes,} \\ &= -L\frac{di}{dt} \text{ volts,} \end{aligned}$$

and, therefore,

$$1 \text{ henry} = 10^9 \text{ c.g.s. units of inductance.} \tag{148}$$

96. Example.—(a) Find the inductance of an endless solenoid in the form of a ring, Fig. 53.

n = number of turns in solenoid = 1,000.
r = mean radius of the ring = 10 cm.
$l = 2\pi r$ = mean length of the flux path through the solenoid.
A = sectional area of the solenoid = 5 sq. cm.
i = current in solenoid in c.g.s. units.

The flux through the solenoid is

$$\Phi = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{4\pi ni}{l/A}\text{ lines,}$$

the flux per unit current is

$$\phi_1 = \frac{\Phi}{i} = \frac{4\pi nA}{l},$$

the number of interlinkages of flux and turns per unit current is

$$\mathcal{L} = n\phi_1 = \frac{4\pi n^2 A}{l}$$

and is equal to the inductance of the circuit in c.g.s. units; thus, the inductance of a circuit is proportional to the square of the number of turns.

Substituting the values given above the inductance of the solenoid is

$$\mathcal{L} = \frac{4 \times 3.14 \times (1{,}000)^2 \times 5}{2 \times 3.14 \times 10} = 10^6 \text{ c.g.s. units;}$$

and the inductance in practical units is

$$L = 10^6 \times 10^{-9} = 0.001 \text{ henry.}$$

(*b*) Find the energy stored in the magnetic field of the solenoid when the current has reached a value $I = 10$ c.g.s. units.

The energy stored when any current i is flowing is equal to the work done in building up the current i against the counter e.m.f. of inductance $e = -\mathcal{L}\frac{di}{dt}$; it is therefore

$$W = \int ei\,dt = \int \mathcal{L}\frac{di}{dt}i\,dt = \mathcal{L}\int_0^I i\,di$$
$$= \left[\mathcal{L}\frac{i^2}{2}\right]_0^I \text{ ergs.}$$

When the current is $i = I = 10$ c.g.s. units, the energy stored is

$$W = \mathcal{L}\frac{I^2}{2} = 10^6 \times \frac{10^2}{2} = 5 \times 10^7 \text{ ergs}$$
$$= 5 \times 10^7 \times 10^{-7} = 5 \text{ watt-sec.}$$

97. Inductance of Circuits Containing Iron.—As stated in Art. 95, the inductance of circuits containing magnetic materials is not constant, since the flux does not vary directly with the current. The relation between flux and current may be represented by a magnetization curve or saturation curve such as those shown in Art. 72, but these curves cannot be represented by mathematical equations and for engineering calculations it is necessary to choose some constant average value to represent the inductance of a circuit, that is, to assume that the material has a constant permeability μ and then to apply a correction to the result.

If a solenoid of n turns is wound on an iron ring of section A

sq. cm. and mean length l cm. and permeability μ, the flux produced by a current i c.g.s. units is

$$\phi = \frac{4\pi ni}{\frac{l}{A\mu}} \text{ lines,}$$

the flux per unit current is

$$\phi_1 = \frac{\phi}{i} = \frac{4\pi nA\mu}{l} \text{ c.g.s. units,}$$

and the inductance is

$$\mathfrak{L} = n\phi_1 = \frac{4\pi n^2 A\mu}{l} \text{ c.g.s. units.}$$

It is proportional to the square of the number of turns, as seen in the last example, but it also depends on the permeability of the material and is therefore not a constant quantity.

When the current in the solenoid is small, the iron is unsaturated, the permeability is high, the flux per unit current is large and therefore the inductance is large. When the current is large, the iron is saturated, the permeability is low, the flux per unit current is small and therefore the inductance is small.

Thus the inductance of circuits containing iron or other magnetic materials is not constant but, above a certain point, decreases as the current increases and the induction density increases.

The permeability curves in Figs. 64 and 65 show the relation between permeability and induction density for cast iron and cast steel.

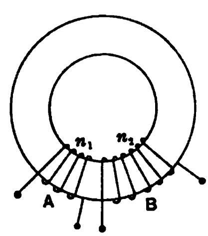


FIG. 76.—Mutual inductance.

98. Mutual Inductance and Self-inductance.—If two electric circuits as A and B in Fig. 76 are interlinked with a magnetic circuit of constant permeability μ, or constant magnetic reluctance $\mathfrak{R}$, the mutual inductance of one circuit with respect to the second circuit may be defined as the number of interlinkages of the second circuit with the flux produced by unit current in the first circuit. It is equal to the mutual inductance of the second circuit with respect to the first and is represented by $\mathfrak{M}$.

The flux produced by unit current in the n_1 turns of the circuit A is

$$\phi_1 = \frac{4\pi n_1}{\mathfrak{R}}$$

and the inductance of A is

$$\mathfrak{L}_1 = n_1\phi_1 = \frac{4\pi n_1^2}{\mathfrak{R}} \text{ c.g.s. units;} \tag{149}$$

if all the flux produced by A passes through B, then the mutual inductance of A with respect to B is

$$\mathfrak{M}_{AB} = n_2\phi_1 = \frac{4\pi n_1 n_2}{\mathfrak{R}} \text{ c.g.s. units.} \tag{150}$$

Similarly, the flux produced by unit current in B is

$$\phi_2 = \frac{4\pi n_2}{\mathfrak{R}}$$

and the inductance of B is

$$\mathfrak{L}_2 = n_2\phi_2 = \frac{4\pi n_2^2}{\mathfrak{R}};$$

again if all the flux produced by B passes through A, the mutual inductance of B with respect to A is

$$\mathfrak{M}_{BA} = n_1\phi_2 = \frac{4\pi n_1 n_2}{\mathfrak{R}},$$

and is equal to the mutual inductance of A with respect to B,

$$\mathfrak{M}_{BA} = \mathfrak{M}_{AB} = \mathfrak{M}.$$

The leakage flux, that is, the flux which passes out between the two circuits and links with only one of them, has been assumed to be very small. In this case

$$\mathfrak{L}_1\mathfrak{L}_2 = \frac{4\pi n_1^2}{\mathfrak{R}} \times \frac{4\pi n_2^2}{\mathfrak{R}} = \left(\frac{4\pi n_1 n_2}{\mathfrak{R}}\right)^2 = \mathfrak{M}^2. \tag{151}$$

If the leakage flux is not negligible, the flux ϕ, produced by unit current in A can be divided into two parts ϕ_{M_1} and ϕ_{S_1}, Fig. 77, of which ϕ_{M_1} interlinks with B and ϕ_{S_1} forms a local leakage circuit about A and does not interlink with B; ϕ_{S_1} is the self-inductive flux produced by unit current in A.

The self-inductance of a circuit is the number of interlinkages with the circuit of the flux, produced by unit current in it, which does not interlink with any other circuit; it is the number of interlinkages of the circuit with the stray flux or the leakage flux produced by unit current in itself.

$$\begin{aligned}
n_1\phi_1 &= \mathfrak{L}_1 = \text{inductance of } A,\\
n_2\phi_{M1} &= \mathfrak{M} = \text{mutual inductance of } A \text{ with respect to } B,\\
n_1\phi_{S_1} &= \mathfrak{L}_{S1} = \text{self-inductance of } A.
\end{aligned}$$

But

$$\phi_1 = \phi_{M_1} + \phi_{S_1}, \text{ and therefore}$$
$$\frac{\mathcal{L}_1}{n_1} = \frac{\mathfrak{M}}{n_2} + \frac{\mathcal{L}_{S1}}{n_1};$$

and similarly,

$$\frac{\mathcal{L}_2}{n_2} = \frac{\mathfrak{M}}{n_1} + \frac{\mathcal{L}_{S2}}{n_2}.$$

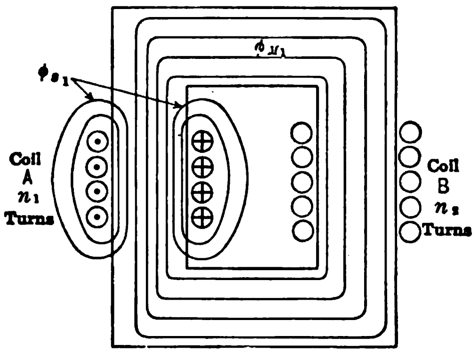


FIG. 77.—Self and mutual inductance.

When any magnetic materials are present the reluctance of the magnetic circuit is not constant but varies with the flux density, that is, with the current and therefore the inductances, the mutual inductance and the self-inductances are all variable quantities.

For any value of current i_1 in A producing a flux ϕ_M linking B the rate of change of flux generates in B an e.m.f. which may be expressed as

$$e_{AB} = n_2 \frac{d\Phi_M}{dt} = \mathfrak{M}_{AB} \frac{di_1}{dt} \tag{152}$$

and the mutual inductance of A with respect to B is

$$\mathfrak{M}_{AB} = n_2 \frac{d\Phi_M}{di_1}. \tag{153}$$

$\frac{d\Phi_M}{di_1}$ can, however, only be evaluated on the assumption that the flux is proportional to the current or that the reluctance of the magnetic circuit is constant. In practical applications allowance must be made for the errors resulting from this assumption.

99. Self-inductance of Continuous-current Circuits.—The self-inductance of continuous-current circuits is apparent only when the current is increasing or decreasing. The two most im-

portant cases are when the current is starting and when it is stopping.

Starting of Current.—When a constant e.m.f. E is impressed on a circuit of resistance R and inductance L the current does not immediately reach a steady value on account of the opposing e.m.f. due to inductance. If at time t after E is impressed the current is changing at the rate $\frac{di}{dt}$; the e.m.f. of inductance is

$$e_b = -L\frac{di}{dt}.$$

By Lenz's law it opposes the impressed e.m.f. and is therefore negative.

The e.m.f. acting on the circuit is

$$E + e_b = E - L\frac{di}{dt},$$

and the current is

$$i = \frac{E - L\frac{di}{dt}}{R}$$

therefore

$$i - \frac{E}{R} = -\frac{L}{R}\frac{di}{dt}$$

or, transposing,

$$\frac{di}{i - \frac{E}{R}} = -\frac{Rdt}{L};$$

the integral of this is

$$\log_\epsilon\left(i - \frac{E}{R}\right) = -\frac{Rt}{L} + \log_\epsilon C,$$

where $\log_\epsilon C$ is the constant of integration. This reduces to

$$i - \frac{E}{R} = C\epsilon^{-\frac{Rt}{L}},$$

or

$$i = \frac{E}{R} + C\epsilon^{-\frac{Rt}{L}},$$

when $t = 0$, $i = 0$, and $C = -\frac{E}{R}$.

Substituting this value the current is

$$i = \frac{E}{R}\left(1 - \epsilon^{-\frac{Rt}{L}}\right). \qquad (154)$$

When $t = \infty$ and $i = \frac{E}{R}$, and the current has reached its steady value, which is independent of the inductance of the circuit; calling this value I and substituting

$$i = I\left(1 - \epsilon^{-\frac{Rt}{L}}\right). \tag{155}$$

The expression $\frac{L}{R}$ is called the time constant of the circuit.

Stopping of Current.—If the impressed e.m.f. E is removed from a circuit of resistance R and inductance L, when it is carrying a current $I = \frac{E}{R}$, and the circuit is closed through a resistance R_1, the current will not at once fall to zero.

The e.m.f. of inductance at time t after the circuit is closed is

$$e_b = -L\frac{di}{dt},$$

and it opposes the decrease of current.

The current in the circuit at this instant is

$$i = -\frac{L\frac{di}{dt}}{R + R_1},$$

or, transposing,

$$\frac{di}{i} = -\frac{(R + R_1)}{L}dt,$$

the integral of this is

$$\log_\epsilon i = -\frac{R + R_1}{L}t + \log C,$$

where $\log C$ is the constant of integration. This reduces to

$$i = C\epsilon^{-\frac{R + R_1}{L}t},$$

when $t = 0$, $i = I$, and $C = I$.

Substituting this value, the current is

$$i = I\epsilon^{-\frac{R + R_1}{L}t}; \tag{156}$$

or

$$i = \frac{E}{R}\epsilon^{-\frac{R + R_1}{L}t}. \tag{157}$$

The e.m.f. generated in the coil, at the time t, is

$$e_b = i(R + R_1) = E\frac{R + R_1}{R}\epsilon^{-\frac{R + R_1}{L}t}. \tag{158}$$

When $t = 0$, the generated e.m.f. is

$$e_b = E\frac{R + R_1}{R}; \tag{159}$$

therefore the e.m.f. generated in the circuit is greater than the impressed e.m.f. in the ratio $\frac{R + R_1}{R}$.

If, when the impressed e.m.f. is removed, the circuit is short-circuited, $R_1 = 0$ and the generated e.m.f. is

$$e_b = E,$$

that is, the e.m.f. does not rise.

But if the circuit is broken $R_1 = \infty$ and $e_b = \infty$, and dangerous e.m.fs. are generated in the circuit. A large value of R_1 causes the current to decrease rapidly, but it also causes a high e.m.f. to be generated.

The energy supplied to the circuit while the current is starting is

$$W = \int_0^{\infty} Ei\,dt,$$

but

$$E = Ri + L\frac{di}{dt};$$

therefore,

$$W = \int_0^{\infty}\left(Ri + L\frac{di}{dt}\right) i\,dt = \int_0^{\infty} Ri^2\,dt + \int_0^{I} Li\,di;$$

of this $\int_0^{\infty} Ri^2\,dt$ is the energy transformed into heat in the electric circuit, and $\int_0^{I} Li\,di$ is the energy stored in the magnetic field.

The energy in the magnetic field is

$$W_M = \int_0^{I} Li\,di = L\frac{I^2}{2}$$

and is independent of the resistance of the circuit.

This magnetic energy is returned to the electric circuit while the current is stopping and prevents it from immediately falling to zero. It is transformed into heat in the circuit and is

$$W = \int_0^{\infty} i^2\,(R + R_1)\,dt,$$

but

$$i = Ie^{-\frac{R + R_1}{L}t}.$$

therefore,

$$W = I^2 (R + R_1) \int_0^\infty e^{-2\frac{(R+R_1)}{L}t}\,dt;$$

the integral of this is

$$W = I^2 (R + R_1) \left\{ - \frac{L}{2(R + R_1)} \right\} \left[e^{-2\left(\frac{R+R_1}{L}\right)t} \right]_0^\infty,$$

and

$$W = L\frac{I^2}{2} \text{ as before.}$$

If the circuit is broken this amount of energy must be discharged and produces a spark.

100. Example.—The shunt field winding of a 6-pole, 125-volt, direct-current generator has 500 turns per pole and carries 12.5 amp.; the resistance of the field winding is 7.5 ohms and the resistance of the field rheostat is $\frac{125}{12.5} - 7.5 = 2.5$ ohms; and the flux per pole is 6.3×10^6 lines; determine:

(*a*) the inductance of the field winding;

(*b*) the time taken by the current to reach one-tenth full value and one-half full value, after the field switch is closed;

(*c*) the initial rise of voltage, if the field circuit carrying a current of 12.5 amp. is closed through a resistance of 25 ohms;

(*d*) the energy stored in the magnetic field when the current is 12.5 amp.

(*a*) The interlinkages of flux and turns with a current of 12.5 amp. or 1.25 c.g.s. units is

$$6 \times 500 \times 6.3 \times 10^6;$$

therefore the inductance is

$$\mathfrak{L} = \frac{6 \times 500 \times 6.3 \times 10^6}{1.25} = 15.1 \times 10^9 \text{ c.g.s. units,}$$

or

$$L = 15.1 \text{ henrys.}$$

(*b*) At time t after the field switch is closed the current is, by equation (155),

$$i = I\left(1 - \epsilon^{-\frac{Rt}{L}}\right),$$

where I is the final value of the current = 12.5 amp. and R is the resistance of the field circuit including the rheostat = 10 ohms; therefore,

$$i = 12.5\left(1 - \epsilon^{-\frac{10t}{15.1}}\right).$$

Let time for the current to reach one-tenth of full value or 1.25 amp. be t_1 sec., then

$$1.25 = 12.5\,(1 - \epsilon^{-0.66t_1})$$

and

$$t_1 = 0.16 \text{ sec.}$$

Let time for the current to reach one-half value or 6.25 amp. be t_2 sec., then

$$6.25 = 12.5\,(1 - \epsilon^{-0.66t_2})$$

and

$$t_2 = 1.05 \text{ sec.}$$

(c) If the field circuit carrying a current of 12.5 amp. is closed through a resistance of 25 ohms, the initial value of the back voltage due to inductance is, by equation (159),

$$e_b = 125\frac{10 + 25}{10} = 437.5 \text{ volts.}$$

(d) When the current is 12.5 amp. the energy stored in the magnetic field is

$$W = L\frac{I^2}{2} = 15.1 \times \frac{(12.5)^2}{2} = 1{,}460 \text{ watt-sec.}$$

101. Inductance of Parallel Conductors.—Determine the inductance of the circuit in Fig. 78 consisting of two parallel wires A and B carrying equal currents but in opposite directions.

R = radius of each wire in centimeters.
D = distance between centers in centimeters.

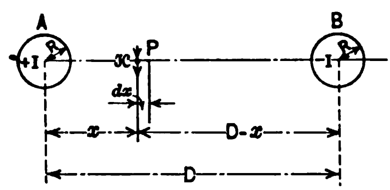


FIG. 78.—Inductance of parallel wires.

When a current I c.g.s. units flows in the wires the field intensity at the point P distant x cm. from A and $D - x$ cm. from B is

$$\mathcal{H} = \frac{2I}{x} + \frac{2I}{D - x} \text{ dynes, by equation (104),}$$

and the flux density is

$$\mathcal{B} = \mathcal{H} = \frac{2I}{x} + \frac{2I}{D - x} \text{ lines per square centimeter;}$$

the flux in a section of width dx and length 1 cm. is

$$d\phi = \mathcal{B}\, dx \text{ lines.}$$

The flux between the wires per centimeter length produced by the current in one wire is

$$\phi_1 = \int_R^D \frac{2I}{x}\, dx = 2I\,[\log x]_R^D = 2I \log \frac{D}{R},$$

and the flux of self-inductance of the circuit per centimeter length is

$$\phi = 2\phi_1 = 4I \log \frac{D}{R},$$

but only half of this flux surrounds each wire and therefore the inductance of each wire per centimeter length is

$$L_1 = \frac{\frac{\phi}{2}}{I} = 2 \log \frac{D}{R} \text{ c.g.s. units.} \tag{160}$$

The flux inside the conductor has been neglected, but its inductive effect can be calculated very easily if it is assumed that the current is distributed uniformly over the section of the wire. In Fig. 79 the section of wire A is shown enlarged.

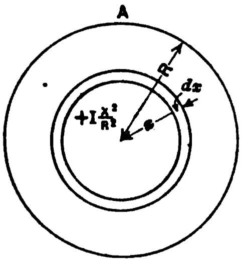


FIG. 79.—Flux inside a conductor.

The flux density at distance x cm. from the center of A is

$$\mathfrak{B} = \frac{2I'}{x},$$

where I' is the current inside the radius x cm. and its value is

$$I' = I \frac{x^2}{R^2} \text{ c.g.s. units.}$$

The flux in the ring of radius x, width dx and length 1 cm. is

$$d\phi' = \frac{2I'}{x} dx = \frac{2Ix}{R^2} dx \text{ lines;}$$

this flux surrounds only the current I' and is equivalent to a smaller flux surrounding the current I, of value

$$d\phi = d\phi' \frac{x^2}{R^2} = \frac{2Ix^3}{R^4} dx,$$

and the flux equivalent to the flux inside the whole section is

$$\phi_2 = \int_0^R d\phi = \int_0^R \frac{2I}{R^4} x^3 \, dx$$
$$= \frac{2I}{R^4} \left[\frac{x^4}{4}\right]_0^R = \frac{I}{2} \text{ lines,}$$

and the inductance per centimeter due to the flux inside the conductor is

$$\mathfrak{L}_2 = \frac{\phi_2}{I} = \frac{1}{2} \text{ c.g.s. units.} \tag{161}$$

The inductance of each wire per centimeter is therefore

$$\mathfrak{L} = \mathfrak{L}_1 + \mathfrak{L}_2 = 2 \log \frac{D}{R} + \frac{1}{2} \text{ c.g.s. units,} \tag{162}$$

and the inductance per mile of wire is

$$L = 2.54 \times 12 \times 5{,}280 \left(2 \times 2.303 \log_{10} \frac{D}{R} + 0.5\right) 10^{-9} \text{ henrys}$$
$$= \left(0.74 \log_{10} \frac{D}{R} + 0.0805\right) 10^{-3} \text{ henrys.} \tag{163}$$

CHAPTER IV

ELECTRIC CIRCUITS (*Continued*)

ALTERNATING-CURRENT CIRCUITS

102. The Sine Wave of Electromotive Force and Current.—If the coil in Fig. 80 rotates with a constant angular velocity in a uniform magnetic field between the poles N and S, an alternating e.m.f. will be generated between the terminals t and t'. Referring to the figure it can be seen that in position (1) the e.m.f. generated in the coil is zero, since the conductors forming it are moving parallel to the lines of magnetic flux and therefore are not cutting them; in position (2) it is positive and increasing; in

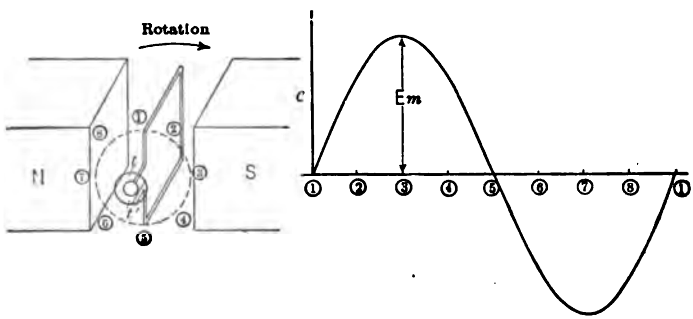


FIG. 80.—Generation of an alternating e.m.f.

(3) it is positive and has reached its maximum value since the conductors are cutting perpendicularly across the flux; in (4) it is positive and decreasing; in (5) it is zero; in (6) it is negative and increasing; in (7) negative and maximum; in (8) negative and decreasing and in (1) is zero again, having gone through one complete cycle. This cycle is represented in Fig. 80.

If Φ = the maximum flux inclosed by the coil, n = the number of turns on the coil and ω = the angular velocity in radians per second, then at time t sec. after the position of maximum inclosure the coil has moved through angle $\theta = \omega t$ radians, and the flux inclosed is

$$\phi = \Phi \cos \omega t;$$

the e.m.f. generated in the coil at this instant is

$$
\begin{aligned}
e &= -n\frac{d}{dt}(\Phi\cos\omega t)\\
&= \omega n\Phi\sin\omega t \text{ absolute units}\\
&= \omega n\Phi\,10^{-8}\sin\omega t \text{ volts.}
\end{aligned}
\tag{164}
$$

When $\theta = \frac{\pi}{2}$, the e.m.f. has its maximum value

$$E_m = \omega n\Phi\,10^{-8} \text{ volts,}$$

and therefore

$$e = E_m \sin\omega t; \tag{165}$$

and the e.m.f. generated in the coil varies as a sine wave.

The number of cycles through which the e.m.f. passes in one second is called its frequency and since one cycle represents 360 electrical degrees, the frequency may be expressed as

$$f = \frac{\omega}{2\pi} \text{ cycles,} \tag{166}$$

and therefore

$$\omega = 2\pi f.$$

Substituting this value of ω in equation (164) gives

$$
\begin{aligned}
e &= 2\pi f n\Phi\,10^{-8}\sin 2\pi ft\\
&= E_m \sin\theta,
\end{aligned}
$$

where $\theta = 2\pi ft$ is the angle turned through in time t sec. after the position of zero e.m.f.; the maximum value of the e.m.f. is

$$E_m = 2\pi f n\Phi\,10^{-8} \text{ volts.} \tag{167}$$

In a two-pole alternating-current generator one cycle corresponds to one revolution of the coil and

$$f = \text{rev. per sec.,}$$

and one electrical space degree is equal to one mechanical space degree. With a p-pole alternator the e.m.f. passes through one cycle for each pair of poles and the frequency is

$$f = \frac{p}{2} \text{ rev. per sec.,}$$

and one electrical space degree is less than one mechanical space degree in the ratio $\frac{2}{p}$.

If a non-inductive resistance of R ohms is connected across the terminals of the coil in Fig. 81 an alternating current will flow in the coil of instantaneous value

$$
\begin{aligned}
i &= \frac{e}{R} = \frac{E_m\sin\theta}{R}\\
&= I_m\sin\theta,
\end{aligned}
$$

where $I_m = \frac{E_m}{R}$ is the maximum value of the current. The current and voltage waves pass through their zero and maximum values together and are therefore in phase. They are represented by the two curves in Fig. 81.

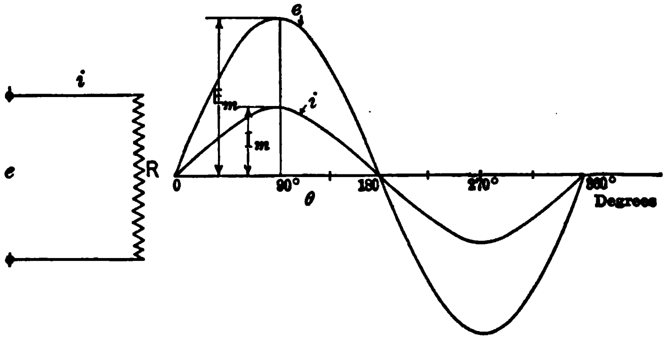


FIG. 81.—Resistance in alternating-current circuits.

103. The Average Value of a Sine Wave.—The average value of the ordinate of a sine wave $i = I_m \sin \theta$ can be found by integrating over one-half wave; it is

$$\begin{aligned} I_{\text{avg.}} &= \frac{1}{\pi}\int_0^\pi I_m \sin \theta \, d\theta \\ &= \frac{I_m}{\pi}\Big[-\cos \theta\Big]_0^\pi \\ &= \frac{I_m}{\pi}[1 + 1] = \frac{2}{\pi} I_m = 0.637 I_m, \end{aligned} \tag{169}$$

that is, the average value of the ordinate of a sine curve is $\frac{2}{\pi}$ times the maximum ordinate.

104. The Effective Value of a Sine Wave.—The effective value of an alternating current is the value of continuous current which would have the same heating effect.

When an alternating current $i = I_m \sin \theta$ flows through a resistance R, energy is transformed into heat at the instantaneous rate i^2R watts; the average rate of transformation of energy is

$$\begin{aligned} \frac{1}{\pi}\int_0^\pi i^2R \, d\theta &= \frac{1}{\pi}\int_0^\pi I_m{}^2R \sin^2 \theta \, d\theta \\ &= \frac{I_m{}^2R}{2\pi}\int_0^\pi (1 - \cos 2\theta) \, d\theta \end{aligned}$$

$$= \frac{I_m{}^2R}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^\pi$$

$$= \frac{I_m{}^2R}{2\pi}[\pi]$$

$$= \frac{I_m{}^2R}{2} = \left(\frac{I_m}{\sqrt{2}}\right)^2 R = I^2R; \qquad (170)$$

thus the alternating current $i = I_m \sin \theta$ has the same average heating effect or consumes the same average power as a continuous current $\frac{I_m}{\sqrt{2}}$; the value $I = \frac{I_m}{\sqrt{2}}$ is therefore called the effective value of the alternating current $i = I_m \sin \theta$

The effective value of the alternating e.m.f. $e = E_m \sin \theta$ is

$$E = \frac{E_m}{\sqrt{2}}. \qquad (171)$$

The effective value of any alternating quantity is the square root of the mean of the squared instantaneous values taken over one complete cycle; it is equal to the maximum value divided by $\sqrt{2}$ only in the case of sine waves.

Alternating-current voltmeters and ammeters indicate the effective values of voltage and current regardless of the wave form.

105. Inductance in Alternating-current Circuits.—When a current flows in a conductor a magnetic field is produced in the space surrounding it; as long as the current remains constant this field does not react on the electric circuit, but when the current varies the flux linking with the circuit also varies and induces in the conductor an e.m.f. opposing the change in the current and consequently the change in the flux. This action is due to the inertia of the magnetic field and is analogous to the action of the flywheel in mechanics. The inertia of the flywheel opposes any change in speed just as the inertia of the magnetic field opposes any change in current. Energy is stored in the flywheel as the speed increases and is given back as it decreases and the only loss of energy is that due to friction. Similarly energy is stored in the magnetic field as the current increases and is returned to the electric circuit as the current decreases, and the only loss of energy is that due to hysteresis and eddy currents in the iron parts of the magnetic circuit.

The energy stored in the flywheel is

$$W = I\frac{\omega^2}{2}, \tag{172}$$

where I is its moment of inertia and ω is its angular velocity.

The energy stored in the magnetic field is

$$W = \frac{Li^2}{2} \text{ watt-sec.,}$$

where L is the inductance of the circuit in henrys and i is the current in amperes.

The inductance of the coil opposes the change in current by generating a back e.m.f.

$$e_b = -L\frac{di}{dt}.$$

If an alternating current $i = I_m \sin\theta = I_m \sin 2\pi ft$ is flowing through a circuit of inductance L henrys and negligible resistance an e.m.f. of inductance will be set up

$$\begin{aligned} e_b &= -L\frac{di}{dt} \\ &= -L\frac{d}{dt}(I_m \sin 2\pi ft) \\ &= -2\pi fLI_m \cos 2\pi ft. \\ &= 2\pi fLI_m \sin(2\pi ft. - 90); \end{aligned} \tag{173}$$

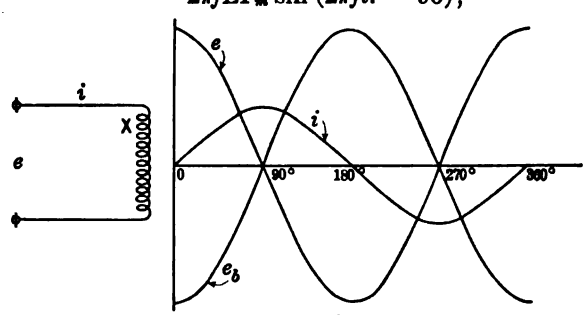


FIG. 82.—Inductance in alternating-current circuits.

it is a sine wave of e.m.f. lagging behind the current wave by 90 degrees (Fig. 82).

In order to drive the current through the circuit an e.m.f. must be applied to the terminals, which at every instant will be equal and opposite to the back e.m.f. e_b; its instantaneous value is

$$e = -e_b = 2\pi fLI_m \sin(2\pi ft + 90), \tag{174}$$

a sine wave of e.m.f. leading the current wave by 90 degrees.

This e.m.f. is consumed by the inductance of the circuit. Its maximum value is

$$E_m = 2\pi f L I_m, \tag{175}$$

where the term $2\pi fL$ is called the inductive reactance of the circuit and is denoted by X. It is of the nature of resistance and is expressed in ohms.

Thus, in a circuit of reactance X and negligible resistance the impressed e.m.f. is 90 degrees ahead of the current, or the current lags 90 degrees behind the impressed e.m.f.

106. Resistance and Reactance in Series.—If a circuit contains a resistance R and a reactance X in series and carries an alternating current $i = I_m \sin 2\pi ft$, determine the value of the impressed e.m.f. and its phase relation with the current (Fig. 83).

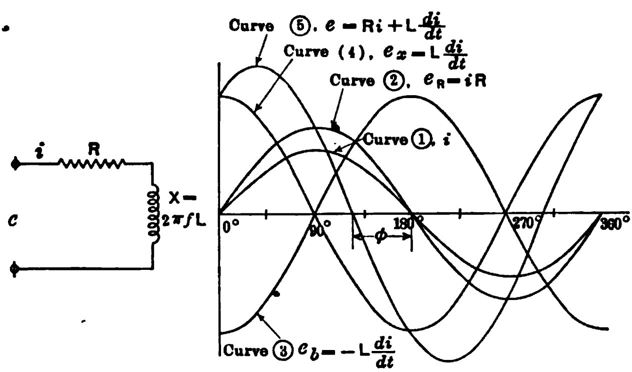


FIG. 83.—Resistance and reactance in series.

To drive a current $i = I_m \sin 2\pi ft$ (curve 1) through a resistance R an e.m.f. is required equal to

$$e_R = iR = I_m R \sin 2\pi ft \text{ (curve 2)},$$

a sine wave in phase with the current with a maximum value $I_m R$.

The inductance of the circuit sets up a back e.m.f.,

$$e_b = -L\frac{di}{dt} = 2\pi f L I_m \sin(2\pi ft - 90) \text{ (curve 3)},$$

a sine wave lagging 90 degrees behind the current with a maximum value $2\pi f L I_m = I_m X$.

To overcome this back e.m.f. due to inductance an e.m.f. must be impressed on the circuit equal and opposite to e_b.

$$\begin{aligned} e_X &= -e_b = L\frac{di}{dt} \\ &= 2\pi f L I_m \sin(2\pi f t + 90) \\ &= I_m X \sin(2\pi f t + 90) \text{ (curve 4).} \end{aligned}$$

This is a sine wave 90 degrees ahead of the current wave, with a maximum value $I_m X$ and is the e.m.f. consumed by the reactance in the circuit.

The instantaneous value of the impressed e.m.f. (curve 5) is the sum of the instantaneous values of e_R and e_X, thus

$$\begin{aligned} e &= e_R + e_X = Ri + L\frac{di}{dt} \qquad (176) \\ &= I_m R \sin 2\pi f t + I_m X \sin(2\pi f t + 90) \\ &= I_m R \sin 2\pi f t + I_m X \cos 2\pi f t \\ &= I_m (R \sin\theta + X \cos\theta) \\ &= I_m \sqrt{R^2 + X^2}\left(\frac{R}{\sqrt{R^2+X^2}}\sin\theta + \frac{X}{\sqrt{R^2+X^2}}\cos\theta\right) \\ &= I_m Z \sin(\theta + \phi), \qquad (177) \end{aligned}$$

where $Z = \sqrt{R^2 + X^2}$ is called the impedance of the circuit and is expressed in ohms and ϕ is the angle of lead of the impressed e.m.f. relative to the current.

$$\sin\phi = \frac{X}{\sqrt{R^2+X^2}} \text{ and } \cos\phi = \frac{R}{\sqrt{R^2+X^2}}.$$

The impressed e.m.f. is therefore a sine wave leading the current by an angle ϕ. Its maximum value is

$$E_m = I_m\sqrt{R^2 + X^2} = I_m Z, \qquad (178)$$

and its effective value is

$$E = \frac{E_m}{\sqrt{2}} = \frac{I_m Z}{\sqrt{2}} = IZ,$$

where I is the effective value of the current.

The component of E in phase with the current is

$$E_1 = IR = \frac{E}{Z}R = E\cos\phi;$$

and the component of E 90 degrees ahead or in quadrature ahead of the current is

$$E_2 = IX = \frac{E}{Z}X = E\sin\phi.$$

107. Capacity in Alternating-current Circuits.—The charge on a condenser or the quantity of electricity stored in it is proportional to the difference of potential between its terminals; thus,

$$q = Ce,$$

where q is the charge,

e is the difference of potential between the terminals,

and C is the capacity of the condenser.

A condenser has a capacity of one farad when one coulomb of electricity stored in it produces a difference of potential of one volt between its terminals.

If an alternating e.m.f. $e = E_m \sin 2\pi ft$ is impressed on the terminals of a condenser of capacity C farads, a current i flows. At any time t the charge on the condenser is

$$q = Ce,$$

but the charge is the amount of electricity which has flowed into the condenser and its value is

$$q = \int_0^t i\,dt,$$

and therefore

$$\int i\,dt = Ce.$$

Differentiating with respect to t gives

$$i = C\frac{de}{dt},$$

and substituting the value of e,

$$\begin{aligned} i &= C\frac{d}{dt}(E_m \sin 2\pi ft) \\ &= 2\pi fCE_m \cos 2\pi ft \\ &= 2\pi fCE_m \sin (2\pi ft + 90), \end{aligned} \tag{179}$$

and thus the current flowing into the condenser is a sine wave leading the impressed e.m.f. by 90 degrees. Its maximum value is

$$\begin{aligned} I_m &= 2\pi fCE_m \\ &= \frac{E_m}{X_c}, \end{aligned} \tag{180}$$

where $X_c = \frac{1}{2\pi fC}$ is called the condensive reactance of the circuit and is expressed in ohms.

In Fig. 84 curve 1 represents the impressed e.m.f. $e = E_m \sin 2\pi ft$ and curve 2 the current $i = I_m \sin (2\pi ft + 90)$. Between the points a and b the current is positive and the e.m.f. is increasing; from b to c the current is negative and the e.m.f. is decreasing.

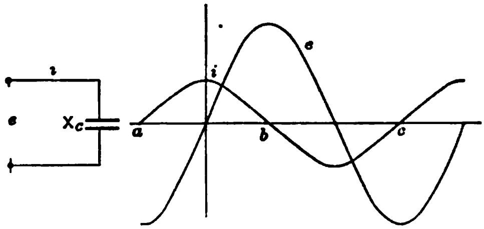


FIG. 84.—Capacity in alternating-current circuits.

108. Resistance and Condensive Reactance in Series.—If an alternating current $i = I_m \sin 2\pi ft$ flows in a circuit consisting of a resistance R in series with a condenser of capacity C farads and reactance of $X_c = \frac{1}{2\pi fC}$ ohms, determine the magnitude of the impressed e.m.f. and its phase relation with the current (Fig. 85).

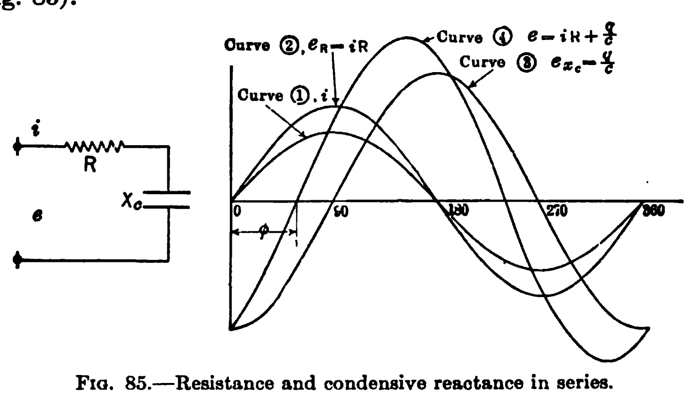


FIG. 85.—Resistance and condensive reactance in series.

The impressed e.m.f. consists of two components; one is required to drive the current i (curve 1) through the resistance of the circuit; it is $e_R = iR$ (curve 2), in phase with i; the other component is the e.m.f. required at the terminals of the condenser or the e.m.f. consumed by the condensive reactance of the circuit; it is $e_{Xc} = \frac{q}{C}$ (curve 3) and was shown to lag 90 degrees behind the current, Art. 107.

The total impressed e.m.f. is (curve 4)

$$
\begin{aligned}
e &= e_R + e_{X_c} \qquad (181)\\
&= iR + \frac{q}{C}\\
&= iR + \frac{\int i\,dt}{C}\\
&= RI_m \sin 2\pi ft + \frac{1}{C}\int I_m \sin (2\pi ft)\,dt\\
&= RI_m \sin 2\pi ft - \frac{I_m}{2\pi fC}\cos 2\pi ft\\
&= I_m (R \sin 2\pi ft - X_c \cos 2\pi ft)\\
&= I_m \sqrt{R^2 + X_c^2}\left(\frac{R}{\sqrt{R^2 + X_c^2}}\sin 2\pi ft - \frac{X_c}{\sqrt{R^2 + X_c^2}}\cos 2\pi ft\right)\\
&= I_m Z \sin (2\pi ft - \phi), \qquad (182)
\end{aligned}
$$

where $Z = \sqrt{R^2 + X_c^2}$ is the impedance of the circuit in ohms, and ϕ is the angle of lag of the impressed e.m.f. behind the current; $\sin\phi = \frac{X_c}{\sqrt{R^2 + X_c^2}}$, $\cos\phi = \frac{R}{\sqrt{R^2 + X_c^2}}$.

109. Resistance, Inductance and Capacity in Series.—If a current $i = I_m \sin 2\pi ft$ flows in a circuit consisting of a resistance R, inductance L and capacity C connected in series, the impressed e.m.f. is

$$e = e_R + e_X + e_{X_c}, \qquad (183)$$

Where $e_R = iR$ is the e.m.f. consumed by the resistance of the circuit,

$e_X = L\frac{di}{dt}$ is the e.m.f. consumed by the inductance of the circuit,

$e_{X_c} = \frac{q}{C}$ is the e.m.f. consumed by the capacity of the circuit.

Assuming that the impressed e.m.f. leads the current by angle ϕ, its equation is

$$e = E_m \sin (2\pi ft + \phi), \qquad (184)$$

and

$$
\begin{aligned}
E_m \sin (2\pi ft + \phi) &= Ri + L\frac{di}{dt} + \frac{q}{C}\\
&= Ri + L\frac{di}{dt} + \frac{\int i\,dt}{C}. \qquad (185)
\end{aligned}
$$

Differentiating with respect to t,

$$2\pi f E_m \cos(2\pi ft + \phi) = R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C}, \tag{186}$$

but

$$i = I_m \sin 2\pi ft, \frac{di}{dt} = 2\pi f I_m \cos 2\pi ft, \frac{d^2i}{dt^2} = -(2\pi f)^2 I_m \sin 2\pi ft$$

when $\quad t = 0, i = 0, \frac{di}{dt} = 2\pi f I_m, \frac{d^2i}{dt^2} = 0,$

when

$$t = \frac{90 - \phi}{2\pi f}, i = I_m \cos\phi, \frac{di}{dt} = 2\pi f I_m \sin\phi, \frac{d^2i}{dt^2} = -(2\pi f)^2 I_m \cos\phi.$$

Substituting these values in equation (186), when $t = 0$,

$$2\pi f E_m \cos\phi = 2\pi f R I_m.$$

and

$$\frac{E_m}{I_m} = \frac{R}{\cos\phi}; \tag{187}$$

when $\quad t = \frac{90 - \phi}{2\pi f},$

$$0 = 2\pi f R I_m \sin\phi - (2\pi f)^2 L I_m \cos\phi + \frac{I_m}{C}\cos\phi,$$

and

$$R \sin\phi = 2\pi f L \cos\phi - \frac{1}{2\pi f C}\cos\phi,$$

therefore,

$$\tan\phi = \frac{2\pi f L - \frac{1}{2\pi f C}}{R}$$

$$= \frac{X - X_c}{R}, \tag{188}$$

$$\sin\phi = \frac{X - X_c}{\sqrt{R^2 + (X - X_c)^2}}, \tag{189}$$

$$\cos\phi = \frac{R}{\sqrt{R^2 + (X - X_c)^2}}, \tag{190}$$

and from equation (162)

$$\frac{E_m}{I_m} = \frac{R}{\cos\phi} = \sqrt{R^2 + (X - X_c)^2} = Z, \tag{191}$$

where

$$Z = \sqrt{R^2 + (X - X_c)^2} = \sqrt{R^2 + \left(2\pi f L - \frac{1}{2\pi f C}\right)^2}$$

is the impedance of the circuit.

The impressed e.m.f. is therefore a sine wave of maximum value $E_m = I_m Z = I_m \sqrt{R^2 + (X - X_c)^2}$, and leads the current wave by an angle $\phi = \tan^{-1} \frac{X - X_c}{R}$.

If $X_c > X$, ϕ is negative and the impressed e.m.f. lags behind the current.

If $X_c = X$, $\phi = 0$ and the e.m.f. is in phase with the current, and the impedance of the circuit is $Z = R$.

110. Vector Representation of Harmonic Quantities.—A sine wave may be obtained from a circular locus as shown in Fig. 86.

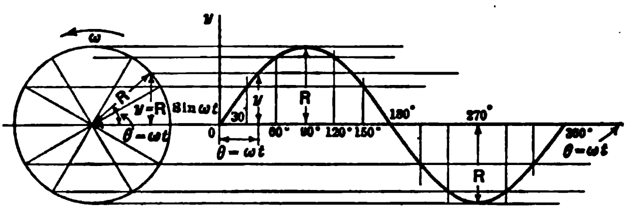


FIG. 86.—Sine wave.

The radius of the circle is the maximum value of the sine function and is called its amplitude. If the radius vector is rotated at uniform angular velocity ω in the counter-clockwise direction, its vertical projection at any time t is

$$y = R \sin \omega t = R \sin \theta,$$

where time is measured from the horizontal. Plotting the values of y on a base of angle θ for a complete revolution of the radius vector gives the cycle shown.

To represent the sine function

$$e = E_m \sin \theta,$$

a circle of radius E_m is taken and the vertical projections of the revolving vector are plotted on base of angle θ (Fig. 87(a)).

To represent a sine function

$$i = I_m \sin (\theta - \phi),$$

a circle of radius I_m is taken, but since i does not pass through zero until angle $\theta = \phi$, the sine wave is displaced to the right by angle ϕ as shown and the e.m.f. wave $e = E_m \sin \theta$ leads the current wave $i = I_m \sin (\theta - \phi)$ by angle ϕ. Instead of using complete circles to represent sine waves, their radii E_m and I_m can

be used as in Fig. 87(*b*) or since alternating quantities are represented by their effective values instead of their maximum values the two vectors OE and OI, Fig. 87(*c*), are used to represent the two sine waves in Fig. 87(*a*). $OE = E = \dfrac{E_m}{\sqrt{2}}$ is the effective value of the e.m.f. $e = E_m \sin \theta$, and $OI = I = \dfrac{I_m}{\sqrt{2}}$ is the effective value of the current $i = I_m \sin (\theta - \phi)$. The vector OI is behind the vector OE by angle ϕ and so indicates the relative phase of the two quantities. The counter-clockwise direction is

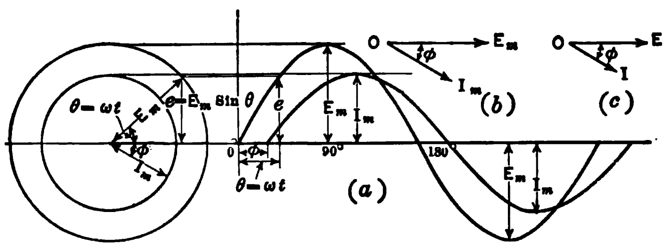


FIG. 87.—Vector representation of harmonic functions.

taken as the direction of advance in phase since it was adopted at the International Electrotechnical Congress at Turin and is now standard.

111. Power and Power Factor.—In continuous-current circuits the power consumed is the product of the impressed e.m.f. and the current, and is

$$P = EI \text{ watts.} \tag{192}$$

In alternating-current circuits the power varies with time; its instantaneous value is

$$p = ei \text{ watts,} \tag{193}$$

where e and i are the instantaneous values of the e.m.f. and current in the circuit.

Two cases will be considered, first, when the current and e.m.f. are in phase and, second, when the current lags behind the e.m.f.

Case I.—If an e.m.f. $e = E_m \sin \theta$ is impressed on the terminals of a circuit of resistance R a current $i = I_m \sin \theta$ will flow in phase with the e.m.f.

The instantaneous power in the circuit is

$$p = ei = E_m I_m \sin^2 \theta = \frac{E_m I_m}{2}(1 - \cos 2\theta). \qquad (194)$$

The values of e, i and p for a complete cycle are plotted in Fig. 88.

The power varies with twice the frequency of the current from O to $E_m I_m$, but never becomes negative. The area beneath the power curve represents the energy consumed in the circuit during one cycle.

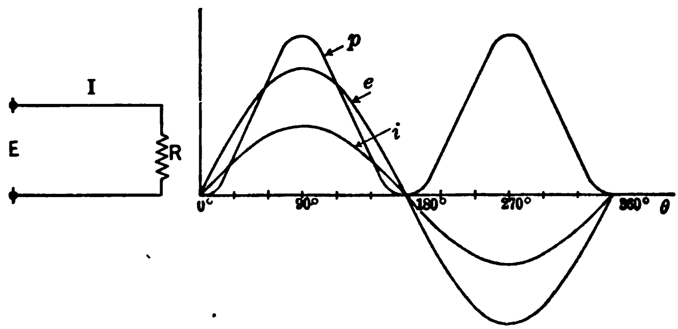


FIG. 88.—Power in a non-inductive circuit.

The average power is

$$\begin{aligned} P &= \frac{1}{\pi}\int_0^\pi p\, d\theta \\ &= \frac{1}{\pi}\int_0^\pi \frac{E_m I_m}{2}(1 - \cos 2\theta)\, d\theta \\ &= \frac{E_m I_m}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^\pi \\ &= \frac{E_m I_m}{2\pi}[\pi] \\ &= \frac{E_m I_m}{2} = \frac{E_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} = EI; \qquad (195) \end{aligned}$$

therefore, the average power in an alternating-current circuit is equal to the product of the effective values of the e.m.f. and the current, if they are in phase.

Since

$$P = EI$$

and

$$E = IR,$$

therefore,

$$P = I^2R, \qquad (196)$$

and the power is equal to the square of the effective value of the current multiplied by the resistance.

Case II.—If an e.m.f. $e = E_m \sin \theta$ is impressed on a circuit containing a resistance R, and an inductive reactance X, the current will lag behind the e.m.f. by an angle $\phi = \tan^{-1}\frac{X}{R}$ and its instantaneous value will be

$$i = I_m \sin(\theta - \phi).$$

The instantaneous power in the circuit is

$$\begin{aligned} p = ei &= E_m I_m \sin\theta \sin(\theta - \phi) \\ &= \frac{E_m I_m}{2}\{\cos\phi - \cos(2\theta - \phi)\}. \end{aligned} \tag{197}$$

The values of e, i and p for one cycle are plotted in Fig. 89. The maximum value of power is $\frac{E_m I_m}{2}(1 + \cos\phi)$, which is less than in Case I, and the power curve falls below the base twice in each cycle. The energy consumed per cycle is the difference between the positive and negative areas intercepted by the power curve.

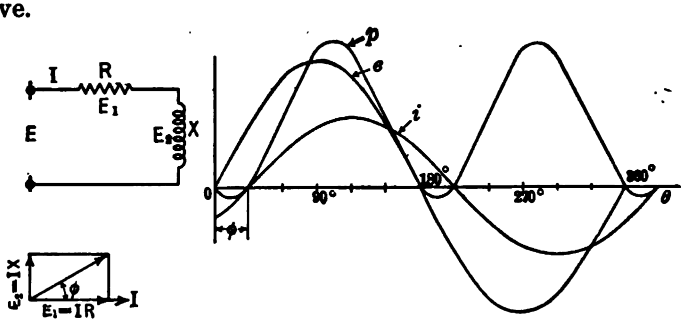


FIG. 89.—Power in an inductive circuit.

The average power is

$$\begin{aligned} P &= \frac{1}{\pi}\int_0^\pi p\,d\theta \\ &= \frac{E_m I_m}{2\pi}\int_0^\pi \{\cos\phi - \cos(2\theta - \phi)\}d\theta \\ &= \frac{E_m I_m}{2\pi}\left[\theta\cdot\cos\phi - \frac{\sin(2\theta - \phi)}{2}\right]_0^\pi \\ &= \frac{E_m I_m}{2\pi}\left[\pi\cdot\cos\phi - \frac{\sin(-\phi)}{2} + \frac{\sin(-\phi)}{2}\right] \\ &= \frac{E_m I_m}{2}\cos\phi \\ &= EI\cos\phi; \end{aligned} \tag{198}$$

therefore, the average power is the product of the effective values of the e.m.f. and current multiplied by the cosine of the angle of phase difference between them.

From the vector diagram, in Fig. 89, it is seen that

$$E \cos \phi = E_1 = IR,$$

and therefore the power is

$$P = E_1 I = I^2 R,$$

and is equal to the active or in-phase component of the e.m.f. multiplied by the current or is equal to the square of the current multiplied by the resistance of the circuit as found in equation

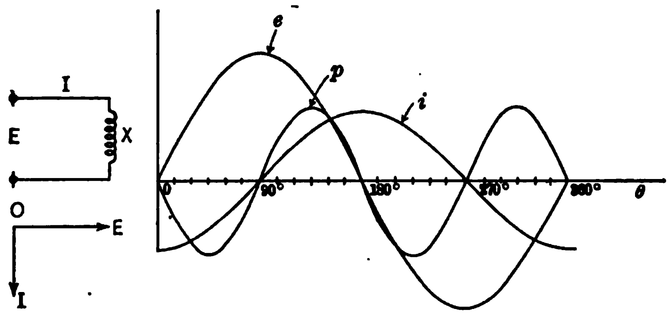


FIG. 90.—Power in an inductive reactance.

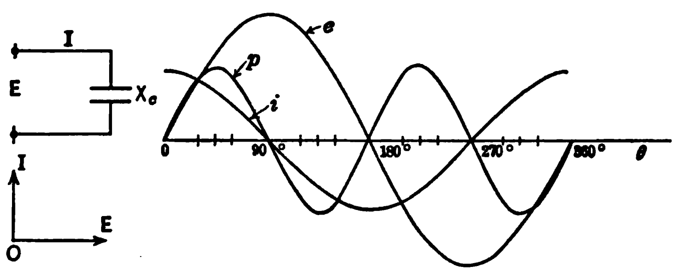


FIG. 91.—Power in a condensive reactance.

196; thus, all the power consumed in the circuit is consumed by the resistance. Reactance or self-inductance does not consume power since the energy stored while the current is increasing is given back while it is decreasing and the e.m.f. consumed by self-inductance is a wattless e.m.f. Similarly condensive reactance does not consume power since the energy stored while the e.m.f. is increasing is given back to the circuit while the e.m.f. is decreasing and the e.m.f. consumed by condensive reactance is wattless.

These results are illustrated in Fig. 90 and Fig. 91.

In Fig. 90 are plotted the values of e, i and p for an inductive circuit without resistance in which the current lags 90 degrees behind the impressed e.m.f. The power curve cuts off equal areas above and below the base line and therefore the average power is zero. The area below the line represents the energy given back by the magnetic field while the current is decreasing from its maximum value I_m to zero and the area above the line represents the energy stored in the magnetic field while the current is increasing again to its maximum value. The amount of energy in each case is $L \frac{I_m^2}{2}$ watt-sec.

In Fig. 91 are plotted the values of e, i and p for a circuit containing a condensive reactance but without resistance. The current leads the impressed e.m.f. by 90 degrees and the average power is again zero, so that no energy is consumed in the circuit. The positive area cut off by the power curve represents the energy stored in the electrostatic field of the condenser while the e.m.f. is increasing and the negative area represents the energy returned to the circuit while the e.m.f. is decreasing. The maximum amount of energy in each case is $C \frac{E_m^2}{2}$ watt-sec. where E_m is the maximum e.m.f.

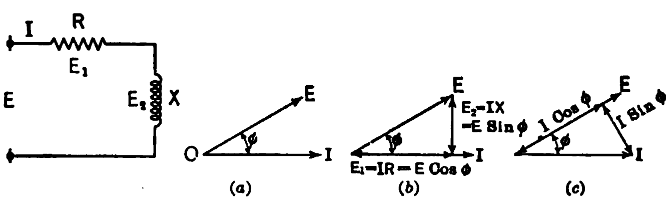


FIG. 92.—Power in alternating-current circuits.

Fig. 92 illustrates various methods of representing the power in a circuit; in (*a*) it is the product of the impressed e.m.f., the current and the cosine of the angle of phase difference between them,

$$P = E \times I \times \cos \phi; \tag{199}$$

in (*b*) it is the product of the current and the in-phase component of the e.m.f.,

$$P = I \times E \cos \phi = I \times E_1 = I^2R; \tag{200}$$

in (*c*) it is the product of the e.m.f. and the in-phase component of the current,

$$P = E \times I \cos \phi \tag{201}$$

The apparent power in a circuit is the product of the impressed e.m.f. and the current and is expressed in volt-amperes or kilovolt-amperes.

The power factor of a circuit is the ratio of the true power to the apparent power and is

$$\frac{P}{EI} = \frac{EI\cos\phi}{EI} = \cos\phi; \tag{202}$$

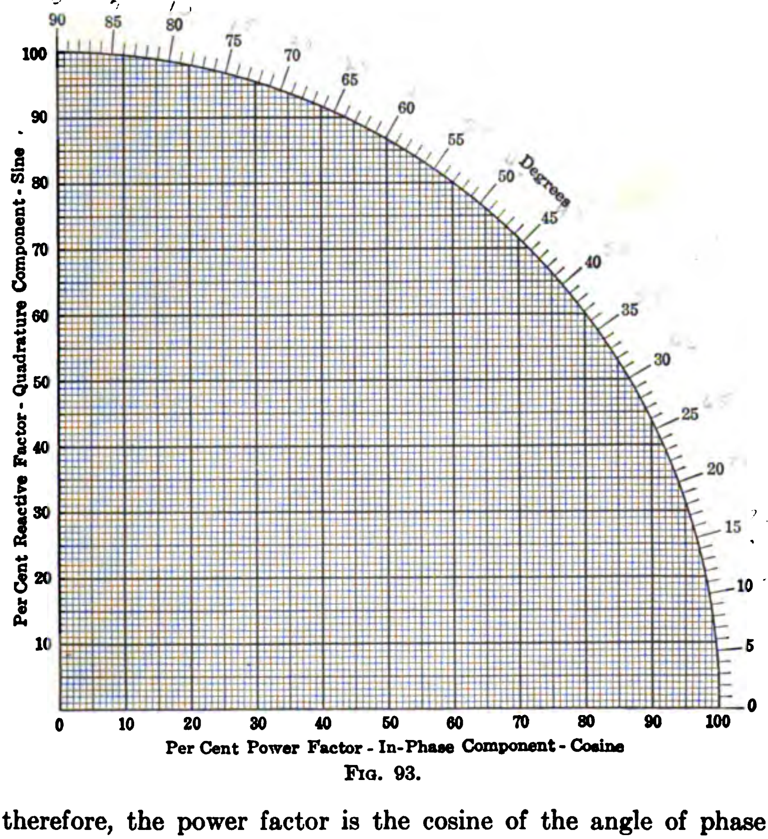


Fig. 93.

therefore, the power factor is the cosine of the angle of phase difference between the current and the impressed e.m.f.; it is usually expressed in per cent. and may be either leading or lagging.

The sine of the angle of phase difference between the current and the impressed e.m.f. is called the reactive factor of the circuit.

When the current is in phase with the e.m.f., the power factor is unity or 100 per cent. and the reactive factor is zero.

When the current leads the e.m.f. by 30 degrees the power factor is cos 30 = 0.866 = 86.6 per cent. leading and the reactive factor is sin 30 = 0.500 = 50 per cent.

When the current lags 60 degrees behind the e.m.f. the power factor is cos 60 = 0.500 = 50 per cent. lagging and the reactive factor is sin 60 = 0.866 = 86.6 per cent.

When the power factor is 90 per cent. the reactive factor is $\sqrt{1 - \overline{0.9}^2} = 0.436 = 43.6$ per cent.

When the power factor is 99 per cent. the reactive factor is $\sqrt{1 - 0.\overline{99}^2} = 0.141 = 14.1$ per cent.

From this last example it may be seen that very great care must be exercised in determining reactive factors from power-factor readings when the power factor is near unity; an error of 1 per cent. in reading the current, e.m.f. or power might cause a reactive component of 14.1 per cent. to be missed entirely.

These and similar problems may be solved quickly by using the chart in Fig. 93.

112. Examples.—1. If an alternating e.m.f. of effective value E is impressed on a non-inductive circuit of resistance R, a current I will flow in phase with e.m.f., where

$$I = \frac{E}{R}.$$

The vector diagram is shown in Fig. 94.

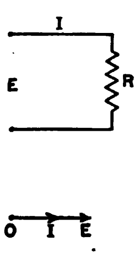


Fig. 94.

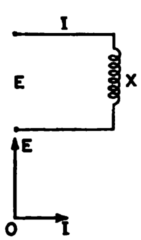


Fig. 95.

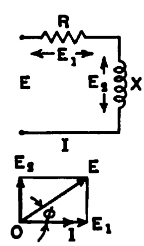


Fig. 96.

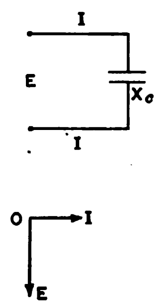


Fig. 97.

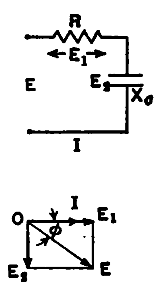


Fig. 98.

2. If an alternating e.m.f. E is impressed on a circuit of reactance X and negligible resistance, a current $I = \frac{E}{X}$ will flow lagging 90 degrees behind the e.m.f. (see Fig. 95).

3. If an alternating e.m.f. E is impressed on a circuit of resistance R and reactance X, a current I will flow lagging behind the e.m.f. by angle ϕ, where

$$\tan \phi = \frac{X}{R}.$$

The vector diagram for the circuit is shown in Fig. 96. The e.m.f. consumed in the resistance is $E_1 = IR$ volts in phase with the current and is represented by the vector OE_1.

The e.m.f. consumed by the reactance is $E_2 = IX$ volts leading the current by 90 degrees represented by OE_2. The impressed e.m.f. is $OE = E$ and is the vector sum of E_1 and E_2; therefore

$$E = \sqrt{E_1^2 + E_2^2} = \sqrt{\overline{IR}^2 + \overline{IX}^2} = I\sqrt{R^2 + X^2} = IZ,$$

where

$$Z = \sqrt{R^2 + X^2} \text{ is the impedance of the circuit.}$$

4. Fig. 97 shows the vector diagram of a circuit containing a condenser of reactance X_c, when an alternating e.m.f. E is impressed on its terminals and a current $I = \frac{E}{X_c}$ flows through it leading the e.m.f. by 90 degrees.

5. Fig. 98 shows the diagram for the same circuit with a resistance R added in series.

The e.m.f. consumed by the resistance is $OE_1 = E_1 = IR$, in phase with $OI = I$.

The e.m.f. consumed by the reactance is $OE_2 = E_2 = IX_c$, lagging 90 degrees behind OI.

The impressed e.m.f. is

$$OE = E = \sqrt{E_1^2 + E_2^2} = I\sqrt{R^2 + X_c^2} = IZ,$$

lagging behind OI by angle ϕ, where

$$\tan \phi = \frac{E_2}{E_1} = \frac{IX_c}{IR} = \frac{X_c}{R}.$$

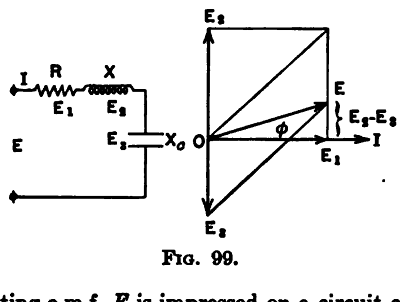


FIG. 99.

6. If an alternating e.m.f. E is impressed on a circuit containing a resistance R, an inductive reactance $X = 2\pi fL$ and a condensive reactance $X_c = \frac{1}{2\pi fC}$ connected in series, determine the magnitude and phase relation of the current and draw the vector diagram for the circuit (Fig. 99).

$OI = I$ is the current taken as horizontal.

$OE_1 = E_1 = IR$ is the e.m.f. consumed by the resistance and is in phase with I.

$OE_2 = E_2 = IX$ is the e.m.f. consumed by the inductive reactance and leads OI by 90 degrees.

$OE_3 = E_3 = IX_c$ is the e.m.f. consumed by the condensive reactance and lags behind OI by 90 degrees.

$OE = E = IZ$ is the e.m.f. impressed on the circuit, or the e.m.f. consumed by the impedance Z. It is the vector sum of the three components E_1, E_2 and E_3 and leads the current by angle ϕ; therefore,

$$\begin{aligned} E &= \sqrt{E_1^2 + (E_2 - E_3)^2} \\ &= I\sqrt{R^2 + (X - X_c)^2} \\ &= IZ, \end{aligned}$$

and

$$\begin{aligned} Z &= \sqrt{R^2 + (X - X_c)^2} \\ &= \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}, \end{aligned}$$

where L is the inductance of the circuit in henrys, C is the capacity in farads and f is the frequency of the impressed e.m.f.

The angle of phase difference between the e.m.f. and current is ϕ, where

$$\tan\phi = \frac{E_2 - E_3}{E_1} = \frac{I(X - X_c)}{IR} = \frac{X - X_c}{R}.$$

If $X > X_c$ the current lags behind the e.m.f.;
if $X = X_c$ the current is in phase with the e.m.f.;
and if $X < X_c$ the current leads the e.m.f.

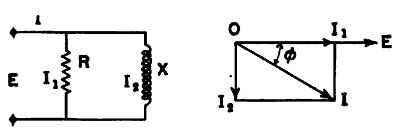


FIG. 100.

7. An alternating e.m.f. E is impressed on the terminals of the circuit in Fig. 100 consisting of a resistance R and an inductive reactance X in parallel.

The main current I has two components,

$$I_1 = \frac{E}{R}, \text{ in phase with } E,$$

and

$$I_2 = \frac{E}{X}, \text{ 90 degrees behind } E.$$

From the vector diagram

$$I = \sqrt{I_1^2 + I_2^2} = E\sqrt{\frac{1}{R^2} + \frac{1}{X^2}} = E\frac{\sqrt{R^2 + X^2}}{RX}$$

and lags behind E by an angle ϕ, where

$$\tan\phi = \frac{I_2}{I_1} = \frac{E/X}{E/R} = \frac{R}{X}.$$

The impedance of the circuit is

$$Z = \frac{E}{I} = \frac{RX}{\sqrt{R^2 + X^2}}.$$

8. In Fig. 101 a third branch is connected in parallel with the two in example (7) containing a condensive reactance $X_c > X$.

The main current I has three components,

$$I_1 = \frac{E}{R}, \text{ in phase with } E,$$

$$I_2 = \frac{E}{X}, \text{ 90 degrees behind } E,$$

and

$$I_3 = \frac{E}{X_c}, \text{ 90 degrees ahead of } E.$$

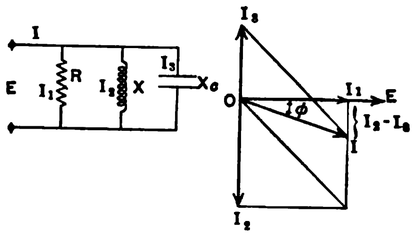


FIG. 101.

From the vector diagram

$$I = \sqrt{I_1^2 + (I_2 - I_3)^2} = E\sqrt{\frac{1}{R^2} + \left(\frac{1}{X} - \frac{1}{X_c}\right)^2}$$

and lags behind E by an angle ϕ, where

$$\tan \phi = \frac{I_2 - I_3}{I_1} = \frac{1/X - 1/X_c}{I/R}.$$

The impedance of the circuit is

$$Z = \frac{E}{I} = \frac{1}{\sqrt{\frac{1}{R^2} + \left(\frac{1}{X} - \frac{1}{X_c}\right)^2}}.$$

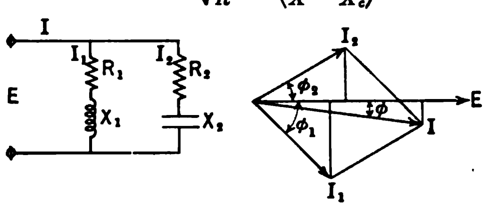


FIG. 102.

9. Find the magnitudes of the currents in the various parts of the circuit in Fig. 102 and their phase relations with the impressed e.m.f.

$I_1 = \frac{E}{\sqrt{R_1^2 + X_1^2}}$ and lags behind the impressed e.m.f. by an angle ϕ_1, where $\tan \phi_1 = \frac{X_1}{R_1}$;

$I_2 = \frac{E}{\sqrt{R_2^2 + X_2^2}}$ and leads the e.m.f. by an angle ϕ_2, where $\tan \phi_2 = \frac{X_2}{R_2}$;

$I = \sqrt{I_1^2 + I_2^2 + 2I_1I_2 \cos(\phi_1 + \phi_2)}$ (see vector diagram) and lags behind the e.m.f. by an angle ϕ, where

$$\tan \phi = \frac{I_1 \sin \phi_1 - I_2 \sin \phi_2}{I_1 \cos \phi_1 + I_2 \cos \phi_2}.$$

The power consumed in the circuit is

$$P = EI \cos \phi$$
$$= I_1^2R_1 + I_2^2R_2 \text{ watts.}$$

113. Numerical Examples.—1. If an alternating e.m.f. of 200 volts at a frequency of 60 cycles per second is impressed on a circuit consisting of a resistance of 10 ohms in series with an inductance of 0.1 henry and a capacity of 100 microfarads, (*a*) determine the current in the circuit and its phase relation with the impressed e.m.f., (*b*) the e.m.f. consumed in each part of the circuit. (*c*) If the impressed e.m.f. is maintained constant and the frequency is varied determine the maximum value of the current. (*d*) Plot the current and the various e.m.fs. on a frequency base.

(*a*) The inductive reactance of the circuit is

$$X = 2\pi fL = 2 \times 3.14 \times 60 \times 0.1 = 37.6 \text{ ohms;}$$

the condensive reactance is

$$X_c = \frac{1}{2\pi fC} = \frac{10^6}{2 \times 3.14 \times 60 \times 100} = 26.4 \text{ ohms;}$$

the impedance of the circuit is

$$Z = \sqrt{R^2 + (X - X_c)^2} = \sqrt{10^2 + (37.6 - 26.4)^2} = 15 \text{ ohms;}$$

and therefore the current is

$$I = \frac{E}{Z} = \frac{200}{15} = 13.3 \text{ amp.}$$

The current lags behind the e.m.f. by an angle ϕ, where

$$\tan \phi = \frac{X - X_c}{R} = \frac{37.6 - 26.4}{10} = 1.12$$

and

$$\phi = 48° \; 18'.$$

(*b*) The e.m.f. consumed in the resistance is

$$E_1 = IR = 13.3 \times 10 = 133 \text{ volts;}$$

the e.m.f. consumed in the inductive reactance is

$$E_2 = IX = 13.3 \times 37.6 = 500 \text{ volts;}$$

and the e.m.f. consumed by the condensive reactance is

$$E_3 = IX_c = 13.3 \times 26.4 = 350 \text{ volts.}$$

The impressed e.m.f. is

$$E = \sqrt{E_1^2 + (E_2 - E_3)^2} = \sqrt{133^2 + (500 - 350)^2} = 200 \text{ volts.}$$

The vector diagram is shown in Fig. 103.

(*c*) The current in the circuit at any frequency is

$$I = \frac{E}{Z} = \frac{E}{\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}}.$$

If E is maintained constant at 200 volts and f is varied, I varies.

When $f = 0$, $\frac{1}{2\pi fC} = \infty$ and $I = 0$;

when $f = \infty$, $2\pi fL = \infty$ and $I = 0$;

when $2\pi fL = \dfrac{1}{2\pi fC}$ or $f = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2 \times 3.14\sqrt{0.1 \times \dfrac{100}{10^6}}} = 50$ cycles

the current has its maximum value

$$I_{\text{max.}} = \frac{E}{R} = \frac{200}{10} = 20 \text{ amp.}$$

and is in phase with the impressed e.m.f.

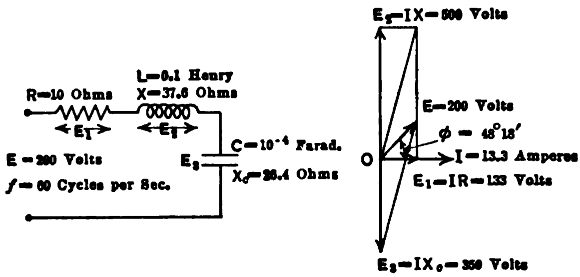


FIG. 103.

The e.m.f. consumed in the resistance is

$$E_1 = IR = 20 \times 10 = 200 \text{ volts;}$$

the e.m.f. consumed in the inductive reactance is

$$E_2 = IX = 20 \times 2 \times 3.14 \times 50 \times 0.1 = 628 \text{ volts;}$$

and the e.m.f. consumed in the condensive reactance is

$$E_3 = IX_c = 20 \times \frac{1}{2 \times 3.14 \times 50 \times \dfrac{100}{10^6}} = 628 \text{ volts.}$$

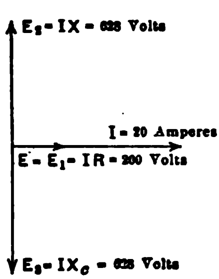


FIG. 104.—Resonant circuit.

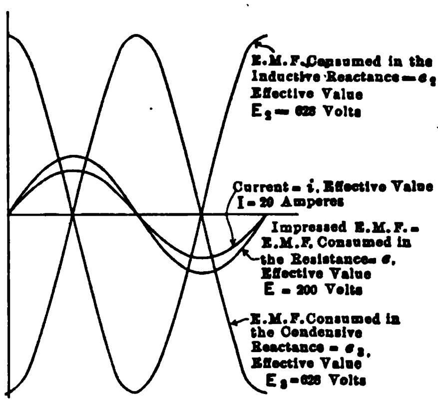


FIG. 105.—E.m.fs. and currents in a resonant circuit.

The vector diagram for the circuit is shown in Fig. 104, and the current and e.m.f. waves are shown in Fig. 105. A series circuit in which the inductive

reactance and the condensive reactance are equal at a certain frequency is said to be in a state of resonance for that frequency. In commercial circuits the capacity is usually so small that resonance cannot occur at ordinary frequencies, but when any high frequency e.m.fs. are produced in the circuit resonance may occur and very large e.m.fs. may appear and break down the insulation.

(*d*) In the table below are given the values of the circuit constants for frequencies from 0 to 200 cycles per second and the corresponding values of I, E_1, E_2, E_3 and cos ϕ.

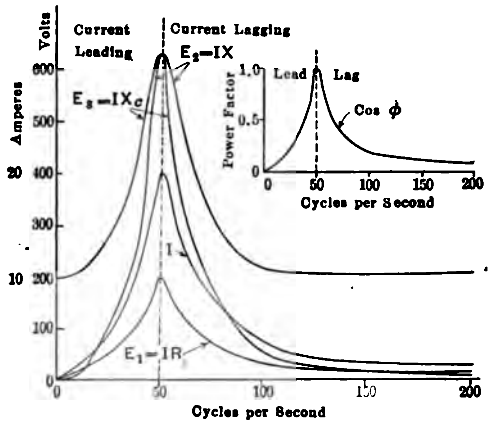


Fig. 106.

f	X	X_c	$X - X_c$	$(X - X_c)^2$	Z^2	Z	I	E_1	E_2	E_3	Cos ϕ
0	0.0	∞	−∞	∞	∞	∞	0.0	0.0	0.0	200.0	0.000
10	6.3	159.0	−152.7	23,400	23,500	153.0	1.3	13.0	8.2	208.0	0.065
40	25.1	39.8	−14.7	216	316	17.8	11.2	112.0	281.0	446.0	0.560
50	31.4	31.4	0.0	0	100	10.0	20.0	200.0	628.0	628.0	1.000
60	37.6	26.4	11.2	126	226	15.0	13.3	133.0	502.0	353.0	0.670
100	62.8	15.9	56.9	3,240	3,340	57.7	3.5	35.0	218.0	55.2	0.170
200	125.6	7.9	117.7	13,800	13,900	118.0	1.7	17.0	213.0	13.4	0.085
∞	∞	0.0	∞	∞	∞	∞	0.0	0.0	200.0	0.0	0.000

These quantities are plotted on a frequency base in Fig. 106. E_2 reaches its max. when $f = 51.4$ and E_3 reaches its max. when $f = 49.2$ cycles per second.

2. If an alternating e.m.f. $E = 200$ volts at a frequency $f = 60$ cycles per second is impressed on the circuit in Fig. 107, determine the value and phase relation of the main current and the currents in the three branches.

The first branch is a resistance $R = 40$ ohms; the second branch is an inductance $L = 0.1$ henry and has a reactance $X = 2\pi fL = 37.6$ ohms at 60 cycles; the third branch is a capacity $C = 100$ microfarads or 10^{-4} farads and has a reactance $X_c = \dfrac{1}{2\pi fC} = 26.4$ ohms.

The current in the resistance is

$$I_1 = \frac{E}{R} = \frac{200}{40} = 5 \text{ amp.},$$

in phase with the impressed e.m.f.; the current in the inductive reactance is

$$I_2 = \frac{E}{X} = \frac{200}{37.6} = 5.3 \text{ amp.},$$

90 degrees behind the impressed e.m.f.; the current in the condensive reactance is

$$I_3 = \frac{E}{X_c} = \frac{200}{26.4} = 7.6 \text{ amp.},$$

90 degrees ahead of the impressed e.m.f.

The main current is

$$I = \sqrt{I_1^2 + (I_2 - I_3)^2} = \sqrt{5^2 + (5.3 - 7.6)^2} = 5.5 \text{ amp.},$$

leading the impressed e.m.f. by an angle ϕ, where

$$\tan \phi = \frac{I_3 - I_2}{I_1} = \frac{2.3}{5} = 0.46,$$

and therefore

$$\phi = 24° \, 42'.$$

If the e.m.f. impressed on the circuit is maintained constant and the frequency is varied find the magnitude of the main current when it is in phase with the e.m.f.

The current at any frequency is

$$I = \sqrt{I_1^2 + (I_2 - I_3)^2}$$
$$= E\sqrt{\frac{1}{R^2} + \left(\frac{1}{X} - \frac{1}{X_c}\right)^2}$$
$$= E\sqrt{\frac{1}{R^2} + \left(\frac{1}{2\pi f L} - 2\pi f C\right)^2};$$

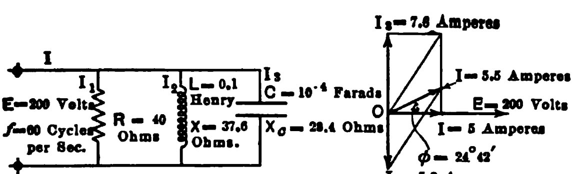


FIG. 107.

the angle of lag of the current behind the e.m.f. is

$$\phi = \tan^{-1}\frac{I_2 - I_3}{I_1} = \tan^{-1}\frac{1/X - 1/X_c}{1/R}.$$

When the current is in phase with e.m.f.

$$\phi = 0 \text{ and } X = X_c \text{ or } 2\pi f L = \frac{1}{2\pi f C};$$

the frequency is therefore

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2 \times 3.14\sqrt{0.1 \times 10^{-4}}} = 50 \text{ cycles.}$$

9

The main current at a frequency of 50 cycles is

$$I = I_1 = \frac{E}{R} = \frac{200}{40} = 5 \text{ amp.}$$

the current in the inductive reactance is

$$I_2 = \frac{E}{X} = \frac{200}{2 \times 3.14 \times 50 \times 0.1} = 6.36 \text{ amp.}$$

the current in the condensive reactance is

$$I_3 = \frac{E}{X_c} = \frac{200}{\dfrac{1}{2 \times 3.14 \times 50 \times 10^{-4}}} = 6.36 \text{ amp.};$$

and the current in the lead between the first and second branches is zero.

3. If an alternating e.m.f. $E = 100$ volts is impressed on a circuit, Fig. 108, made up of two parallel branches 1 and 2, the first with resistance $R_1 = 4$ ohms and inductive reactance $X_1 = 3$ ohms and the second with resistance $R_2 = 6$ ohms and inductive reactance $X_2 = 8$ ohms; determine the magnitude of the main current I and of the two branch currents I_1 and I_2 and their phase relations with the impressed e.m.f. E. The vector diagram is shown in Fig. 109.

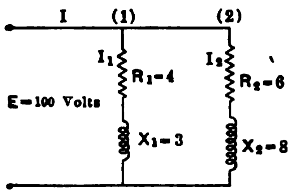


FIG. 108.

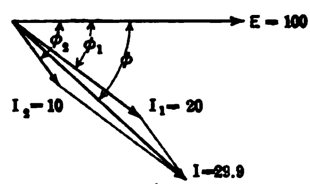


FIG. 109.

The current in branch 1 is

$$I_1 = \frac{E}{\sqrt{R_1^2 + X_1^2}} = \frac{100}{\sqrt{4^2 + 3^2}} = 20 \text{ amp.},$$

and it lags behind the impressed e.m.f. by angle

$$\phi_1 = \tan^{-1}\frac{X_1}{R_1} = \tan^{-1} 3/4 = \sin^{-1} 3/5 = \cos^{-1} 4/5.$$

The current in branch 2 is

$$I_2 = \frac{E}{\sqrt{R_2^2 + X_2^2}} = \frac{100}{\sqrt{6^2 + 8^2}} = 10 \text{ amp.},$$

and it lags behind the impressed e.m.f. by angle

$$\phi_2 = \tan^{-1}\frac{X_2}{R_2} = \tan^{-1} 8/6 = \tan^{-1} 4/3 = \sin^{-1} 4/5 = \cos^{-1} 3/5.$$

The main current I is the vector sum of I_1 and I_2 and may be found as

$$I = \sqrt{(I_1 \sin\phi_1 + I_2 \sin\phi_2)^2 + (I_1 \cos\phi_1 + I_2 \cos\phi_2)^2}$$

$$= \sqrt{(20 \times 3/5 + 10 \times 4/5)^2 + (20 \times 4/5 + 10 \times 3/5)^2} = \sqrt{20^2 + 22^2} = 29.9 \text{ amp.}$$

and it lags behind the impressed e.m.f. E by angle

$$\phi = \tan^{-1}\frac{I_1 \sin\phi_1 + I_2 \sin\phi_2}{I_1 \cos\phi_1 + I_2 \cos\phi_2} = \tan^{-1}\frac{20}{22} = \tan^{-1} 0.909 = 42^\circ\, 18'.$$

114. Circuit Constants.—A continuous-current circuit has two constants:

$$\text{resistance } R, r = \frac{\text{impressed e.m.f.}}{\text{current}},$$

$$\text{conductance } G, g = \frac{\text{current}}{\text{impressed e.m.f.}},$$

and the conductance is the reciprocal of the resistance or

$$G = \frac{1}{R}.$$

Continuous-current circuits also have inductance and electrostatic capacity, but these do not affect the flow of current except at the instant of opening or closing the circuit.

An alternating-current circuit has six so-called constants:

1. resistance R, r,
2. reactance X, x,
3. impedance Z, z,
4. admittance Y, y,
5. conductance G, g,
6. susceptance B, b.

1. The resistance of a circuit consumes a component of e.m.f. in phase with the current and so consumes power. In circuits which are partially inclosed in iron an alternating magnetic flux is produced in the iron and a loss of power occurs due to hysteresis and eddy currents. These iron losses are sometimes included with the copper loss and charged against the resistance. This gives a value of resistance greater than the true ohmic resistance and is called the effective resistance of the circuit. Since the hysteresis and eddy-current losses vary both with the frequency and the induction density in the iron, the effective resistance is not a constant quantity.

The active component of the impressed e.m.f. or the component in phase with the current is

$$E_1 = IR,$$

and the resistance is

$$R = \frac{E_1}{I} = \frac{\text{active component of impressed e.m.f.}}{\text{current}}$$

$$= \frac{\text{in-phase component of impressed e.m.f.}}{\text{current}} \qquad (203)$$

2. The reactance of a circuit consumes a component of e.m.f. in quadrature with the current, leading in the case of circuits of

large inductance and lagging in circuits of large electrostatic capacity, but it does not consume any power.

The inductive reactance of a circuit is

$$X = 2\pi fL \text{ ohms,}$$

where f is the frequency of the impressed e.m.f. and L is the inductance in henrys. Commercial circuits are operated at a fixed frequency and so f is constant.

The inductance of a circuit in air or any non-magnetic material is constant but in an iron-clad circuit it varies with the current, decreasing as the current increases since the permeability of the iron decreases as the flux density in it increases.

Since inductive reactance consumes a component of e.m.f. in quadrature ahead of the current it is taken as a positive reactance.

The condensive reactance of a circuit is

$$X_C = \frac{1}{2\pi fC} \text{ ohms,}$$

where C is the capacity of the circuit in farads. The capacity of a circuit does not vary with the current or e.m.f. and thus the condensive reactance is constant so long as the frequency is constant.

Condensive reactance is taken as a negative reactance since it consumes a component of e.m.f. in quadrature behind the current. Thus when inductive reactance and condensive reactance are connected in series they oppose and the reactance of the circuit is

$$X = X_L - X_C = 2\pi fL - \frac{1}{2\pi fC} \text{ ohms.}$$

In series-parallel circuits the reactance is a complex function of the resistances and reactances of the various branches.

The reactance of any circuit is

$$X = \frac{\text{reactive component of impressed e.m.f.}}{\text{current}}$$
$$= \frac{\text{quadrature component of impressed e.m.f.}}{\text{current}}. \quad (204)$$

3. The impedance of a circuit includes both the resistance and the reactance; it is

$$Z = \sqrt{R^2 + X^2} = \frac{\text{impressed e.m.f}}{\text{current}}. \quad (205)$$

4. The admittance of a circuit is

$$Y = \frac{\text{current}}{\text{impressed e.m.f.}}; \tag{206}$$

it is the reciprocal of the impedance and thus

$$Y = \frac{1}{Z} = \frac{1}{\sqrt{R^2 + X^2}}. \tag{207}$$

The admittance has two components, conductance and susceptance.

Fig. 110 shows a circuit of impedance $Z = \sqrt{R^2 + X^2}$ in which the current lags behind the impressed e.m.f. by an angle

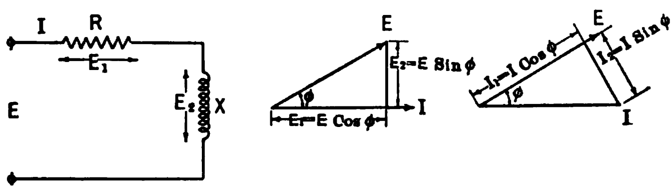


FIG. 110.

ϕ. The current I can be resolved into two components in phase and in quadrature with the impressed e.m.f. E. The in-phase or active component of current is

$$I_1 = I \cos\phi = \frac{E}{Z}\cos\phi = E\frac{R}{Z^2} = EG, \tag{208}$$

where

$$G = \frac{R}{Z^2} = \frac{R}{R^2 + X^2} \tag{209}$$

is the conductance of the circuit.

The quadrature of reactive component of current is

$$I_2 = I \sin\phi = \frac{E}{Z}\sin\phi = E\frac{X}{Z^2} = EB, \tag{210}$$

where

$$B = \frac{X}{Z^2} = \frac{X}{R^2 + X^2} \tag{211}$$

is the susceptance of the circuit.

The total current

$$I = \sqrt{I_1^2 + I_2^2} = \sqrt{\overline{EG}^2 + \overline{EB}^2} = E\sqrt{G^2 + B^2},$$

but, by equation (206), $I = EY$, and therefore

$$Y = \sqrt{G^2 + B^2}. \tag{212}$$

5. The conductance of a circuit is, from equation (208),

$$G = \frac{\text{active component of current}}{\text{impressed e.m.f.}}. \qquad (213)$$

6. The susceptance of a circuit is, from equation (210),

$$B = \frac{\text{reactive component of current}}{\text{impressed e.m.f.}}. \qquad (214)$$

In the solution of series circuits it is not necessary to employ the terms admittance, conductance and susceptance but the solution of series-parallel circuits is very much simplified by their use.

115. Rectangular Coördinates.—The simplest method of dealing with alternating-current phenomena is to express the e.m.fs., currents, etc., as the sum of two components, one along a chosen axis and the other perpendicular to it.

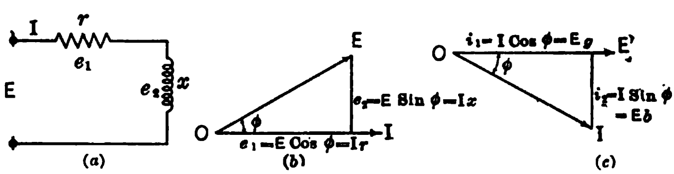


FIG. 111.

In Fig. 111(*b*) which represents the e.m.f. and current in a circuit of impedance $Z = \sqrt{r^2 + x^2}$ the axis is chosen in the direction of the current and the e.m.f. is resolved into two components, e_1 in phase with the current and e_2 in quadrature ahead of the current.

The absolute value of the e.m.f. is

$$E = \sqrt{e_1^2 + e_2^2},$$

and it leads the current, which was chosen as axis by an angle ϕ, where

$$\tan\phi = \frac{e_2}{e_1}.$$

Thus when the e.m.f. is expressed as the sum of two components at right angles both its magnitude and its phase are known.

To distinguish between horizontal and vertical components the prefix $j = \sqrt{-1}$ is added to all vertical components and the expression for the e.m.f. above is

$$\underset{\cdot}{E} = e_1 + je_2. \qquad (215)$$

The dot is placed under the E to show that it is expressed in rectangular coördinates and serves to distinguish it from its absolute value.

Since $e_1 = E \cos \phi = Ir$ and $e_2 = E \sin \phi = Ix$,

$$\underset{\cdot}{E} = E \cos \phi + jE \sin \phi$$
$$= Ir + jIx$$
$$= I\,(r + jx), \tag{216}$$

and therefore the impedance in rectangular coördinates is

$$\underset{\cdot}{Z} = r + jx. \tag{217}$$

In Fig. 111(c) the e.m.f. is chosen as axis and the current is behind it in phase by an angle ϕ and has two components i_1 in phase with the e.m.f. and i_2 in quadrature behind it.

The current may be written

$$\underset{\cdot}{I} = i_1 - ji_2, \tag{218}$$

and this equation indicates that the current has a value

$$I = \sqrt{i_1^2 + i_2^2},$$

and that it is behind the chosen axis (in this case the e.m.f.) in phase by an angle ϕ, where

$$\tan \phi = \frac{i_2}{i_1}.$$

Since $i_1 = I \cos \phi = Eg$ and $i_2 = I \sin \phi = Eb$, where $Y = \sqrt{g^2 + b^2}$ is the admittance of the circuit, equation (218) may be written

$$\underset{\cdot}{I} = I \cos \phi - jI \sin \phi$$
$$= Eg - jEb$$
$$= E\,(g - jb), \tag{219}$$

and therefore the admittance in rectangular coördinates is

$$\underset{\cdot}{Y} = g - jb.$$

Admittance and impedance are not alternating quantities and their components are independent of the axis of reference, but they can be represented in rectangular coördinates.

A current multiplied by an impedance gives an e.m.f. displaced from it in phase by an angle whose tangent is the ratio of the reactance to the resistance.

A current divided by an admittance gives an e.m.f. displaced in phase by an angle whose tangent is the ratio of the susceptance to the conductance.

Similarly an e.m.f. divided by an impedance or multiplied by an admittance gives a current.

By definition a vector multiplied by j is turned through 90 degrees in the counter-clockwise direction; when multiplied by j^2 it is turned through 180 degrees and its sign is reversed.

Therefore

$$j^2 = -1$$

and

$$j = \sqrt{-1}. \tag{220}$$

Taking this value for j alternating quantities expressed in rectangular coördinates referred to a given axis can be added, subtracted, multiplied and divided and the results obtained are expressed in rectangular coördinates referred to the same axis.

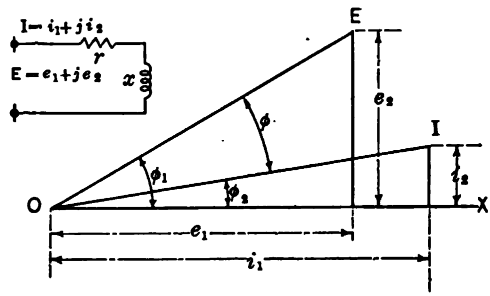


Fig. 112.

It is not necessary to choose either the current in a circuit or the e.m.f. as axis and any other line may be taken as shown in Fig. 112, but the e.m.f. must now be expressed as the sum of two components along and perpendicular to the new axis, thus

$$\dot{E} = e_1 + je_2,$$

and similarly the current is

$$\dot{I} = i_1 + ji_2.$$

The e.m.f. has an absolute value

$$E = \sqrt{e_1^2 + e_2^2}$$

and is ahead of the axis by an angle

$$\phi_1 = \tan^{-1}\frac{e_2}{e_1} = \cos^{-1}\frac{e_1}{E}.$$

The current has an absolute value

$$I = \sqrt{i_1^2 + i_2^2}$$

and is ahead of the axis by an angle

$$\phi_2 = \tan^{-1}\frac{i_2}{i_1} = \cos^{-1}\frac{i_1}{I}.$$

The e.m.f. leads the current by an angle

$$\phi = \phi_1 - \phi_2.$$

The impedance of the circuit in rectangular coördinates is

$$\begin{aligned} \dot{Z} &= \frac{\dot{E}}{\dot{I}} = \frac{e_1 + je_2}{i_1 + ji_2} = \frac{e_1 + je_2}{i_1 + ji_2} \times \frac{i_1 - ji_2}{i_1 - ji_2} \\ &= \frac{e_1 i_1 + e_2 i_2}{i_1^2 + i_2^2} + j\frac{e_2 i_1 - e_1 i_2}{i_1^2 + i_2^2} \\ &= r + jx, \end{aligned}$$

where the resistance of the circuit is

$$r = \frac{e_1 i_1 + e_2 i_2}{i_1^2 + i_2^2} = \frac{e_1 i_1 + e_2 i_2}{I^2}$$

and the reactance of the circuit is

$$x = \frac{e_2 i_1 - e_1 i_2}{i_1^2 + i_2^2} = \frac{e_2 i_1 - e_1 i_2}{I^2}.$$

The admittance of the circuit is

$$\begin{aligned} \dot{Y} &= \frac{\dot{I}}{\dot{E}} = \frac{i_1 + ji_2}{e_1 + je_2} = \frac{i_1 + ji_2}{e_1 + je_2} \times \frac{e_1 - je_2}{e_1 - je_2} \\ &= \frac{e_1 i_1 + e_2 i_2}{e_1^2 + e_2^2} - j\frac{e_2 i_1 - e_1 i_2}{e_1^2 + e_2^2} \\ &= g - jb; \end{aligned}$$

the conductance is

$$g = \frac{e_1 i_1 + e_2 i_2}{e_1^2 + e_2^2} = \frac{e_1 i_1 + e_2 i_2}{E^2}$$

and the susceptance is

$$b = \frac{e_2 i_1 - e_1 i_2}{e_1^2 + e_2^2} = \frac{e_2 i_1 - e_1 i_2}{E^2}.$$

The power factor of the circuit is

$$\begin{aligned} \cos\phi &= \cos(\phi_1 - \phi_2) \\ &= \cos\phi_1 \cos\phi_2 + \sin\phi_1 \sin\phi_2 \\ &= \frac{e_1 i_1 + e_2 i_2}{EI}. \end{aligned}$$

The power consumed in the circuit is

$$\begin{aligned} P &= EI\cos\phi \\ &= EI\frac{e_1 i_1 + e_2 i_2}{EI} \\ &= e_1 i_1 + e_2 i_2, \end{aligned} \tag{221}$$

and is the sum of the products of the components of the e.m.f. and current which are in phase. The products of the components of the e.m.f. and current which are in quadrature, namely, $e_1 i_2$ and $e_2 i_1$, do not represent power consumed.

The power may also be represented as

$$P = I^2r = e_1i_1 + e_2i_2,$$

or

$$P = E^2g = e_1i_1 + e_2i_2. \tag{222}$$

116. Examples in Rectangular Coördinates.—1. Find the current in the circuit in Fig. 113 in terms of the impressed e.m.f. and the constants of the circuit.

$$\dot{E}_1 = \dot{I}\,(r_1 + jx_1).$$
$$\dot{E}_2 = \dot{I}\,(r_2 + jx_2).$$
$$\dot{E} = \dot{E}_1 + \dot{E}_2 = \dot{I}\,\{(r_1 + r_2) + j(x_1 + x_2)\}.$$

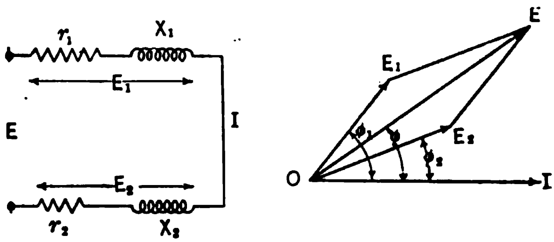


FIG. 113.

The impedance of the circuit is

$$\dot{Z} = (r_1 + r_2) + j(x_1 + x_2)$$

and its absolute value is

$$Z = \sqrt{(r_1 + r_2)^2 + (x_1 + x_2)^2};$$

and the absolute value of the current is

$$I = \frac{E}{Z} = \frac{E}{\sqrt{(r_1 + r_2)^2 + (x_1 + x_2)^2}};$$

the power factor of the circuit is

$$\cos\phi = \frac{r_1 + r_2}{Z}.$$

The vector diagram is drawn taking the current as the axis.

2. Solve the circuit in Fig. 114.

$$\dot{I}_1 = \frac{\dot{E}}{r_1 + jx_1} = \dot{E}\,(g_1 - jb_1)$$

where

$$g_1 = \frac{r_1}{r_1^2 + x_1^2} \text{ and } b_1 = \frac{x_1}{r_1^2 + x_1^2};$$

$$\dot{I}_2 = \frac{\dot{E}}{r_2 + jx_2} = \dot{E}\,(g_2 - jb_2);$$

the main current is

$$I = I_1 + I_2 = E\{(g_1 + g_2) - j(b_1 + b_2)\}$$

and its absolute value is

$$I = E\sqrt{(g_1 + g_2)^2 + (b_1 + b_2)^2}.$$

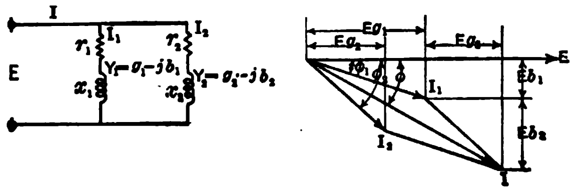


FIG. 114.

The admittance of the circuit is

$$Y = (g_1 + g_2) - j(b_1 + b_2)$$

and its absolute value is

$$Y = \sqrt{(g_1 + g_2)^2 + (b_1 + b_2)^2}.$$

The power factor of the circuit is

$$\cos\phi = \frac{g_1 + g_2}{Y}.$$

The vector diagram is drawn with the e.m.f. as axis.

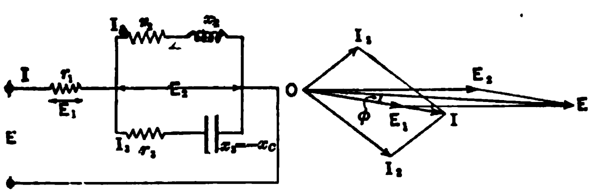


FIG. 115.

3. Solve the circuit in Fig. 115.

$$I_2 = \frac{E_2}{r_2 + jx_2} = E_2(g_2 - jb_2),$$

$$I_3 = \frac{E_2}{r_3 + jx_3} = E_2(g_3 - jb_3),$$

$$I = I_2 + I_3 = E_2\{(g_2 + g_3) - j(b_2 + b_3)\},$$

and therefore

$$E_2 = \frac{I}{(g_2 + g_3) - j(b_2 + b_3)} = I(R + jX),$$

where

$$R = \frac{g_2 + g_3}{(g_2 + g_3)^2 + (b_2 + b_3)^2} \text{ and } X = \frac{b_2 + b_3}{(g_2 + g_3)^2 + (b_2 + b_3)^2},$$

$$\dot{E}_1 = \dot{I}r_1$$

and

$$\dot{E} = \dot{E}_1 + \dot{E}_2 = \dot{I}\,(r_1 + R + jX).$$

The impedance of the circuit is

$$\dot{Z} = r_1 + R + jX$$

and its absolute value is

$$Z = \sqrt{(r_1 + R)^2 + X^2}.$$

The absolute value of the current is

$$I = \frac{E}{Z}.$$

The power factor of the circuit is

$$\cos\phi = \frac{r_1 + R}{Z}.$$

The vector diagram is drawn with E_2 as the axis.

4. If an e.m.f. $\dot{E} = 14 + j38$ is impressed on a circuit and a current $\dot{I} = 6 + j2$ flows, find the impedance of the circuit.

The impedance is

$$\dot{Z} = \frac{\dot{E}}{\dot{I}} = \frac{14 + j38}{6 + j2}$$

$$= \frac{14 + j38}{6 + j2} \times \frac{6 - j2}{6 - j2}$$

$$= \frac{160 + j200}{40} = 4 + j5;$$

the resistance of the circuit is 4 ohms and the inductive reactance is 5 ohms.

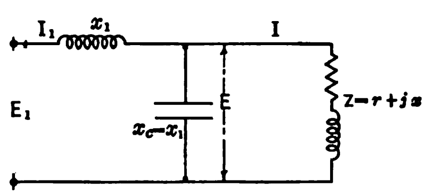


FIG. 116.—Constant potential to constant current.

5. In Fig. 116 a condensive reactance x_c is connected across the terminals of a receiver circuit of variable impedance $\dot{Z} = r + jx$. If an inductive reactance $x_1 = x_c$ is connected in the supply lines and a constant e.m.f. E_1 is impressed on the terminals, show

that the current in the receiver circuit is constant independent of the impedance and power factor.

$$\dot{E} = \text{e.m.f. at terminals of receiver,}$$

$$\dot{I} = \frac{\dot{E}}{\dot{Z}} = \text{current in receiver,}$$

$$\dot{I}_c = \frac{\dot{E}}{-jx_c} = \text{current in condensive reactance,}$$

$$\dot{I}_1 = \dot{I} + \dot{I}_c = \text{current in the line,}$$

$$\dot{E}_1 = \dot{E} + j\dot{I}_1 x_1 = \text{e.m.f. impressed.}$$

Substituting the values above

$$\begin{aligned}
\dot{E}_1 &= \dot{E} + jx_1(\dot{I} + \dot{I}_c) \\
&= \dot{E} + jx_1\left(\frac{\dot{E}}{\dot{Z}} - \frac{\dot{E}}{jx_c}\right) \\
&= \dot{E}\left(1 + j\frac{x_1}{\dot{Z}} - \frac{jx_1}{jx_c}\right) \\
&= j\frac{\dot{E}}{\dot{Z}}x_1, \text{ since } x_c = x_1 \\
&= j\dot{I}x_1
\end{aligned}$$

or in absolute values

$$E_1 = Ix_1$$

and

$$I = \frac{E_1}{x_1}.$$

Since E_1 is constant I is constant independent of the impedance and the power factor of the receiver circuit. This circuit therefore transforms power from constant potential to constant current.

117. Kirchoff's Laws Applied to Alternating-current Circuits.—Kirchoff's two laws enunciated in Art. 90 apply directly to alternating-current circuits when dealing with instantaneous values of e.m.fs. and currents; they also apply to the effective values of e.m.fs. and currents when combined in their proper phase relations. Thus the vector sum of all currents at a junction is zero; and the vector sum of all the e.m.fs. around a closed circuit is equal to the sum of the e.m.fs. consumed by the resistances in the circuit.

CHAPTER V

COMPLEX ALTERNATING-CURRENT WAVES

118. Complex Alternating Waves.—The waves of alternating current and voltage dealt with in the preceding articles have been assumed to be true sine waves. The majority of alternating waves met with in electrical engineering approach a sine form but comparatively few are true sine waves. The ordinary wave consists of a fundamental sine wave of the generator frequency with certain higher harmonics superimposed on it.

The maximum, average and effective values of such waves do not bear the same ratios to one another as in the case of the sine wave.

The maximum value is found as the maximum ordinate of the plotted wave or it may be determined mathematically if the equation of the wave is known.

The average value is found as the area under one-half wave divided by the base, π radians,

$$E_{\text{avg.}} = \frac{1}{\pi}\int_0^\pi e d\theta.$$

The effective value or root-mean-square value is

$$E_{\text{eff.}} = \sqrt{\frac{1}{\pi}\int_0^\pi e^2 d\theta}.$$

The form factor of the wave is defined as the ratio of the effective value to the average value and is denoted by γ,

$$\gamma = \frac{E_{\text{eff.}}}{E_{\text{avg.}}} = \frac{\sqrt{\frac{1}{\pi}\int_0^\pi e^2 d\theta}}{\frac{1}{\pi}\int_0^\pi e d\theta}. \tag{223}$$

In practice the complex waves are replaced by "equivalent" sine waves.

A sine wave of e.m.f. which is "equivalent" to a complex wave is one which has the same effective value and the same frequency as its fundamental and which also in conjunction with the current wave represents the same average power.

In the circuit in Fig. 117, e and i are complex waves of e.m.f. and current. The voltmeter V indicates the effective value of the e.m.f. $= E$ and the ammeter A indicates the effective value of the current $= I$. The wattmeter W indicates the average power consumed in the circuit $= P$. The equivalent sine wave of e.m.f. has an effective value E, the equivalent sine wave of current has an effective value I and these two waves are displaced by an angle ϕ which is such that $P = EI \cos \phi$ or $\phi = \frac{P}{EI}$.

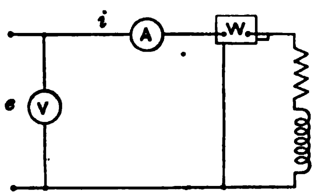


FIG. 117.

119. Examples.—Following are a number of problems dealing with complex alternating waves.

1. If an e.m.f. wave consists of a fundamental sine wave of effective value 100 and a fifth harmonic of effective value 10 passing through zero in the same direction together, find the maximum effective and average values of the wave.

The equation of the e.m.f. wave is

$$e = 100\sqrt{2} \sin \theta + 10\sqrt{2} \sin 5\theta.$$

From the original assumption the maximum value is the sum of the maxima of the fundamental and the harmonic and its value is $100\sqrt{2} + 10\sqrt{2} = 155.5$. In any other case it would be found by solving for the maximum value in the ordinary way or by plotting the wave.

The average value of the e.m.f. is

$$E_{avg.} = \frac{1}{\pi}\int_0^\pi e d\theta = \frac{1}{\pi}\int_0^\pi (100\sqrt{2} \sin \theta + 10\sqrt{2} \text{ in } 5\theta) d\theta.$$

$$= \frac{10\sqrt{2}}{\pi}\left[-10 \cos \theta - \frac{\cos 5\theta}{5}\right]_0^\pi = \frac{10\sqrt{2}}{\pi}\left[20 + 0.4\right] = 91.9.$$

The effective value is

$$E_{eff.} = \sqrt{\frac{1}{\pi}\int_0^\pi e^2 d\theta} = \sqrt{\frac{1}{\pi}\int_0^\pi (100\sqrt{2} \sin \theta + 10\sqrt{2} \sin 5\theta)^2 d\theta}$$

$$= 10\sqrt{2}\sqrt{\frac{1}{\pi}\int_0^\pi (100 \sin^2 \theta + 20 \sin \theta \sin 5\theta + \sin^2 5\theta) d\theta}$$

$$= 10\sqrt{2}\sqrt{\frac{1}{\pi}\int_0^\pi \left\{\frac{100}{2}(1 - \cos 2\theta) + \frac{20}{2}(\cos 4\theta - \cos 6\theta) + \tfrac{1}{2}(1 - \cos 10\theta)\right\} d\theta}$$

$$= 10\sqrt{2}\sqrt{\frac{1}{\pi}\left[50\left(\theta - \frac{\sin 2\theta}{2}\right) + 10\left(\frac{\sin 4\theta}{4} - \frac{\sin 6\theta}{6}\right) + \tfrac{1}{2}\left(\theta - \frac{\sin 10\theta}{10}\right)\right]_0^{\pi}}$$

$$= 10\sqrt{2}\sqrt{\frac{1}{\pi}\left[50\pi + \tfrac{1}{2}\pi\right]} = 10\sqrt{2}\sqrt{50.5} = 100.5.$$

The form factor of the wave is

$$\gamma = \frac{E_{\text{eff.}}}{E_{\text{avg.}}} = \frac{100.5}{91.9} = 1.093.$$

2. If the e.m.f. of the last example is impressed on the terminals of the circuit CD in Fig. 118(a) determine the current flowing. The fundamental frequency is 50 cycles per second.

The resistance of the circuit is 1 ohm, the inductive reactance is 2 ohms at 50 cycles and 10 ohms at 250 cycles and the condensive reactance is 50 ohms at 50 cycles and 10 ohms at 250 cycles.

The current wave is made up of a fundamental of $\dfrac{100}{\sqrt{1 + (50 - 2)^2}} = \dfrac{100}{48}$ $= 2.08$ amp. leading the fundamental e.m.f. by nearly 90 degrees and a fifth harmonic of $\dfrac{10}{\sqrt{1 + (10 - 10)^2}} = 10$ amp.

The circuit is resonant for the fifth harmonic of e.m.f. at 250 cycles and so the fifth harmonic current is very much exaggerated. The current is plotted in Fig. 118(b).

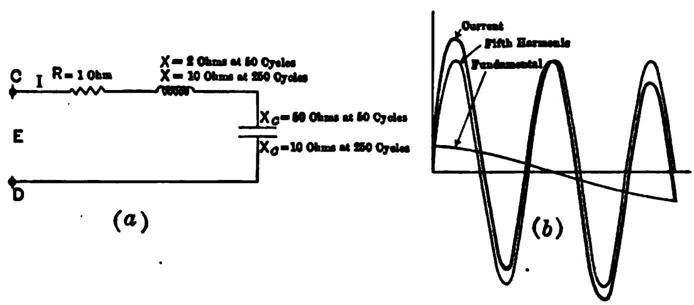


FIG. 118.

If the e.m.f. wave maintains its shape and value while the frequency of the fundamental is varied from 0 to 500 cycles per second, plot the effective values of the current on a frequency base.

In the table below are given the effective values of the fundamental and fifth harmonic currents at various frequencies and also the effective values of the resultant current.

The values and phase relations of the fundamental and fifth harmonic currents are found by treating the fundamental and fifth harmonic e.m.fs. separately. The equation of the resultant current can then be written down and the effective value found as in example 1.

At 50 cycles the equation of the current wave is $i = 2.08\sqrt{2} \sin (\theta + 90) + 10\sqrt{2} \sin 5\theta$ and its effective value is 10.21 amp.

f	$5f$	Fundamental current	Fifth harmonic current	Resultant current
0	0	0.00	0.00	0.00
50	250	2.08	10.00	10.21
100	500	4.76	6.60	8.12
200	1,000	21.70	0.27	21.70
250	1,250	100.00	0.21	100.00
300	1,500	26.70	0.17	26.70
400	2,000	10.20	0.13	10.20
500	2,500	6.70	0.10	6.70

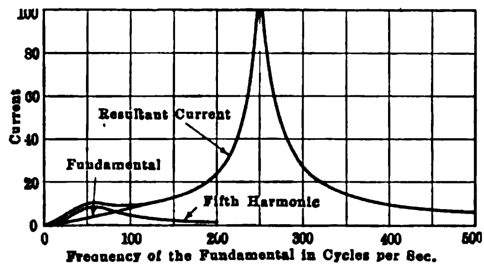


FIG. 119.

These values of current are plotted on a frequency base in Fig. 119. There are two peaks on the curve, first at 50 cycles where the circuit becomes resonant for the fifth harmonic and again at 250 cycles where the circuit becomes resonant for the fundamental.

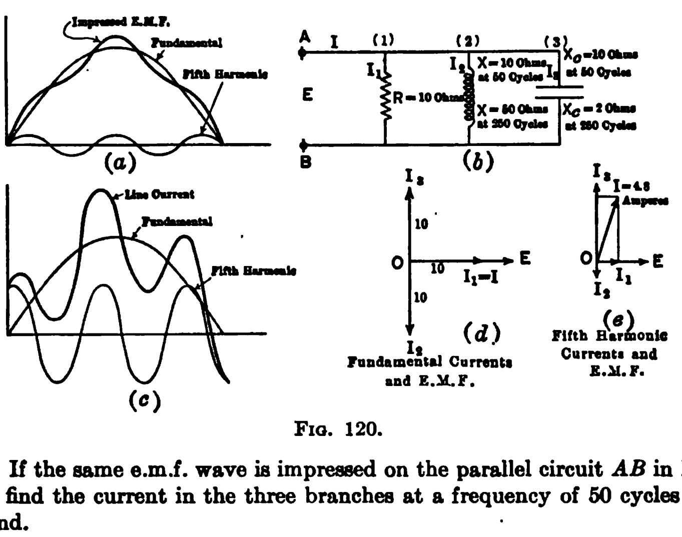


FIG. 120.

3. If the same e.m.f. wave is impressed on the parallel circuit AB in Fig. 120, find the current in the three branches at a frequency of 50 cycles per second.

The resistance of branch (1) is 10 ohms; the reactance of (2) is 10 ohms at

50 cycles and 50 ohms at 250 cycles; the condensive reactance of (3) is 10 ohms at 50 cycles and 2 ohms at 250 cycles.

The current I_1 in (1) consists of a fundamental of $\frac{100}{10} = 10$ amp. and a fifth harmonic of $\frac{10}{10} = 1$ amp. in phase with their respective e.m.fs. The equation of the current is

$$i_1 = 10\sqrt{2} \sin \theta + \sqrt{2} \sin 5\theta.$$

The wave shape is the same as that of the impressed e.m.f. Resistance, therefore, does not affect the wave shape.

The current I_2 in (2) consists of a fundamental of $\frac{100}{10} = 10$ amp. and a fifth harmonic of $\frac{10}{50} = 0.2$ amp. in quadrature behind the e.m.fs. producing them. The equation of the current is

$$i_2 = 10\sqrt{2} \sin (\theta - 90) + 0.2 \times \sqrt{2} \sin (5\theta - 90)$$

The fifth harmonic is not nearly so prominent as in the resistance circuit. Inductive reactance thus tends to eliminate the harmonics in irregular waves and makes them more sinusoidal in form.

The current I_3 in (3) consists of a fundamental of $\frac{100}{10} = 10$ amp. and a fifth harmonic $\frac{10}{2} = 5$ amp. in quadrature ahead of the e.m.fs. producing them. The equation of the current is

$$i_3 = 10\sqrt{2} \sin (\theta + 90) + 5\sqrt{2} \sin (5\theta + 90).$$

The fifth harmonic is much more prominent in the capacity circuit than in either of the others. Capacity, therefore, tends to exaggerate the harmonics in a peaked wave.

The main current I consists of a fundamental and a fifth harmonic. The fundamental is the resultant of the fundamental currents in the three branches and from Fig. 120(d) is found to be 10 amp. in phase with the fundamental e.m.f. The fifth harmonic is the resultant of the fifth harmonic currents in the three branches and from Fig. 120(e) it is found to be 4.8 amp., leading the fifth harmonic e.m.f. by approximately a quarter of a cycle. The equation of the resultant current is

$$i = 10\sqrt{2} \sin \theta + 4.8 \times \sqrt{2} \sin (5\theta + 90).$$

This current is shown in Fig. 120(c).

20. Analysis of Alternating Waves.—The majority of alternating-current problems are solved on the assumption that the waves of e.m.f. and current are sine waves or such that they can be represented by sine waves with sufficient accuracy. In certain cases, however, it is necessary to know the values of the more important harmonics in order to understand the phenomena completely.

The most general expression for a univalent periodic function may be given in the form of an infinite trigonometric series or Fourier series,

$$y = a_0 + a_1 \cos \theta + a_2 \cos 2\theta + + a_n \cos n\theta + b_1 \sin \theta + b_2 \sin 2\theta + + b_n \sin n\theta \quad (224)$$

where a_0, a_1, a_2, b_1, b_2, etc., are constants; or combining the sine and cosine terms it may be expressed as

$$y = a_0 + c_1 \sin(\theta + \phi_1) + c_2 \sin(2\theta + \phi_2) + + \\ c_n \sin(n\theta + \phi_n) \quad (225)$$

where

$$c_n = \sqrt{a_n^2 + b_n^2}, \quad \sin\phi_n = \frac{a_n}{\sqrt{a_n^2 + b_n^2}}, \quad \cos\phi_n = \frac{b_n}{\sqrt{a_n^2 + b_n^2}},$$

$$\tan\phi_n = \frac{a_n}{b_n}. \quad (226)$$

If the ordinates of such a function for one complete cycle of the fundamental, are given in form of a curve as obtained from an oscillograph record or tabulated as on page 149, the constants a_n and b_n for any harmonic can be found as explained below.

To find the coefficient a_0 integrate equation (224), from 0 to 2π.

$$A = \int_0^{2\pi} y\,d\theta = \int_0^{2\pi} (a_0 + a_1 \cos\theta + a_2 \cos 2\theta + + + a_n \cos n\theta + b_1 \sin\theta + b_2 \sin 2\theta + + + b_n \sin n\theta)d\theta = a_0[\theta]_0^{2\pi} = 2\pi a_0,$$

since all the other terms vanish between these limits.

Thus $a_0 = \dfrac{A}{2\pi}$ = average value of the ordinate between θ and 2π. The coefficient a_n is found by multiplying all the terms in equation (224) by $\cos n\theta$ and integrating from 0 to 2π. This gives

$$\begin{aligned} A_n = \int_0^{2\pi} y \cos n\theta\,d\theta &= \int_0^{2\pi} (a_0 \cos n\theta + a_1 \cos n\theta \cdot \cos\theta + + + \\ &\quad a_n \cos^2 n\theta + b_1 \cos n\theta \sin\theta + + + b_n \cos n\theta \sin n\theta)d\theta \\ &= \int_0^{2\pi} \Big[a_0 \cos n\theta + \frac{a_1}{2}\{\cos(n-1)\theta + \cos(n+1)\theta\} + + + \\ &\quad \frac{a_n}{2}(1 + \cos 2n\theta) + \frac{b_1}{2}\{\sin(n+1)\theta - \sin(n-1)\theta\} + + + \\ &\quad \frac{b_n}{2}(\sin 2n\theta)\Big]d\theta \\ &= \frac{a_n}{2}[\theta]_0^{2\pi} = 2\pi\frac{a_n}{2} \end{aligned}$$

and

$$a_n = 2\left(\frac{A_n}{2\pi}\right) = 2 \text{ avg. } (y \cos n\theta)_0^{2\pi}$$

b_n is found by multiplying equation (224) by $\sin n\theta$ and integrating as before.

$$\begin{aligned} B_n = \int_0^{2\pi} y \sin n\theta\,d\theta &= \int_0^{2\pi} (a_0 \sin n\theta + a_1 \sin n\theta \cdot \cos\theta + + + \\ &\quad a_n \sin n\theta \cos n\theta + b_1 \sin n\theta \cdot \sin\theta + + + b_n \sin^2 n\theta)d\theta \end{aligned}$$

$$= \int_0^{2\pi} \Big[a_0 \sin n\theta + \frac{a_1}{2}\{\sin (n+1)\theta + \sin (n-1)\theta\} + + + \frac{a_n}{2}(\sin 2n\theta) + \frac{b_1}{2}\{\cos (n-1)\theta - \cos (n+1)\theta\} + + + \frac{b_n}{2}(1 - \cos 2n\theta)\Big] d\theta$$

$$= \frac{b_n}{2}\Big[\theta\Big]_0^{2\pi} = 2\pi\frac{b_n}{2}$$

and

$$b_n = 2\left(\frac{B_n}{2\pi}\right) = 2 \text{ avg. } (y \sin n\theta)_0^{2\pi}$$

Applying these results,

$$a_0 = \text{avg. } (y)_0^{2\pi}$$

$$a_1 = 2 \text{ avg. } (y \cos \theta)_0^{2\pi} \qquad b_1 = 2 \text{ avg. } (y \sin \theta)_0^{2\pi}$$

$$a_2 = 2 \text{ avg. } (y \cos 2\theta)_0^{2\pi} \qquad b_2 = 2 \text{ avg. } (y \sin 2\theta)_0^{2\pi}$$

$$a_n = 2 \text{ avg. } (y \cos n\theta)_0^{2\pi} \qquad b_n = 2 \text{ avg. } (y \sin n\theta)_0^{2\pi}$$

Practically all of the waves met with in electrical engineering are symmetrical waves having the two half waves of the same shape but opposite in sign. The values of the ordinates from 180 to 360 degrees are the same as those from 0 to 180 degrees but their signs are different.

In symmetrical waves only those harmonics can exist which reverse in sign when θ is increased by 180 degrees. Therefore, all the even harmonics are absent and there is no constant term.

The general expression for a symmetrical alternating wave may be put in the form

$$y = a_1 \cos \theta + a_3 \cos 3\theta + a_5 \cos 5\theta + + + b_1 \sin \theta + b_3 \sin 3\theta + + \qquad (227)$$

or combining the sine and cosine terms

$$y = c_1 \sin (\theta + \phi_1) + c_3 \sin (3\theta + \phi_3) + c_5 \sin (3\theta + \phi_5) \qquad (228)$$

where the values c_1, c_3, ϕ_1, ϕ_3, etc., are found as indicated in equation 226.

In order to analyze such a wave into its component harmonics it is necessary to know the values of the ordinates for one-half wave only and the constants are determined from the following equations,

$$a_1 = 2 \text{ avg. } (y \cos \theta)_0^{\pi} \qquad b_1 = 2 \text{ avg. } (y \sin \theta)_0^{\pi}$$

$$a_3 = 2 \text{ avg. } (y \cos 3\theta)_0^{\pi} \qquad b_3 = 2 \text{ avg. } (y \sin 3\theta)_0^{\pi}$$

$$a_5 = 2 \text{ avg } (y \cos 5\theta)_0^{\pi} \qquad b_5 = 2 \text{ avg. } (y \sin 5\theta)_0^{\pi}$$

121. Example of Analysis.—One-half of an alternating-current wave is shown as curve 1, Fig. 121, and it may be represented by the expression,

$$y = a_1 \cos\theta + a_3 \cos 3\theta + a_5 \cos 5\theta + + + b_1 \sin\theta + b_3 \sin 3\theta + b_5 \sin 5\theta + +$$

or by

$$y = c_1 \sin(\theta + \phi_1) + c_3 \sin(3\theta + \phi_3) + c_5 \sin(5\theta + \phi_5) + \text{higher harmonics.}$$

The values required to determine the fundamental, and third and fifth harmonics are tabulated below.

$\theta°$	y	y^2	$\cos\theta$	$\sin\theta$	$y\cos\theta$	$y\sin\theta$	$\cos 3\theta$	$\sin 3\theta$	$y\cos 3\theta$	$y\sin 3\theta$	$\cos 5\theta$	$\sin 5\theta$	$y\cos 5\theta$	$y\sin 5\theta$
0	0.0	0.0	1.000	0.000	0.00	0.00	1.000	0.000	0.00	0.00	1.000	0.000	0.00	0.00
10	5.0	25.0	0.985	0.174	4.85	0.87	0.866	0.500	4.33	2.50	0.643	0.766	3.22	3.84
20	8.2	67.2	0.940	0.342	7.70	2.80	0.500	0.866	4.10	7.10	−0.174	0.985	− 1.43	8.07
30	9.7	94.0	0.866	0.500	8.40	4.85	0.000	1.000	0.00	9.70	−0.866	0.500	− 8.40	4.85
40	11.0	121.0	0.766	0.643	8.42	7.07	−0.500	0.866	− 5.50	9.53	−0.940	−0.342	−10.34	− 3.76
50	12.0	144.0	0.643	0.766	7.71	9.19	−0.866	0.500	−10.40	6.00	−0.342	−0.940	− 4.10	−11.28
60	13.7	187.7	0.500	0.866	6.85	11.86	−1.000	0.000	−13.70	0.00	0.500	−0.866	6.85	−11.85
70	15.1	228.0	0.342	0.940	5.16	14.15	−0.866	−0.500	−13.06	− 7.55	0.985	−0.174	14.87	− 2.62
80	17.7	313.3	0.174	0.985	3.70	17.40	−0.500	−0.866	− 8.85	−15.32	0.766	0.643	13.56	11.37
90	25.7	660.5	0.000	1.000	0.00	25.70	0.000	−1.000	0.00	−25.70	0.000	1.000	0.00	25.70
100	30.5	930.2	−0.174	0.985	− 5.28	30.00	0.500	−0.866	15.25	−26.40	−0.766	0.643	−23.40	19.60
110	38.5	1482.2	−0.342	0.940	− 13.30	36.00	0.866	−0,500	33.35	−19.25	−0.985	−0.174	−37.90	− 6.70
120	44.0	1936.0	−0.500	0.866	− 22.00	38.10	1.000	0.000	44.00	0.00	−0.500	−0.866	−22.00	−38.20
130	46.0	2116.0	−0.643	0.766	− 28.60	35.30	0.866	0.500	39.85	23.00	0.342	−0.940	15.70	−43.20
140	40.8	1664.6	−0.766	0.643	− 31.30	26.20	0.500	0.866	20.40	35.30	0.940	−0.342	38.35	−13.83
150	30.0	900.0	−0.866	0.500	− 26.00	15.00	0.000	1.000	0.00	30.00	0.866	0.500	25.98	15.00
160	19.0	361.0	−0.940	0.342	− 17.87	6.50	−0.500	0.866	− 9.50	16.46	0.174	0.985	3.30	18.70
170	10.0	100.0	−0.985	0.174	− 9.85	1.74	−0.866	0.500	− 8.66	5.00	−0.643	0.766	− 6.43	7.66
Total	376.9	11330.7	.....	.....	−101.41	282.73	....	....	91.59	50.37	....	....	7.83	−16.65

$$a_1 = 2\,\text{avg.}\,(y\cos\theta)_0^\pi = 2\left(\frac{-101.41}{18}\right) = -11.27$$

$$b_1 = 2\,\text{avg.}\,(y\sin\theta)_0^\pi = 2\left(\frac{282.73}{18}\right) = 31.41$$

$$c_1 = \sqrt{a_1^2 + b_1^2} = 33.3$$

$$\sin\phi_1 = \frac{a_1}{c_1} = \frac{-11.27}{33.3} = -0.338$$

$$\cos\phi_1 = \frac{b_1}{c_1} = \frac{31.41}{33.3} = 0.944$$

$$\tan\phi_1 = \frac{a_1}{b_1} = \frac{-11.27}{31.41} = -0.359$$

$$\phi_1 = -19°\,45'$$

$$a_3 = 2\,\text{avg.}\,(y\cos 3\theta)_0^\pi = 2\left(\frac{91.59}{18}\right) = 10.17$$

$$b_3 = 2\,\text{avg.}\,(y\sin 3\theta)_0^\pi = 2\left(\frac{50.37}{18}\right) = 5.596$$

$$c_3 = \sqrt{a_3^2 + b_3^2} = 11.6$$

$$\sin\phi_3 = \frac{a_3}{c_3} = \frac{10.17}{11.6} = 0.877$$

$$\cos\phi_3 = \frac{b_3}{c_3} = \frac{5.596}{11.6} = 0.483$$

$$\tan\phi_3 = \frac{a_3}{b_3} = \frac{10.17}{5.596} = 1.82$$

$$\phi_3 = 61°\,10'$$

$$a_5 = 2\,\text{avg.}\,(y\cos 5\theta)_0^\pi = 2\left(\frac{7.83}{18}\right) = 0.87$$

$$b_5 = 2\,\text{avg.}\,(y\sin 5\theta)_0^\pi = 2\left(\frac{-16.65}{18}\right) = -1.85$$

$$c_5 = \sqrt{a_5^2 + b_5^2} = 2.05$$

$$\sin\phi_5 = \frac{a_5}{c_5} = \frac{0.87}{2.05} = 0.434$$

$$\cos\phi_5 = \frac{b_5}{c_5} = \frac{-1.85}{2.05} = -0.902$$

$$\tan\phi_5 = \frac{a_5}{b_5} = \frac{0.87}{-1.85} = -0.47$$

$$\phi_5 = 154°\,49'$$

The fundamental is $y_1 = c_1 \sin(\theta + \phi_1) = 33.3 \sin(\theta - 19°45')$, Curve 2, Fig. 121; the third harmonic is $y_3 = c_3 \sin(3\theta + \phi_3) = 11.6 \sin(3\theta + 61° 10')$, Curve 3, Fig. 121; the fifth harmonic is $y_5 = c_5 \sin(5\theta + \phi_5) = 2.05 \sin(5\theta + 154° 49')$, Curve 4, Fig.

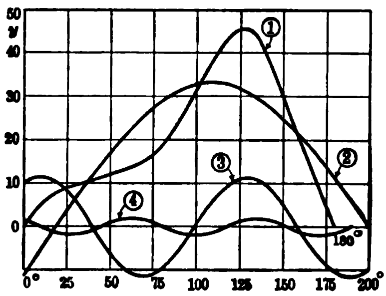


FIG. 121.

121; and the complete expression for the alternating-current wave is $y = 33.3 \sin(\theta - 19° 45') + 11.6 \sin(3\theta + 61° 10') + 2.05 \sin(5\theta + 154° 49')$ + higher harmonics which are negligible.

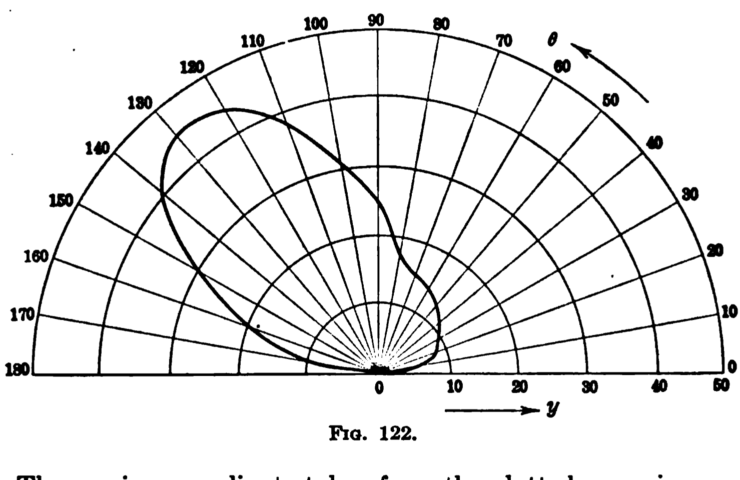


FIG. 122.

The maximum ordinate taken from the plotted curve is

$$y_{max.} = 46;$$

The average ordinate is found from column (1)

$$y_{avg.} = \frac{376.9}{18} = 20.9;$$

the effective value is obtained from column (3)

$$y_{\text{eff.}} = \sqrt{\frac{11,330.7}{18}} = 25.2$$

and the form factor is

$$\gamma = \frac{25.2}{20.9} = 1.20.$$

The effective value may also be found by plotting the values of y on polar coördinate paper as in Fig. 122 and measuring the area of the curve A_p by means of a planimeter.

The area of a polar curve is

$$A_p = \frac{1}{2}\int_0^{\pi} y^2 d\theta,$$

and therefore the effective value is

$$y_{\text{eff.}} = \sqrt{\frac{1}{\pi}\int_0^{\pi} y^2 d\theta} = \sqrt{\frac{2A_p}{\pi}}.$$

The wave analyzed above was taken from an oscillograph record of the exciting current of a transformer.

CHAPTER VI

POLYPHASE ALTERNATING-CURRENT CIRCUITS

122. Polyphase Alternating-current Circuits.—In Art. 102 the generation of an alternating e.m.f. was discussed. Fig. 123 represents a simple single-phase alternator. The field poles are excited by direct current and an alternating e.m.f. is produced in the winding which is carried on the armature. It does not make

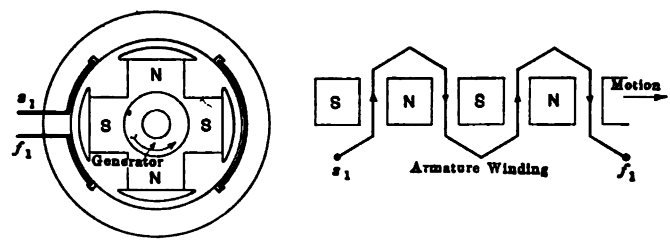


FIG. 123.—Single-phase alternator, revolving field type.

any difference whether the poles are fixed and the armature winding moves or the winding is fixed and the poles move.

The winding starts at s_1 and ends at f_1 and the equation of the e.m.f. between s_1 and f_1 is

$$e_1 = E_m \sin \theta, \text{ effective value } = \frac{E_m}{\sqrt{2}} = E = E_1.$$

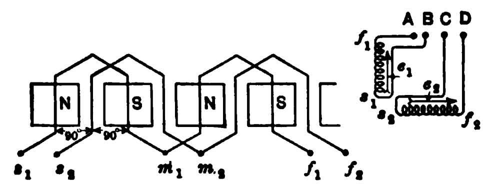


FIG. 124.—Two-phase winding.

The positive direction of e_1 is from s_1 to f_1. It is here assumed that the flux in the air gap is so distributed that the e.m.f. is a sine wave.

If a second winding is placed on the same armature, Fig. 124, but displaced 90 electrical degrees from the first, it will have an

e.m.f. generated in it of the same value and wave form as the first but displaced from it in phase by 90 degrees (Figs. 125 and 126).

Phase 2 starts at s_2 and ends at f_2 and the e.m.f. generated in it is

$$e_2 = E_m \sin (\theta - 90), \text{ effective value } = \frac{E_m}{\sqrt{2}} = E = E_2.$$

The machine is called a two-phase alternator.

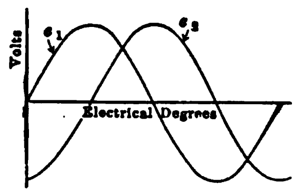


FIG. 125.—E.m.f. waves of a two-phase alternator.

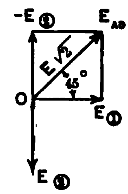


FIG. 126.

The two windings are usually entirely separate and their ends are brought out to four terminals $ABCD$.

If any two terminals as B and C are joined, the e.m.f. between A and D is

$$\begin{aligned} e_{AD} = e_1 - e_2 = E_m \sin \theta - E_m \sin (\theta - 90) \\ = E_m (\sin \theta + \cos \theta) = \sqrt{2} E_m \sin(\theta + 45); \end{aligned}$$

it leads e_1 in phase by 45 degrees, its maximum value is $\sqrt{2}E_m$ and its effective value is $E_{AD} = \frac{\sqrt{2}E_m}{\sqrt{2}} = E_m = \sqrt{2}E$. This value can also be obtained by subtracting the two vectors as shown in Fig. 126.

The middle points m_1 and m_2 of the two windings are sometimes connected together and a fifth terminal F used as shown in Fig. 127. The common terminal F is called the neutral point of the winding and may be connected to earth. The e.m.f. from each of the other terminals to F is the same,

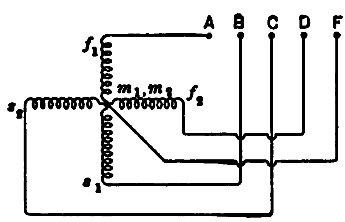


FIG. 127.

$$E_{AF} = E_{BF} = E_{CF} = E_{DF} = \frac{E}{2}.$$

The four e.m.fs. E_{AC}, E_{CB}, E_{BD} and E_{DA} are equal, since each is the vector difference of two e.m.fs. of effective value $\frac{E}{2}$ at right angles to one another, and these four e.m.fs. are also at right angles to one another and form a four-phase or quarter-phase system.

The effective value of each of the four e.m.fs. is

$$\sqrt{\left(\frac{E}{2}\right)^2 + \left(\frac{E}{2}\right)^2} = \frac{E}{\sqrt{2}}.$$

123. Three-phase Circuits.—If three similar windings are placed on the same alternator armature displaced 120 electrical degrees from one another, Fig. 128, and the ends of the windings

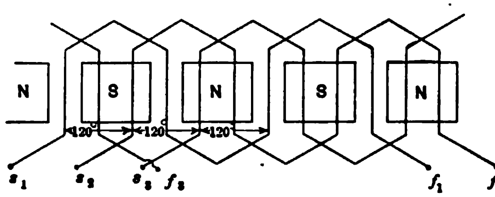


FIG. 128.—Three-phase winding.

are brought out to terminals, the machine is a three-phase alternator. The e.m.fs. generated in the three windings are displaced 120 degrees.

The e.m.f. in phase 1 is $e_1 = E_m \sin \theta$, effective value $E = \frac{E_m}{\sqrt{2}}$; the e.m.f. in phase 2 is $e_2 = E_m \sin (\theta - 120)$, effective value $E = \frac{E_m}{\sqrt{2}}$; and the e.m.f. in phase 3 is $e_3 = E_m \sin (\theta - 240)$, effective value $E = \frac{E_m}{\sqrt{2}}$.

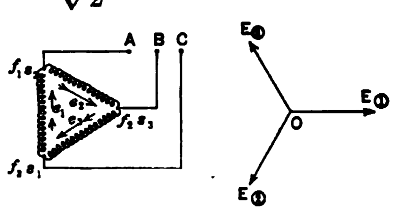


FIG. 129.—Delta connection.

The windings may be interconnected in two ways. (1) Join f_1 to s_2, f_2 to s_3 and f_3 to s_1, and connect the three junctions to the terminals A, B and C (Fig. 129) This is called the "delta" connection or ring connection and is represented by Δ.

The resultant e.m.f. around the closed circuit at any instant is

$$\begin{aligned} e_1 + e_2 + e_3 &= E_m \sin\theta + E_m \sin(\theta - 120) + E_m \sin(\theta - 240) \\ &= E_m (\sin\theta + \sin\theta\cos 120 - \cos\theta\sin 120 \\ &\quad + \sin\theta\cos 240 - \cos\theta\sin 240) \\ &= E_m \left(\sin\theta - 2\sin\theta - \frac{\sqrt{3}}{2}\cos\theta - \tfrac{1}{2}\sin\theta \right. \\ &\quad \left. + \frac{\sqrt{3}}{2}\cos\theta\right) = 0. \end{aligned}$$

2. If the ends s_1, s_2 and s_3 are connected together and the ends f_1, f_2 and f_3 are joined to the three terminals A, B and C, Fig. 130, the windings are connected Y or "star."

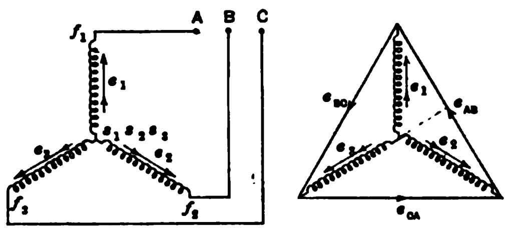


FIG. 130.—Star or "Y" connection.

The e.m.f. between A and B is

$$\begin{aligned} e_{AB} &= e_1 - e_2 = E_m \sin\theta - E_m \sin(\theta - 120) \\ &= E_m(\sin\theta - \sin\theta\cdot\cos 120^\circ + \cos\theta\cdot\sin 120) \\ &= E_m\left(\sin\theta + \tfrac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta\right) \\ &= \sqrt{3}\, E_m\left(\frac{\sqrt{3}}{2}\sin\theta + \tfrac{1}{2}\cos\theta\right) \\ &= \sqrt{3}\, E_m \sin(\theta + 30); \end{aligned} \tag{230}$$

its maximum value is $\sqrt{3}\, E_m$ and its effective value is

$$\sqrt{3}\frac{E_m}{\sqrt{2}} = \sqrt{3}\, E$$

Similarly the e.m.f. between B and C is

$$\begin{aligned} e_{BC} &= e_2 - e_3 = E_m \{\sin(\theta - 120) - \sin(\theta - 240)\} \\ &= \sqrt{3}E_m \sin(\theta - 90) \end{aligned} \tag{231}$$

of maximum value $\sqrt{3}\, E_m$ and effective value $\sqrt{3}E$.

The e.m.f. between C and A is

$$\begin{aligned} e_{CA} &= e_3 - e_1 = E_m \{\sin(\theta - 240) - \sin\theta\} \\ &= \sqrt{3}E_m \sin(\theta - 210) \end{aligned} \tag{232}$$

of maximum value $\sqrt{3}E_m$ and effective value $\sqrt{3}E$.

Thus the three e.m.fs. between terminals are equal to one another and are displaced in phase by 120 degrees.

A fourth terminal is usually connected to the neutral point O and it may be grounded.

124. Electromotive Forces, Currents and Power in Three-phase Circuits.—Fig. 131 shows a delta-connected three-phase system.

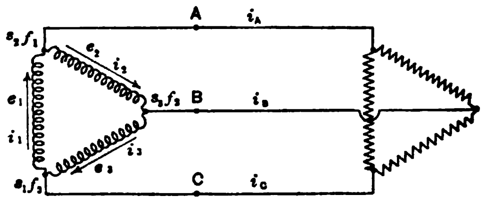


FIG. 131.

The e.m.f. in phase 1 is

$$e_1 = E_m \sin \theta, \text{ effective value } E_1 = \frac{E_m}{\sqrt{2}};$$

the e.m.f. in phase 2 is

$$e_2 = E_m \sin (\theta - 120), \text{ effective value } E_2 = \frac{E_m}{\sqrt{2}};$$

the e.m.f. in phase 3 is

$$e_3 = E_m \sin (\theta - 240), \text{ effective value } E_3 = \frac{E_m}{\sqrt{2}}.$$

The effective values of the e.m.fs. in the three phases are equal and are the terminal e.m.fs. of the alternator or the e.m.fs. between lines,

$$E_t = E_1 = E_2 = E_3 = \frac{E_m}{\sqrt{2}}.$$

Assume that the loads on the three phases are balanced, that is, that the currents have the same effective value and are displaced from their respective e.m.fs. by the same angle ϕ.

The current in phase 1 is

$$i_1 = I_m \sin (\theta - \phi), \text{ effective value } I_1 = \frac{I_m}{\sqrt{2}};$$

the current in phase 2 is

$$i_2 = I_m \sin (\theta - \phi - 120), \text{ effective value } I_2 = \frac{I_m}{\sqrt{2}};$$

the current in phase 3 is

$$i_3 = I_m \sin (\theta - \phi - 240), \text{ effective value } I_3 = \frac{I_m}{\sqrt{2}}.$$

The effective value of the currents in the three phases is the same and is $I = \frac{I_m}{\sqrt{2}}$.

The current in line A is

$$i_A = i_1 - i_2 = I_m \sin(\theta - \phi) - I_m \sin(\theta - \phi - 120)$$
$$= \sqrt{3} I_m \sin(\theta - \phi + 30), \qquad (233)$$

and its effective value is

$$I_A = \sqrt{3}\,\frac{I_m}{\sqrt{2}} = \sqrt{3}\,I.$$

The current in line B is

$$i_B = i_2 - i_3 = I_m \sin(\theta - \phi - 120) - I_m \sin(\theta - \phi - 240)$$
$$= \sqrt{3} I_m \sin(\theta - \phi - 90), \qquad (234)$$

and its effective value is

$$I_B = \sqrt{3} I.$$

The current in line C is

$$i_C = i_3 - i_1 = I_m \sin(\theta - \phi - 240) - I_m \sin(\theta - \phi)$$
$$= \sqrt{3} I_m \sin(\theta - \phi - 210), \qquad (235)$$

and its effective value is

$$I_C = \sqrt{3} I.$$

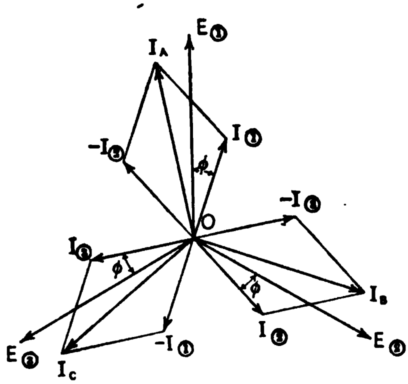


Fig. 132.

Thus the currents in the three lines are equal in magnitude and are 120 degrees out of phase with one another. If the effective value of the current in each of the lines is represented by I_l then

$$I_l = I_A = I_B = I_C = \sqrt{3} I.$$

The effective values of all these quantities are shown in the vector diagram in Fig. 132.

The power supplied by the alternator is

$$P = E_1 I_1 \cos\phi + E_2 I_2 \cos\phi + E_3 I_3 \cos\phi$$
$$= 3EI \cos\phi \qquad (236)$$
$$= \sqrt{3} E_t I_l \cos\phi \qquad (237)$$

and is equal to $\sqrt{3}$ times the product of the terminal e.m.f., the line current and the power factor.

If the system is not balanced, that is, if either the currents or the power factors in the three phases differ from one another the line currents will not be equal and they will not be displaced in phase by 120 degrees.

If the current in phase 1 is

$$i_1 = I_{m_1} \sin(\theta - \phi_1)$$

and the current in phase 2 is

$$i_2 = I_{m_2} \sin(\theta - \phi_2 - 120)$$

the current in line A is

$$i_A = i_1 - i_2$$
$$= I_{m_1} \sin(\theta - \phi_1) - I_{m_2} \sin(\theta - \phi_2 - 120). \qquad (238)$$

Fig. 133 shows a star- or Y-connected three-phase system

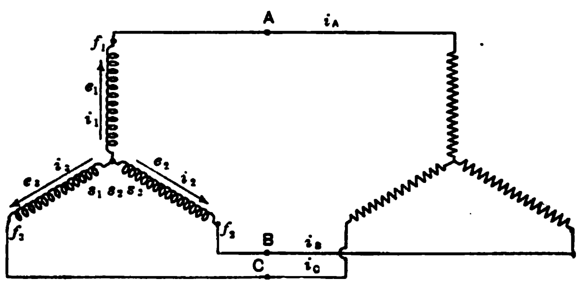


FIG. 133.

Using the same notation as before and taking the results obtained in Art. 123,

the e.m.f. between lines A and B is

$e_{AB} = e_1 - e_2 = \sqrt{3} E_m \sin(\theta + 30)$, effective value $E_{AB} = \sqrt{3} E = E_t$;

the e.m.f. between lines B and C is

$e_{BC} = e_2 - e_3 = \sqrt{3} E_m \sin(\theta - 90)$, effective value $E_{BC} = \sqrt{3} E = E_t$;

the e.m.f. between lines C and A is

$e_{CA} = e_3 - e_1 = \sqrt{3} E_m \sin(\theta - 210)$, effective value $E_{CA} = \sqrt{3} E = E_t$;

the current in line A is

$$i_A = i_1 = I_m \sin(\theta - \phi), \text{ effective value } I_A = \frac{I_m}{\sqrt{2}} = I_l;$$

the current in line B is

$$i_B = i_2 = I_m \sin(\theta - \phi - 120), \text{ effective value } I_B = \frac{I_m}{\sqrt{2}} = I_l;$$

the current in line C is

$$i_C = i_3 = I_m \sin(\theta - \phi - 240), \text{ effective value } I_C = \frac{I_m}{\sqrt{2}} = I_l.$$

Fig. 134 is a vector diagram for this circuit.

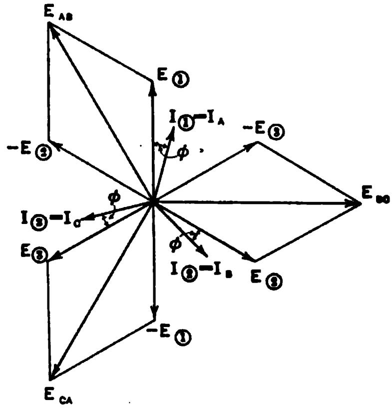


FIG. 134.

125. Measurement of Power in Polyphase Circuits.—The power in two-phase circuits (Fig. 135) can be measured by connecting wattmeters in the two phases and taking the sum of

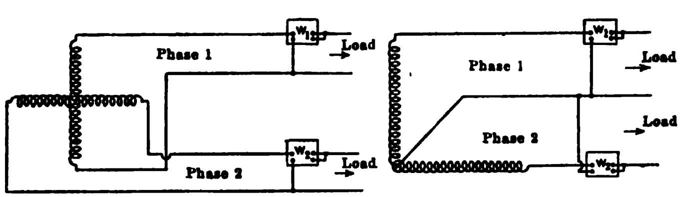


FIG. 135.—Measurement of power in two-phase circuits.

the readings. If the two phases are similarly loaded a single meter will suffice if its reading is doubled.

In the three-phase circuits (Fig. 136) the total power is given by the sum of the two wattmeter readings, if they are connected,

as shown at a, with their current coils in two of the lines and their voltage coils connected across from these lines to the third line.

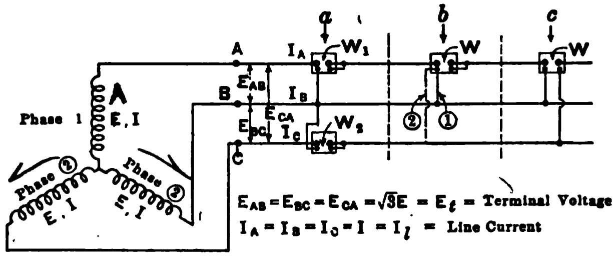


FIG. 136.—Power in balanced three-phase circuits.

Consider first the case where the loads on the three phases are balanced. Referring to the vector diagram, Fig. 137, for the Y—connected circuit,

$$W_1 = E_{AB}I_A \cos (30 + \phi) = \sqrt{3}EI \cos (30 + \phi).$$
$$W_2 = E_{CB}I_C \cos (30 - \phi) = \sqrt{3}EI \cos (30 - \phi).$$

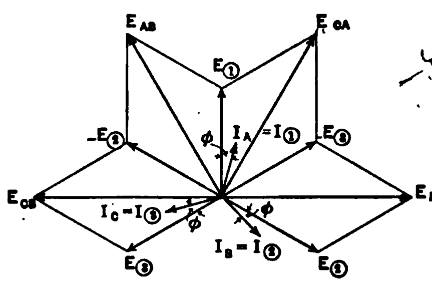


FIG. 137.

Adding,

$$W_1 + W_2 = \sqrt{3}EI\ \{\cos (30 + \phi) + \cos (30 - \phi)\}$$
$$= \sqrt{3}EI\ \{(\cos 30 \cos \phi - \sin 30 \sin \phi) + (\cos 30 \cos \phi + \sin 30 \sin \phi)\}$$
$$= \sqrt{3}EI(2 \cos 30 \cos \phi) = 3EI \cos \phi = \sqrt{3}E_tI_l \cos \phi$$

and this is the total power in the circuit.

The power factor of the circuit can also be found from the two wattmeter readings.

Subtracting,

$$W_2 - W_1 = \sqrt{3}EI\ (2 \sin 30 \sin \phi) = \sqrt{3}EI \sin \phi.$$

and
$$\frac{W_2 - W_1}{W_2 + W_1} = \frac{\sqrt{3}EI \sin \phi}{3EI \cos \phi} = \frac{1}{\sqrt{3}} \tan \phi$$

or

$$\tan \phi = \sqrt{3}\,\frac{W_2 - W_1}{W_2 + W_1} \tag{239}$$

and

$$\cos \phi = \frac{W_2 + W_1}{\sqrt{(W_2 + W_1)^2 + 3(W_2 - W_1)^2}}. \tag{240}$$

When the power factor is 50 per cent., $\phi = 60$ degrees and the reading of wattmeter W_1 becomes zero,

$$W_1 = E_{AB}I_A \cos (30 + 60) = 0.$$

For lower power factors the reading of W_1 would be negative and the total power in the circuit is given by the difference of the readings.

In some cases it may be difficult to tell whether the readings should be added or subtracted. The curve in Fig. 138 makes it possible to determine the sign of W_1. It shows the relation between power factor and the ratio of the readings of the two meters, $\cos \phi$ vs. $\frac{W_1}{W_2}$.

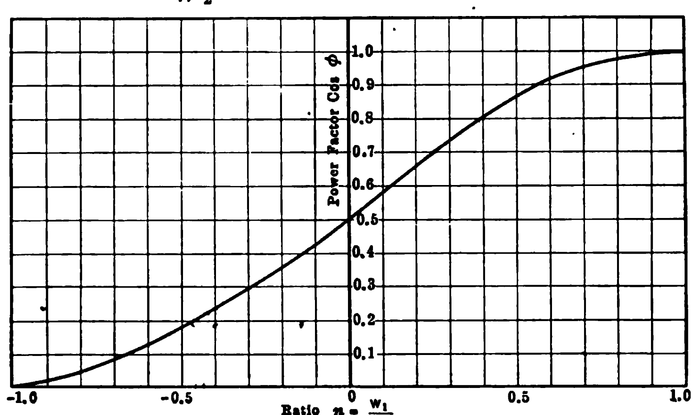


FIG. 138.—Watt-ratio curve.

The equation of the curve may be obtained from equation (240).

$$\operatorname{Cos} \phi = \frac{W_2 + W_1}{\sqrt{(W_2 + W_1)^2 + 3(W_2 - W_1)^2}} = \frac{1}{\sqrt{1 + 3\left(\frac{W_2 - W_1}{W_2 + W_1}\right)^2}}$$

$$= \frac{1}{\sqrt{1 + 3\frac{\left(1 - \frac{W_1}{W_2}\right)^2}{\left(1 + \frac{W_1}{W_2}\right)^2}}} \tag{241}$$

Above 50 per cent. power factor W_1 is positive and $\frac{W_1}{W_2}$ is positive. At 50 per cent. power factor W_1 and $\frac{W_1}{W_2}$ become zero. Below 50 per cent. power factor W_1 and $\frac{W_1}{W_2}$ are negative. If in any case it is doubtful whether W_1 is positive or negative, try it both ways and calculate the corresponding value of cos ϕ from equation (241) and then referring to Fig. 138 see which assumption agrees with the watt-ratio curve.

If the current and voltage readings are also given the determination of cos ϕ is simplified.

$$\text{Power factor} = \cos\phi = \frac{\text{True power}}{\text{Apparent power}} = \frac{W_2 \pm W_1}{\sqrt{3}E_lI_l}.$$

The following readings taken on a three-phase induction motor running light show the value of the watt ratio curve.

E	I	W_1	W_2	$\frac{W_1}{W_2}$	$\text{Cos}\,\phi = \frac{W_2 \pm W_1}{\sqrt{3}EI}$	Cos ϕ from Fig. 138	Correct value of W_1
20	1.01	+4	19	+0.21	0.656	0.66	+4
		−4		−0.21	0.428	0.35	
40	1.24	+5	40	+0.125	0.528	0.60	
		−5		−0.125	0.410	0.41	−5

In the first case the wattmeter readings must be added while in the second they must be subtracted.

It is possible to determine the total power in a balanced three-phase circuit with one wattmeter without opening the circuit to change the current coil from one line to the other. This may be done in two ways. (1) Referring to Fig. 136(*b*), first connect the current coil in line A with the potential coil between A and B, the reading is

$$W_1 = E_{AB}I_A \cos(30 + \phi) = \sqrt{3}EI \cos(30 + \phi);$$

then with the current coil still in line A connect the potential coil from B to C; the reading is

$$W_2 = E_{AC}I_A \cos(30 - \phi),$$

and the power is the sum of the tw readings.

2. The second method requires an ammeter and voltmeter in addition to the wattmeter. Connect the wattmeter with its current coil in line A and its potential coil from B to C; the reading is

$$W = E_{BC}I_A \cos(90 - \phi), \qquad \text{Figs. 136}(c) \text{ and 137,}$$
$$= E_tI_l \sin \phi.$$

$\text{Sin } \phi = \frac{W}{E_tI_l}$ is therefore known and $\cos \phi$ can be calculated. The power in the circuit is $\sqrt{3}E_tI_l \cos \phi$.

This latter method does away with the difficulty encountered in the case of low power factors.

The two wattmeters connected as in Figs. 136(a) and 139, give the total power in the circuit, whether the loads on the

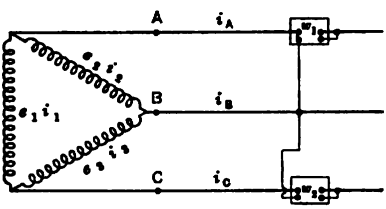


FIG. 139.

phases are balanced or unbalanced. Take the case of the delta-connected circuit, Fig. 139, and consider the instantaneous values of voltages and currents and power.

$$w_1 = e_2i_A = e_2(i_2 - i_1) = e_2i_2 - e_2i_1$$
$$w_2 = e_3i_C = e_3(i_3 - i_1) = e_3i_3 - e_3i_1$$

and

$$w_1 + w_2 = e_2i_2 + e_3i_3 - (e_2 + e_3)i_1$$

but

$$e_1 + e_2 + e_3 = 0, \text{ and } e_1 = -(e_2 + e_3),$$

therefore $w_1 + w_2 = e_1i_1 + e_2i_2 + e_3i_3 =$ the sum of the instantaneous values of power in the three phases.

When measuring power in a three-phase, four-wire system, the two-wattmeter method will give accurate results only when the loads are balanced. For unbalanced loads three wattmeters must be used connected in the three lines and with their potential coils connected between the lines and neutral.

126. Example.—A 220-volt, three-phase alternator supplies the following loads:

1. Twenty-five kilowatts at 100 per cent. power factor for lighting.
2. A 100-hp. (output) induction motor of 92 per cent. efficiency and 90 per cent. power factor.
3. A number of small induction motors with a combined output of 50 hp., average efficiency = 80 per cent. and average power factor = 75 per cent.

If the loads on the three phases are balanced, find:

(a) The output of the alternator in kilowatts.
(b) The line current.
(c) The power factor of the combined load.

(a) The output of the alternator $= 25.0 + \frac{100}{0.92} \times \frac{746}{1,000} + \frac{50}{0.80} \times \frac{746}{1,000}$

$= 25.0 + 81.2 + 46.7 = 152.9$ kw.

(b) The in-phase current per line for (1) $= \frac{25 \times 1,000}{\sqrt{3} \times 220} = 66$ amp.

The quadrature current per line for (1) $= 0$

The in-phase current per line for (2) $= \frac{81.2 \times 1,000}{\sqrt{3} \times 220} = 213$ amp.

The total current per line for (2) $= \frac{213}{0.9} = 237$ amp.

The quadrature current per line for (2) $= \sqrt{(237)^2 - (213)^2} = 103$ amp.

The in-phase current per line for (3) $= \frac{46.7 \times 1,000}{\sqrt{3} \times 220} = 123$ amp.

The total current per line for (3) $= \frac{123}{0.75} = 164$ amp.

The quadrature current per line for (3) $= \sqrt{(164)^2 - (123)^2} = 108$ amp.

The total in-phase current per line $= 66 + 213 + 123 = 402$ amp.

The total quadrature current per line $= 0 + 103 + 108 = 211$ amp.

The current per line $= \sqrt{(402)^2 + (211)^2} = 455$ amp.

(c) The power factor of the load is $\frac{402}{455} \times 100 = 88.3$ per cent.

CHAPTER VII

DIRECT-CURRENT MACHINERY

127. The Direct-current Dynamo.—A direct-current dynamo consists of an electric circuit, connected to a commutator and tapped by brushes, revolving in a magnetic field which is produced by stationary electric circuits.

Such a machine is illustrated in Fig. 140 and comprises the following parts:

$$\left.\begin{array}{l}\text{1. Yoke}\\ \text{2. Pole pieces}\\ \text{3. Armature core}\end{array}\right\}\text{Magnetic circuit.}$$

$$\left.\begin{array}{l}\text{4. Armature winding}\\ \text{5. Commutator}\end{array}\right\}\text{Revolving electric circuit.}$$

6. Brushes and brush holders. Collecting apparatus.
7. Field winding. Stationary exciting circuit.

128. Yoke.—The yoke serves mechanically as the frame of the machine and magnetically to carry the flux from pole to pole; in small machines it is usually made of cast iron but in machines where great weight is objectionable it is made of cast steel which has greater strength and permeability.

129. Pole Pieces.—The pole pieces or pole cores are usually made of cast steel or sheet steel and are bolted to the yoke. For small machines the yoke and poles are sometimes cast in one piece. All solid poles must have laminated pole faces bolted to them in order to reduce the eddy-current loss due to local variations of the magnetic density in the pole faces as the armature teeth move across them.

The pole cores carry the field windings of the machine. Solid poles are made circular and so have the greatest section for a given perimeter and require the smallest length of field copper. Laminated poles must, however, be made rectangular.

The section of the pole face is made much greater than that of the pole core in order to reduce the flux density in the air gap.

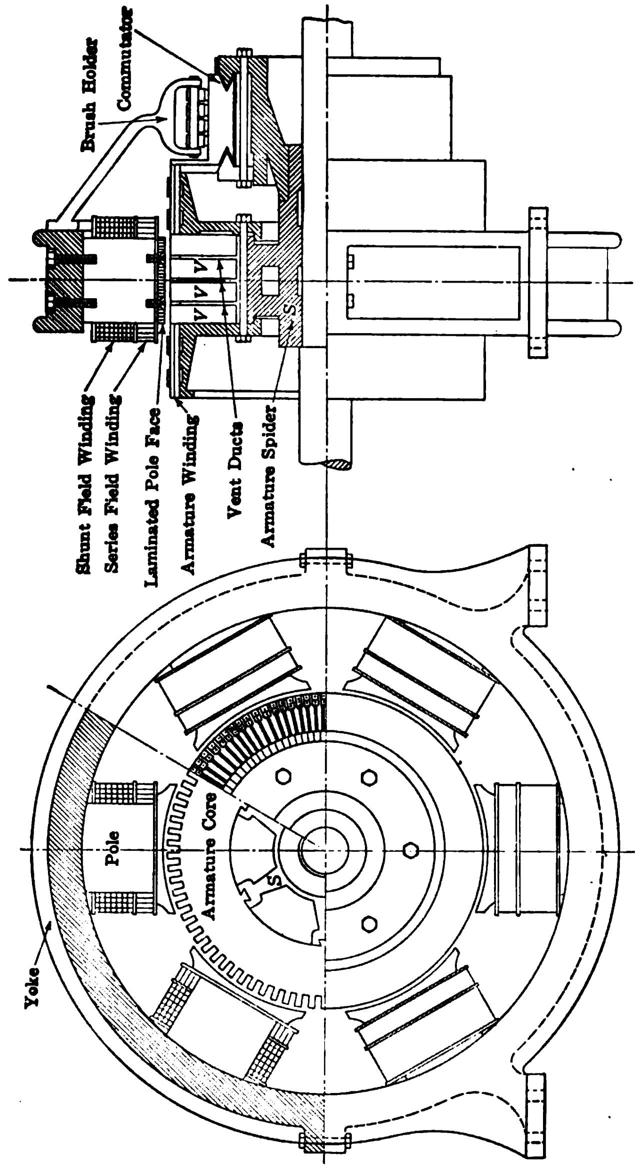


FIG. 140.—Direct-current dynamo.

130. Armature Core.—The armature core carries the rotating electric circuit in slots punched out on its periphery. It is built up of sheets of steel about 0.014 in. in thickness. Alternate sheets are coated with an insulating varnish to increase the resistance in the path of the induced eddy currents. Open spaces are left in the core, called vent ducts (*V.V.*, Fig. 140), which allow air to circulate through the armature and carry off the heat generated due to the iron and copper losses. The number of vent ducts required depends on the length of the armature.

The armature punchings are carried on a spider *S* and are kept in place by heavy end plates which have projections on their outer edges to support the end connections of the armature coils.

131. Armature Winding.—The armature winding is the seat of the generated e.m.f. It must be tapped at certain points by brushes, in order that the machine may supply power to an external receiver circuit. The winding consists of a number of coils of one or more turns, connected together to form a continuous winding; leads are run from their junctions to the commutator bars from which the current is collected by the brushes. The coils forming the winding must be so connected together that the e.m.fs. generated in coils between brushes of opposite polarity will all act in the same direction.

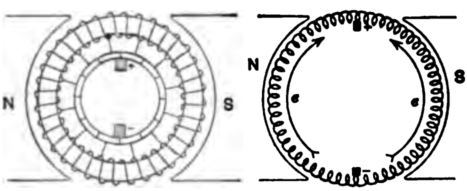


FIG. 141.—Bipolar ring winding.

The earliest type of armature winding was the ring winding, Figs. 141 and 142, but this has been replaced by the various forms of drum windings, a few of which are illustrated in Figs. 144 to 151.

132. Ring Windings.—In the bipolar ring winding, Fig. 141, all the conductors on each half of the armature are connected in series between the brushes. When the brushes are placed on the neutral line, that is, in such a position that the coil being commutated is not generating any e.m.f., the e.m.fs. generated in all conductors under one pole will act in the same direction and will combine to give the terminal e.m.f. of the generator. The e.m.fs. generated under the other pole will be equal in magnitude but

will act in the opposite direction. Thus, there is no e.m.f. tending to cause current to circulate through the winding at no load and there are two paths in multiple for the current flowing through the armature.

The connection from one conductor to the next is run through inside the armature, where it cannot cut magnetic flux, and consequently one-half of the winding is not effective in generating e.m.f. This extra wire increases the resistance of the armature and adds to the weight and cost of the machine. The ring winding has the further disadvantage, that it is very difficult to replace injured coils. On the other hand, the voltage between adjacent coils is so low that very little insulation is required between them. This type of winding is now obsolete.

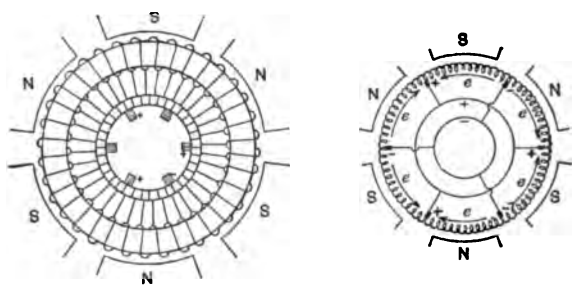


FIG. 142.—Six-pole ring winding.

Fig. 142 shows a six-pole ring winding with 36 coils connected to a commutator with 36 bars. This winding must be tapped at six equidistant points by brushes; there are six paths in parallel through the armature from positive to negative terminals and the voltage of the machine is that generated in one-sixth of the winding.

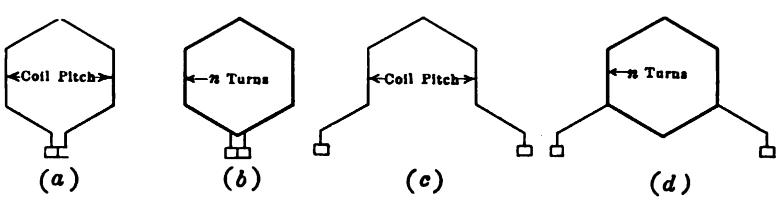


FIG. 143.—Armature coils.

133. Drum Winding.—In drum-wound machines the whole of the armature winding is carried in slots on the outside of the armature core and both sides of any coil are effective in generat-

ing e.m.f. The single coils are of the shapes shown in Fig. 143 and may consist of one or more turns.

The conductors forming the two sides of a coil must be situated in fields of opposite polarity in order that the e.m.fs. generated

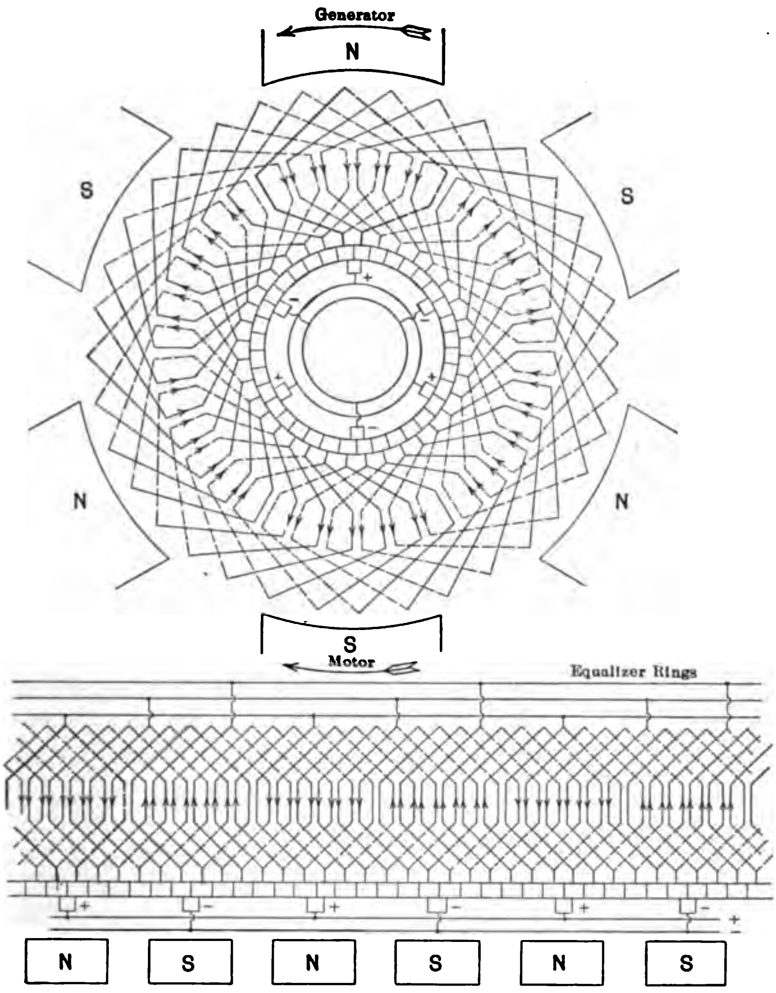


FIG. 144 AND 145.—Six-pole multiple-drum winding.

in them may act in the same direction. One side of a coil is placed in the top of a slot and the other side in the bottom of a slot in a similar position under the next pole.

According to the way in which the end connections are brought out to the commutator bars and the coils are connected together, drum windings are divided into two classes, multiple or lap wind-

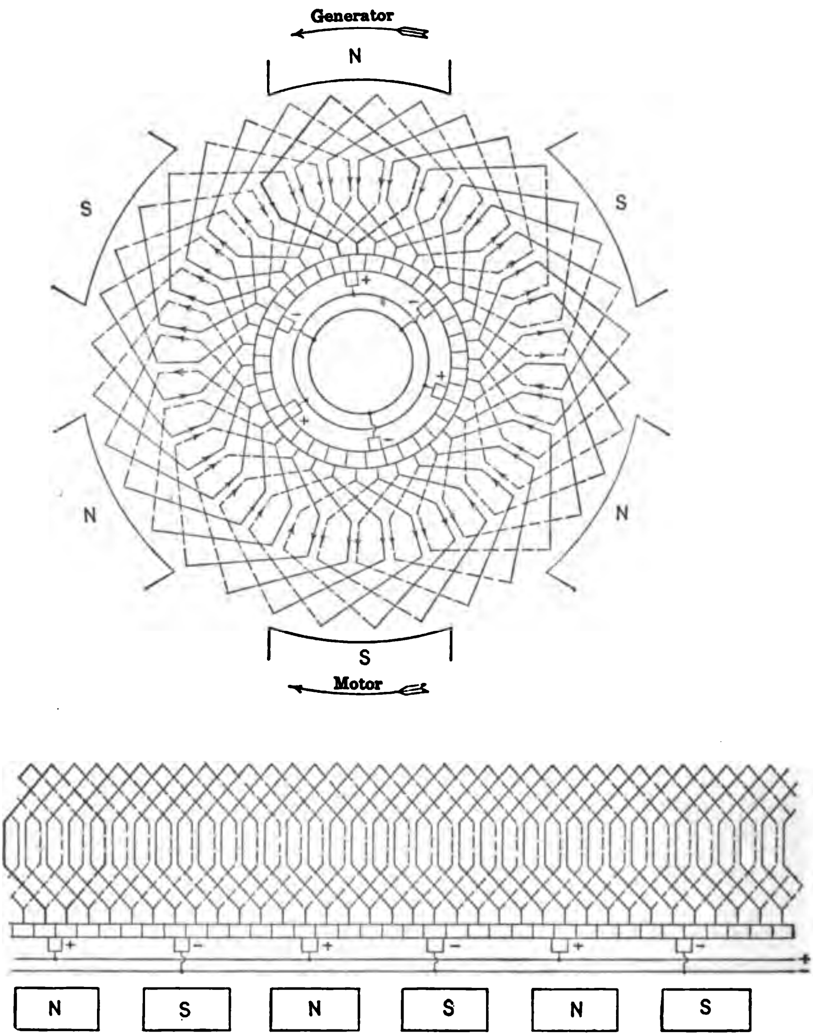


FIG. 146.—Six-pole multiple-drum winding. Fractional pitch.

ings, as illustrated by coils *a* and *b* in Fig. 143 and the windings in Figs. 144 to 146 and series or two-circuit windings, as illustrated by the coils *c* and *d* in Fig. 143 and the windings in Figs. 149 and 150.

134. Multiple-drum Windings.—In the multiple winding the two terminals of a coil are connected to adjacent commutator bars. Fig. 144 represents a multiple winding for a six-pole machine with 72 conductors and 36 slots. The sides of a coil are placed in slots 1 and 7 and the terminals are connected to bars 1 and 2. The same winding is shown in Fig. 145 and the directions of the currents are shown by arrowheads. The brushes are placed on the no-load neutral points and therefore directly under the centers of the poles and as many sets of brushes are required as there are poles.

Tracing through the winding from a positive to a negative brush only one-sixth of the conductors are taken and there are therefore six paths in multiple through the armature winding from the positive to the negative terminal of the machine and each conductor carries only one-sixth of the current passing through the armature. The number of paths is equal to the number of poles as in the ring winding.

Generally in multiple windings the coil pitch is almost equal to the pole pitch and the windings are called full-pitch windings. When the coil pitch is less by one or more teeth than the pole pitch the winding is called a fractional pitch or short-chord winding. Fig. 146 shows a fractional pitch, multiple-drum winding. Fractional-pitch windings have shorter end connections than full-pitch windings and have a smaller inductive e.m.f. generated during commutation since the two coils in one slot are not commutated at the same time and thus the inductive flux is that due to half the ampere-turns acting in the case of a full-pitch winding.

135. Equalizer Rings.—In multiple-wound machines, if there is any irregularity in spacing the brushes or if the air gaps under all the poles are not of the same depth, the e.m.fs. generated in the different sections of the winding will not be equal and the unbalanced e.m.f. will tend to cause current to circulate through the brushes even when the machine is not carrying any load. To keep these circulating currents out of the brushes similar points under the different pairs of poles which should normally be at the same potential are joined together by heavy copper connections called equalizer rings and these provide a path of very low resistance which the circulating currents follow in preference to the comparatively high-resistance path through the brushes (Figs. 144 and 145).

Equalizer rings should be provided on all multiple-wound

machines except the smallest sizes. They are connected to every third or fourth coil. Windings to be used with equalizer rings must have the number of coils a multiple of the number of pairs of poles.

Fig. 147 shows the circulating currents in a four-pole machine, in which the gaps under the two lower poles have been shortened, due to wear of the bearings. The voltages generated in the two lower sections of the winding are assumed to be 10 per cent. higher than those in the upper sections. Fig. 148 shows three equalizer rings connected to this machine to keep the circulating currents out of the brushes.

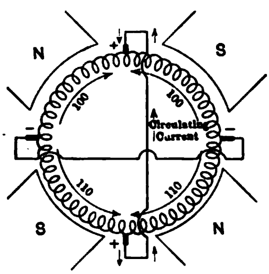


FIG. 147.

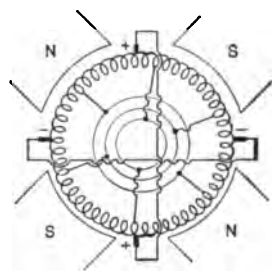


FIG. 148.

136. Series-drum Windings.—In the series winding the terminals of a coil are connected to two commutator bars approximately twice the pole pitch apart. Fig. 149 represents a series or two-circuit winding for a six-pole machine with 44 conductors and 22 slots. One side of a coil is placed in the top of slot 1 and the other side in the bottom of slot 5 and the terminals of the coil are connected to commutator bars 1 and 8.

Tracing out the winding from the positive brush B_1 to the negative brush B_2 one-half of the armature conductors are taken in. There are therefore but two circuits in multiple between terminals independent of the number of poles and the winding is called a two-circuit or series winding.

Only two sets of brushes are required to collect the current but when the current is large it is usual to employ other sets of brushes as shown at B_3, B_4, B_5 and B_6 and as many sets of brushes as there are poles may be used.

Series windings are used in small high-voltage machines or where it is desirable to use only two sets of brushes, as in small railway motors; but in large multipolar machines with many sets

of brushes the current does not divide equally between brushes of the same polarity and commutation is unsatisfactory.

The number of coils in a series winding must be one more or one less than a multiple of the number of pairs of poles, or

$$N = C\frac{p}{2} \pm 1,$$

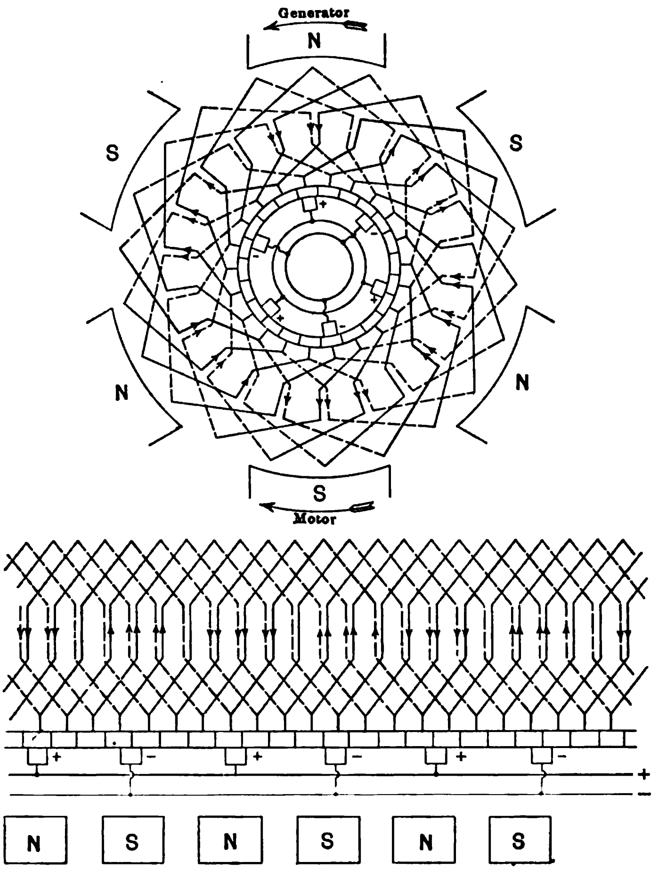


FIG. 149.—Six-pole series-drum winding, retrogressive.

where N = number of armature coils,
$p/2$ = numbers of pairs of poles,

and C = a constant whole number = avg. winding pitch.

Each coil may have any number of turns.

If

$$N = C\frac{p}{2} - 1, \tag{241}$$

the winding starting from bar 1 goes once around the armature and is connected to bar 2. It is therefore called a progressive winding (Fig. 150).

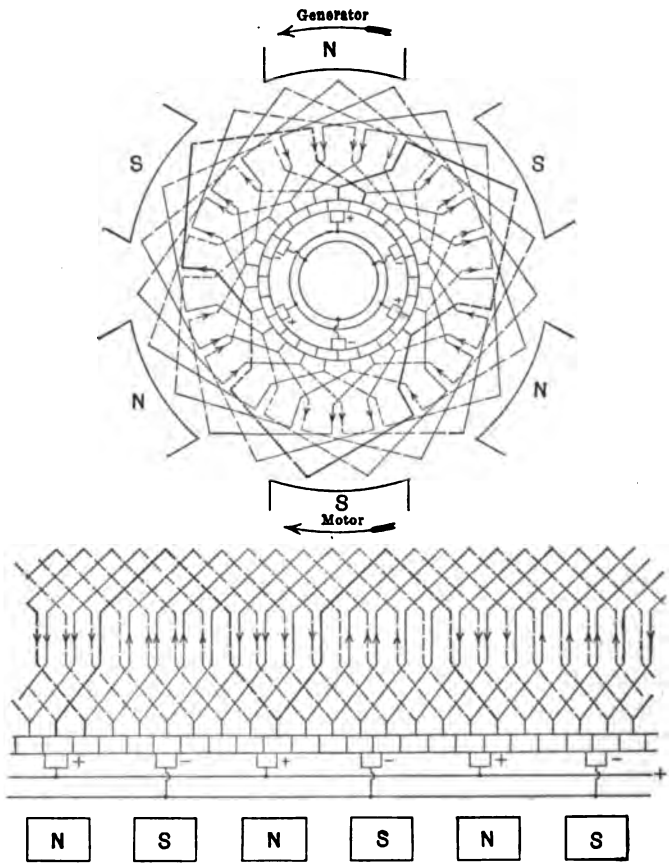


FIG. 150.—Six-pole series-drum winding, progressive.

If

$$N = C\frac{p}{2} + 1, \tag{242}$$

the winding starting from bar 1 and going once around the armature is connected to the bar before 1 and it is called a retrogressive winding (Fig. 149).

137. Double Windings.—If space is left between adjacent coils of a multiple winding, a second winding may be placed on

the same core. The second winding may be entirely separate from the first, that is, each of the two windings closed upon itself; or after passing through the first winding the circuit enters the second and after passing through the second reënters the first.

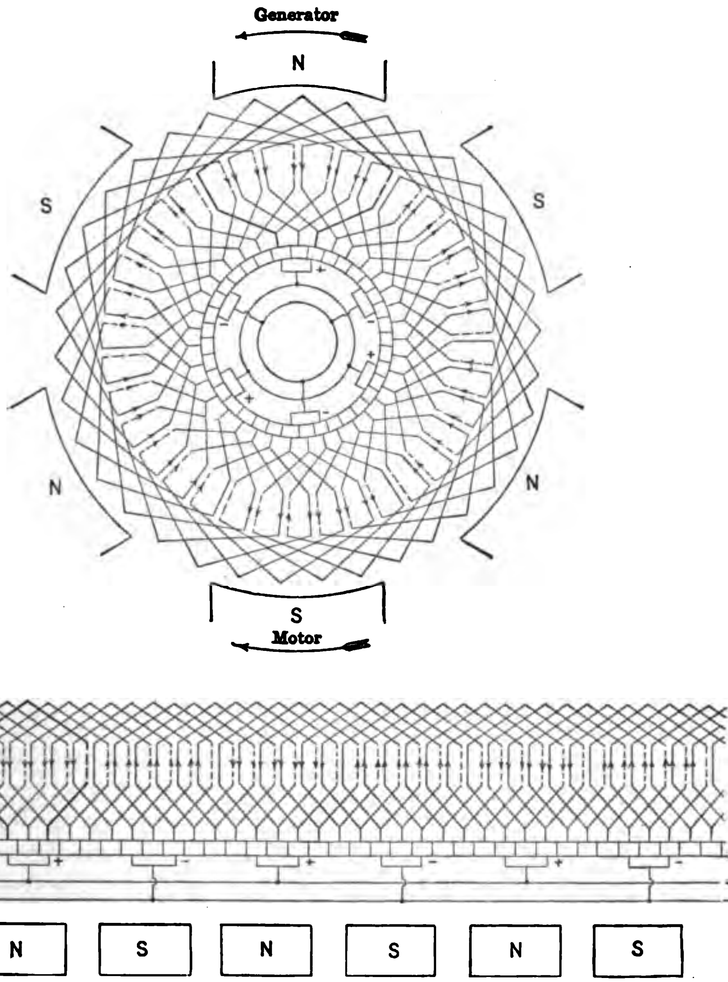


FIG. 151.—Sixpole multiple-drum winding, duplex, doubly reëntrant.

In the first case the winding is duplex doubly reëntrant and in the second case duplex singly reëntrant. Duplex multiple windings have twice as many circuits in multiple between terminals as there are poles. Such windings are suitable for large low-voltage ma-

chines used in electrolytic work. The brushes must be wide enough to collect current from both sections of the winding at the same time. Fig. 151 shows a duplex doubly reëntrant winding for a six-pole machine with 72 conductors and 36 slots.

Similarly the series winding may be made double by placing a second winding in alternate slots and connecting it to alternate commutator bars. The second winding is in multiple with the first and there are four paths in multiple between terminals.

138. Commutator.—The commutator is one of the most important parts of a direct-current machine. It consists of a number of bars of hard-drawn copper, insulated from one another by thin sheets of mica or other insulating material, and built up into the form of a cylinder (Fig. 152). The bars are held together by

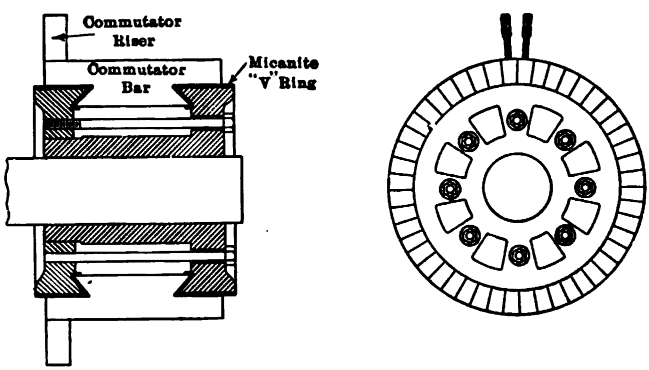


FIG. 152.—Commutator.

a cast-iron spider from which they are insulated by micanite "V" rings. The terminals of the coils forming the armature winding are connected to the bars either directly by soldering them into slots in the bars or by means of vertical connectors called commutator risers. In order that the brushes, which collect the current from the commutator, may run smoothly without vibrating or chattering the commutator surface must be perfectly round and smooth.

The function of the commutator is illustrated in Fig. 153. The current from the machine is I amp. and the current in each conductor is $I_c = \frac{I}{2}$ amp. During the time taken for the brush to move across the insulation between bars 1 and 2 the current

in coil c must change from I_c in one direction to I_c in the opposite direction. This reversal of the current is called commutation and to be satisfactory it must be effected without sparking. The brushes are shown placed on the neutral line and the coils short-circuited are not cutting any flux and therefore have no e.m.f. generated in them due to rotation. If the short-circuited coil had no inductance the current would reverse completely due to the contact resistance between the brush and the commutator.

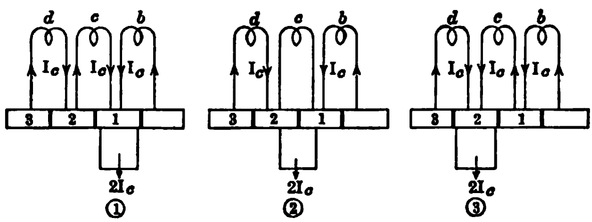


FIG. 153.—Commutation.

In (1) the current in coil c is I_c; in (2) it is zero since current I_c from bar 1 goes through one-half of the brush-contact area and current I_c from bar 2 goes through the other half, and the drop of voltage on both sides is the same and therefore there is no voltage available to drive the current through the resistance of the coil. Between (1) and (2) the resistance from bar 2 to the brush is greater than the resistance from bar 1 to the brush and so part of the current from 2 flows through the coil c. Between (2) and (3) the resistance from bar 2 to the brush is less than from bar 1 and part of the current from 1 flows through the coil c. In (3) the current in c is I_c but in the opposite direction from that in (1) and commutation is complete.

The self-inductance L of the coil opposes any change of current by generating a back e.m.f. $L\frac{di}{dt}$ volts; when the current is large and the time of commutation short this back e.m.f. is large and the current will not be reversed when the brush breaks contact with bar 1 and sparking will occur.

To counteract the effect of self-inductance the brushes in a generator are moved ahead of the neutral in the direction of rotation and back in a motor. The short-circuited coil is then in a field which generates in it an e.m.f. due to rotation which opposes the back e.m.f. of self-inductance, or, as it is usually called, "the re-

actance voltage of the coil," and assists commutation. The problem of commutation is discussed fully in Art. 192.

139. Brushes and Brush Holders.—The brushes collect the current from the moving commutator and from them it passes to the receiver circuit.

Brushes were at first made of copper because it had a low resistance and large current-carrying capacity but commutation of large currents was not satisfactory. Carbon brushes were then introduced and commutation was greatly improved due to the action of the high-resistance contact film between the brush and commutator. A much better contact surface was also obtained and the wear on the commutator was reduced. But since carbon will carry only about 30 to 50 amp. per square inch while copper will carry 150 to 200 amp. per square inch a much larger brush area is required and a larger commutator.

In order to maintain a good contact between the brush and commutator a spring is used exerting on the brush a pressure of about 1½ to 2 lb. per square inch of contact area.

The brush holders are made of brass and carry part of the current but leads are connected directly from the brushes to the main leads of the machine to prevent any drop of voltage which might occur due to poor contact between brushes and holders.

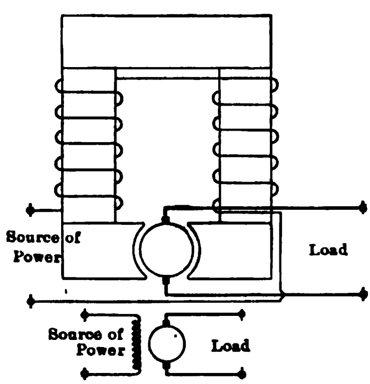


FIG. 154.—Separate excitation.

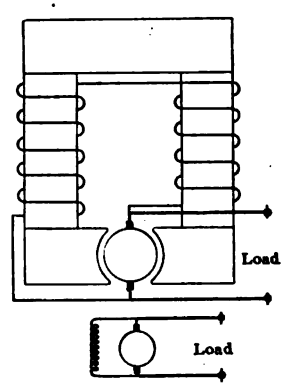


FIG. 155.—Shunt or self-excitation.

140. Field Windings.—The field winding is a stationary electric circuit consisting of one or more coils of wire placed on each of the field poles. They are supplied with current and provide the m.m.f. necessary to drive the magnetic flux through the

machine. The methods used in calculating the number of ampere-turns required to produce the flux in a machine is worked out in Art. 227.

According to the manner of exciting the fields, direct-current machines are divided into magneto machines in which the flux is produced by permanent magnets; separately excited machines (Fig. 154) in which the flux is produced by a winding supplied with current from some source outside the machine; shunt machines in which the field winding is connected across the armature terminals and receives a small current at the full-machine voltage (Fig. 155); series machines in which the field winding is connected in series with the armature and carries the full armature current (Fig. 156); and compound machines in which the field has both a shunt and a series winding (Fig. 157).

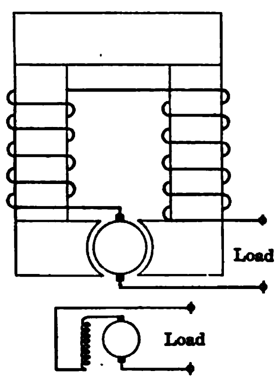
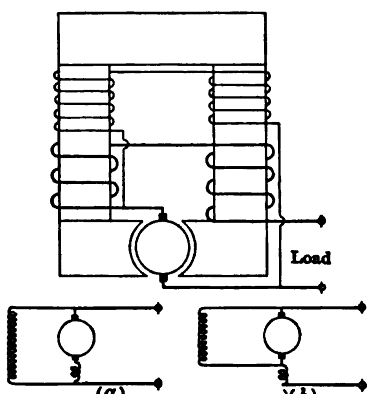


FIG. 156.—Series excitation.



(a) Long Shunt (b) Short Shunt

FIG. 157.—Compound excitation.

According to the number of poles machines are divided into bipolar and multipolar machines but the bipolar type is adapted only for small sizes.

141. Direction of Rotation of Generators and Motors.—Fig. 158 represents either a generator or motor. The directions of the currents in the armature are shown by the dots and crosses and the directions of rotation by arrows.

With currents as shown, the direction of rotation of a generator is counter-clockwise and of a motor is clockwise.

These results are obtained by examining the fields produced by the armature currents.

Take for example the conductor A. Its field combines with the main field and produces a strong field below the conductor and a weak field above it. There is therefore a force f acting on the conductor tending to move it up. This is the force that must be overcome by the engine driving the generator in order to develop electric power and therefore the rotation of a generator is against this force and is counter-clockwise.

In the case of the motor, the force f on the conductor is the mechanical force developed and the rotation is in the direction of this force and is clockwise.

To reverse the direction of rotation of a motor it is necessary to reverse either the amature current or the field current but not both.

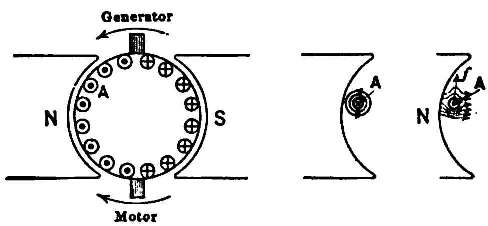


FIG. 158.—Direction of rotation.

142. Generation of Electromotive Force.—The e.m.f. generated in the armature of a direct-current generator or motor is

$$\mathcal{E} = Zn\Phi \frac{p}{p_1} 10^{-8} \text{ volts,}$$

where Z = number of conductors on the armature,
n = speed of armature in r.p.s.,
Φ = flux crossing the air gap from one pole,
p = number of poles,
and p_1 = number of paths in multiple between terminals.

In 1 sec. each conductor cuts $n\Phi p$ lines of force and thus the average e.m.f. generated in each of the Z conductors is

$$e = n\Phi p \, 10^{-8} \text{ volts.}$$

Between the terminals there are $\frac{Z}{p_1}$ conductors connected in series and therefore the e.m.f. between terminals is

$$\mathcal{E} = Zn\Phi \frac{p}{p_1} 10^{-8} \text{ volts.} \qquad (243)$$

This is, the e.m.f. equation of a direct-current generator; it may be written

$$\mathcal{E} = Kn\Phi, \tag{244}$$

where $K = Z \frac{p}{p_1} 10^{-8}$ is a constant of the machine.

The e.m.f. is therefore directly proportional to the speed and to the flux crossing the air gap.

Fig. 159 shows the relation between e.m.f. and speed for a constant value of Φ. The locus is a straight line passing through the origin.

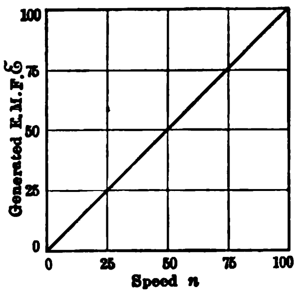


FIG. 159.—Variation of generated e.m.f. with speed.

In a generator the current flows in the direction of the generated e.m.f. but in a motor an e.m.f. is impressed on the terminals of the armature and drives a current against the e.m.f. generated in the motor. The e.m.f. generated in a motor armature is therefore called a back e.m.f.

143. Effect of Moving the Brushes.—The equation

$$\mathcal{E} = Zn\Phi \frac{p}{p_1} 10^{-8}$$

holds only if the brushes are on the no-load neutral points. When the brushes are moved ahead of the neutral points or behind them the e.m.f. between terminals is decreased. This may be seen by reference to Fig. 160. With the brushes on the neutral points the e.m.fs. generated in all the conductors in series between terminals act in the same direction and combine to give the maximum e.m.f. When the brushes are moved ahead the e.m.f. between terminals is only that generated in conductors a–b or d–c since the resultant of the e.m.fs. generated in conductors a–d and b–c is zero. Thus advancing the brushes corresponds to a decrease in the number of armature conductors.

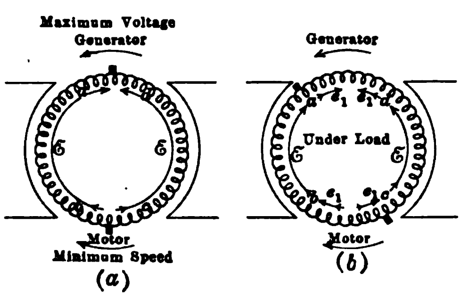


FIG. 160.—Effect of moving the brushes.

144. Building up of Electromotive Force in a Self-excited Generator.—In a self-excited generator at rest there is no flux crossing the gap except the residual magnetism. When the armature is rotated only a small e.m.f. is generated in it and a very small current is produced in the field winding. If the m.m.f. of this current is in the direction of the residual magnetism, it will increase the flux and the e.m.f. will increase and gradually build up to its full value.

If, however, the current opposes the residual magnetism, it will cause it to decrease and the e.m.f. will not build up until the field winding is reversed.

If there is no residual magnetism, the e.m.f. cannot build up until power is supplied to the winding from some outside source to start the flux.

145. Armature Reaction and Distribution of Magnetic Flux.—Fig. 161 shows approximately the distribution of flux in a two-pole machine with the fields excited but without current in the

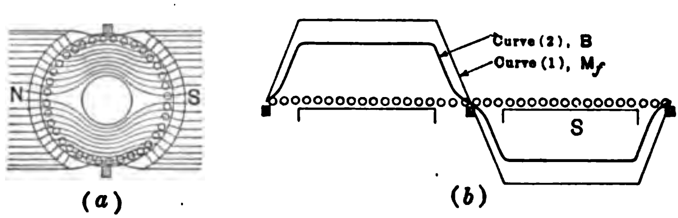


FIG. 161.—Distribution of flux and m.m.f. at no load.

armature winding. Curve 1 shows the m.m.f. acting at each point of the armature circumference; under the north pole it is positive and has a constant value; under the south pole it is negative but of the same magnitude; beyond the pole tips its value may be represented by the straight line which passes through zero midway between the poles. The m.m.f. is expressed in ampere-turns and is denoted by M_f.

The flux density produced by this m.m.f. is at every point directly proportional to the m.m.f. and is inversely proportional to the reluctance of the path. It is of constant value over the pole face if the air gap is uniform but falls off rapidly beyond the pole tips due to the increased reluctance of the air path and to the decrease in the m.m.f. acting. Midway between the poles it is zero. It is represented by B and its values are plotted in curve 2. The total flux entering the armature is represented by the

area under curve 2 and this is the value of Φ which appears in the e.m.f. equation.

When, however, the armature is carrying current it exerts a m.m.f. called armature reaction, which combines with the m.m.f. of the field winding and changes both the distribution and the total value of the flux entering the armature.

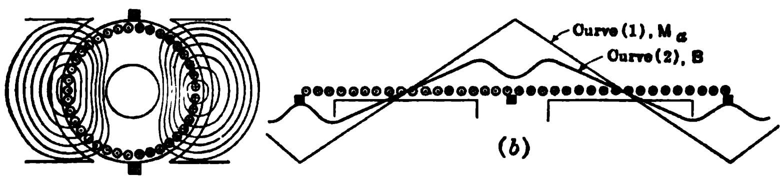


FIG. 162.—Armature m.m.f. and flux.

Fig. 162(*a*) shows the distribution of flux produced by the armature m.m.f. acting alone, and the values of the m.m.f. of the armature at all points around the circumference are plotted in curve 1, Fig. 162(*b*). The brushes are placed on the no-load neutral points. The armature m.m.f. M_a is a maximum in line with the brushes and falls off as a linear function to zero under the center of the poles. The distribution of the flux produced by the armature m.m.f. is shown in curve 2, Fig. 162(*b*).

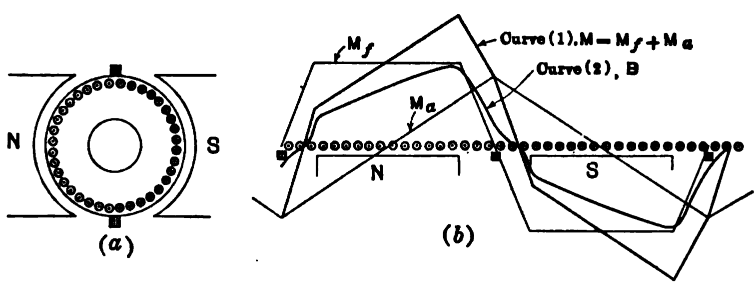


FIG. 163.—Distribution of flux and m.m.f. under load, with the brushes on the no-load neutral points.

Fig. 163(*a*) represents the conditions when the m.m.fs. of field and armature are acting together and with the brushes still on the no-load neutral points. Curve 1, Fig. 163(*b*), shows the resultant m.m.f. acting at each point. Its ordinates are represented by M and they are the sum of the corresponding ordinates M_f and M_a.

The m.m.f. across the pole face is no longer constant but is

decreased over one-half and increased over the other half by the same amount. The flux density B (curve 2) at each point is still proportional to the m.m.f. and inversely proportional to the reluctance of the path, but, since part of the path is made up of a magnetic material, due to the effect of saturation, the increase of flux under one-half of the pole is less than the decrease under the other half and consequently the total flux is decreased.

If it were not for the effect of saturation the ordinates of curve 2, Fig. 163(*b*), could be obtained by adding the corresponding ordinates of Fig. 161(*b*) and Fig. 162(*b*).

The neutral points are no longer midway between the poles but have been shifted in the counter-clockwise direction in Fig. 163(*a*) and to the right in Fig. 163(*b*). To prevent sparking the brushes must be moved up to or a little beyond the load neutral points. In a generator the brushes must be moved forward in the direction of rotation and in a motor must be moved backward against the direction of rotation as indicated in Fig. 164.

With the brushes midway between the poles the direction of the armature m.m.f. is at right angles to the field m.m.f. and it therefore does not directly oppose it but causes a distortion of the flux and a decrease due to saturation. In this case the m.m.f. of the armature is cross-magnetizing.

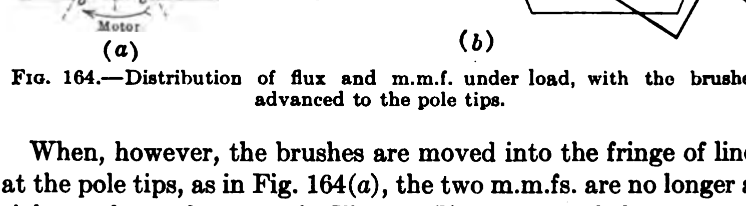


FIG. 164.—Distribution of flux and m.m.f. under load, with the brushes advanced to the pole tips.

When, however, the brushes are moved into the fringe of lines at the pole tips, as in Fig. 164(*a*), the two m.m.fs. are no longer at right angles and as seen in Fig. 164(*b*) the part of the armature m.m.f. which is subtracted from the field m.m.f. is much greater than the part which is added to it and therefore the resultant m.m.f. is reduced and the flux is both distorted and decreased.

Referring to Fig. 164(*a*) the armature conductors may be separated into two groups; namely those between *a* and *d* included

in the double angle of advance α with their return conductors from c to b and those under the pole between b and a with their return conductors between d and c. The first group acts in direct opposition to the field m.m.f. and decreases the flux crossing the air gap. They are therefore called the demagnetizing ampere-turns of the armature. This demagnetizing m.m.f. increases as the shift of the brushes is increased and it also increases directly with the armature current.

The second group exerts a m.m.f. at right angles to the field m.m.f. and distorts the flux as in Fig. 163(a) and causes a decrease due to saturation. They are called the cross-magnetizing ampere-turns of the armature.

For sparkless commutation without the use of interpoles the brushes must be moved ahead of the neutral points in order that the coils short-circuited by them may be cutting the fringe of flux at the pole tips. E.m.fs. are thus generated in the coils opposing the back e.m.fs. due to inductance, and they aid in reversing the current. As the armature current is increased a point is finally reached where the armature m.m.f. is so strong that it over-

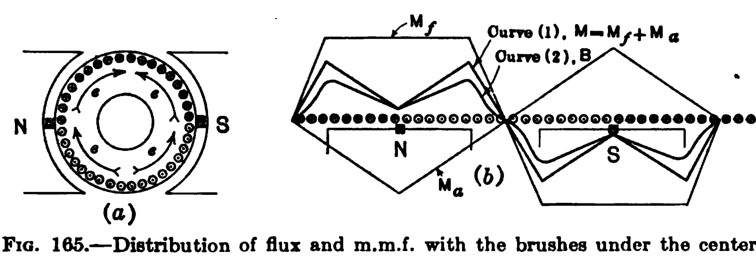


FIG. 165.—Distribution of flux and m.m.f. with the brushes under the centers of the poles.

balances the field m.m.f. at the pole tips and therefore no reversing field is left and commutation is not possible without interpoles. It is of no use to move the brushes further ahead because that only increases the demagnetizing component of armature m.m.f. and decreases the flux more. In direct-current machines without interpoles the armature ampere-turns per pole at full load should not exceed 70 per cent. of the field ampere-turns per pole.

Fig. 165 shows the effect of moving the brushes to the center of the poles. The whole armature m.m.f. is demagnetizing and the flux is reduced to a small value. There is no difference of potential between the brushes since the sum of the e.m.fs. generated in one-half of the conductors in series between the

brushes is exactly equal and opposite to that generated in the other half.

146. No-load Saturation Curve.—The no-load saturation curve of a generator shows the relation between the voltage generated at no load and the field current. It is sometimes plotted with the values of Φ_0, the useful flux as ordinates and the field m.m.f. or field ampere-turns per pole as abscissæ.

To obtain the no-load saturation curve, the generator is run at constant speed and a variable resistance is connected in series with the field winding. As the current is raised from a very low value the flux and the generated voltage increase directly as the current while the iron parts of the magnetic circuit are unsaturated, but as the flux density increases the magnetic circuit becomes saturated and a greater increase in current is required to produce a given increase in voltage than on the lower part of the curve (Fig. 166).

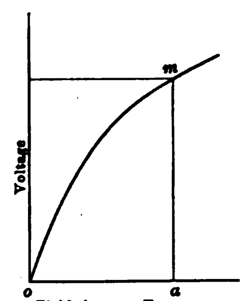


FIG. 166.—No-load saturation curve.

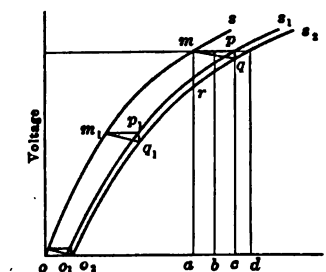


FIG. 167.—Full-load saturation curve.

Shunt-excited machines are operated at a point slightly above the knee of the saturation curve to insure stability, that is, in order that slight changes of speed may not cause large changes in voltage.

Machines with compound windings must be operated at a lower point on the saturation curve in order that the required increase of voltage may be obtained without supplying too large a m.m.f. in the series field windings.

If the field current is gradually decreased from its maximum the flux and the generated voltage fall off, but they come down on a curve slightly above the original curve, which cuts the vertical axis at a point above the origin. The value of flux represented by this intercept is the residual magnetism in the machine.

147. Load Saturation Curves.—The curve showing the relation between the terminal voltage and the field current, when the generator is delivering a given load current is called a load saturation curve (Fig. 167). The load saturation curves may be obtained from the no-load saturation curve if the values of the demagnetizing and cross-magnetizing armature ampere-turns are known and also the resistance of the armature.

If the brushes are moved ahead of the no-load neutral position by an angle α electrical degrees the demagnetizing ampere-turns per pole $= \frac{2\alpha}{180} \times$ armature ampere-turns per pole (Fig. 164). If $\alpha = 18$ degrees or 10 per cent. of the pole pitch, the demagnetizing ampere-turns per pole $= 0.2 \times$ armature ampere-turns per pole.

To overcome the effect of the demagnetizing ampere-turns an equal number of ampere-turns must be added to the field winding. This number should be increased 15 or 20 per cent. to make allowance for the leakage of flux between the pole and armature (Art. 219).

The remaining armature ampere-turns per pole are cross-magnetizing. They are $\frac{180 - 2\alpha}{180} \times$ armature ampere-turns per pole, or when $\alpha = 18$ degrees, the cross-magnetizing turns $=$ $0.8 \times$ armature ampere-turns per pole.

The cross-magnetizing ampere-turns produce a very high saturation in the teeth under one-half of the pole tips and so increase the reluctance of the magnetic circuit as a whole and decrease the flux. This effect is very difficult to determine accurately but in normally designed machines in which the field ampere-turns per pole = approximately 150 per cent. of the armature ampere-turns per pole, the cross-magnetizing ampere-turns may be replaced by 40 per cent. of their number of demagnetizing ampere-turns.

This value can be applied only for points on the no-load saturation curve just above the knee. Where the saturation is very high the effect of the cross-magnetizing turns is relatively larger and a larger percentage than 40 must be used. On the other hand, for points below the knee of the saturation curve much smaller percentages must be used.

In the case of the generator designed in Chapter VIII, the armature ampere-turns per pole = 4,860 and the angle of brush lead

α = 18 electrical degrees; the demagnetizing ampere-turns per pole, = 0.2 × 4,860 = 972, and the cross-magnetizing turns = 0.8 × 4,860 = 3,888. Here the full voltage point is just at the knee of the saturation curve and 30 per cent. of 3,888 = 1,166 demagnetizing ampere-turns are taken to represent the cross-magnetizing effect.

The demagnetizing effect varies directly with the current but the cross-magnetizing effect increases at a more rapid rate. If the two effects are equal at full load, then on overloads the cross-magnetizing effect will be greater than the demagnetizing effect while at light loads it will be less.

In Fig. 167, *os* is the no-load saturation curve plotted on a base of field ampere-turns. At no load the field m.m.f. required for rated voltage is *oa*. Let *ab* be the demagnetizing ampere-turns per pole at full load and *bc* be the demagnetizing ampere-turns equivalent to the cross-magnetizing turns. In order to generate the same voltage under load as at no load the field m.m.f. per pole must be increased by the amount $ac = mp$ and p is a point on the curve o_1s_1 showing the relation between the voltage generated under load and the field m.m.f. Curve o_1s_1 would be parallel to *os* if the cross-magnetizing effect were constant but at the lower points on the curve it becomes very small and the intercept oo_1 may be taken as $= ab$.

The terminal voltage E will be less than the generated voltage $\mathcal{E}$ by the resistance drop in the armature and series field, $Ir = pq$. The curve o_2s_2 is drawn parallel to the curve o_1s_1 at a distance $pq = Ir$ below it. This is the full-load saturation curve for the generator.

A saturation curve for half load could be obtained by joining the center points of all the lines such as mq, m_1q_1, etc.

148. Voltage Characteristic or Regulation Curve.—The voltage characteristic of a direct-current generator is the curve showing the relation between the terminal voltage and the current output or load.

The voltage generated in the armature is

$$\mathcal{E} = Zn\Phi \frac{p}{p_1} 10^{-8} = Kn\Phi \text{ volts.} \qquad \text{(Art. 142)}$$

Take first the case of a separately excited generator in which the speed n and the field current I_f are both kept constant.

At no load the flux crossing the air gap under each pole is Φ_0 and the voltage generated is

$$\mathcal{E}_0 = Kn\Phi_0;$$

this is also the terminal voltage at no load.

As the generator is loaded the terminal voltage decreases due to two causes: (*a*) armature reaction, and (*b*) armature resistance.

(*a*) When current flows in the armature, the armature m.m.f., which is partly demagnetizing and partly cross-magnetizing, reduces the flux crossing the gap and thus reduces the generated voltage. The loss of flux increases at a more rapid rate than the current. The consequent loss of voltage is called the drop due to armature reaction.

(*b*) It is necessary to distinguish between the voltage, $\mathcal{E}$, generated in the armature and the terminal voltage, E. At no load they are the same but when current flows in the armature part of the generated voltage is consumed in driving the armature current I through the resistance of the armature winding and brushes r. This resistance drop is Ir and increases directly with the current.

The terminal voltage is $$E = \mathcal{E} - Ir. \tag{245}$$

In Fig. 168 are shown the no-load and load saturation curves plotted on the same base. The constant exciting ampere-turns $= oa$. A vertical line through a cuts the various saturation curves at m, n, r and t which represent the terminal voltages corresponding to the respective loads. These points are plotted on a load base as curve m_1r and this is the voltage characteristic of the generator. It is sometimes called the external characteristic to distinguish it from the corresponding curve of generated voltages m_1k.

The straight line ol represents the armature resistance drops at all loads $= Ir$.

The voltage generated in the armature at any load is $\mathcal{E} = E + Ir$ and by adding the ordinates of ol to those of m_1r the curve m_1k is obtained showing the relation between the generated voltage and the load. This is called the internal voltage characteristic.

At full load the total drop of voltage below the no-load value is mr and it is made up of two parts, mk the drop due to armature reaction and kr the armature resistance drop.

149. Field Characteristic or Compounding Curve.—The field characteristic of a generator is the curve showing the increase of field current required under load to maintain a constant terminal voltage at constant speed.

In a separately excited or self-excited generator the terminal voltage can be maintained constant as the load current increases if the field current is increased to such a value that the increase in field m.m.f. will not only overcome the effect of armature m.m.f. but will produce an increase in flux to provide the extra voltage to supply the armature resistance drop.

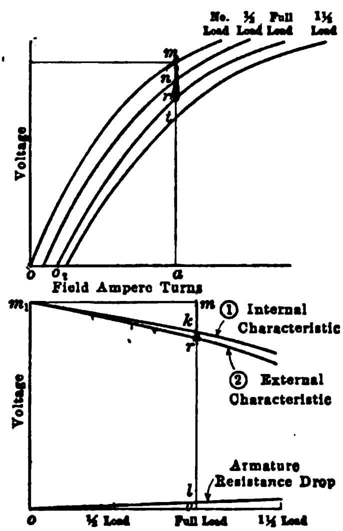


FIG. 168.—Voltage characteristics separately excited.

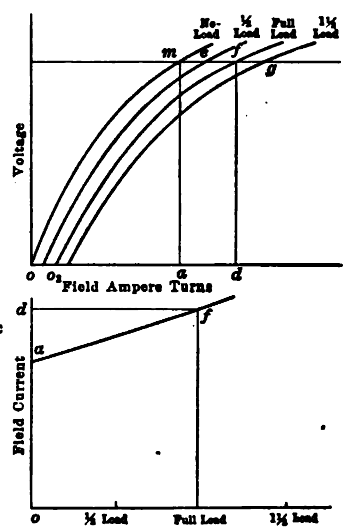


FIG. 169.—Field characteristics.

Referring to Fig. 167 the terminal voltage at no load = *am* and the field m.m.f. = *oa*; to obtain an equal terminal voltage at full load the field m.m.f. must be increased by an amount *ad*; of this *ac* is required to overcome armature reaction and *cd* is required to supply the armature resistance drop.

In Fig. 169 the horizontal line through the no-load voltage point *m* cuts the various load saturation curves at *e*, *f* and *g*. The corresponding values of field m.m.f. or field current plotted on a load base give the curve *af* which is the field characteristic of the generator.

The increase of field current under load is obtained by gradually cutting out resistance from the field rheostat in series with the field winding (Fig. 170). This regulation of the voltage must be done by hand and cannot take care of sudden changes of load.

150. Voltage Characteristic of a Shunt Generator.—In the shunt generator, armature reaction and armature resistance cause a decrease in terminal voltage under load, but a third condition must also be taken into account. The field circuit is connected across the terminals of the armature and the current in it is proportional to the terminal voltage; thus, when the terminal voltage decreases due to armature reaction and armature resistance, the field current also decreases and causes a further decrease in the flux; and therefore the terminal voltage of a shunt-excited generator is less than it would be if the machine were separately excited (Fig. 171).

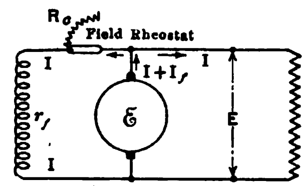


FIG. 170.—Shunt generator.

The voltage characteristic of a shunt generator may be obtained from the load saturation curves, Fig. 171, by drawing a straight line *om* from the origin through the no-load voltage point *m*. The slope of this line represents the resistance of the shunt field

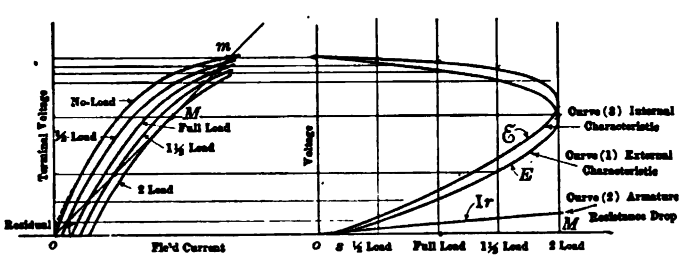


FIG. 171.—Voltage characteristics of a shunt generator.

winding and its intersections with the various saturation curves give the terminal voltages corresponding to the respective loads. There are two values of voltage corresponding to each value of load. The method of obtaining the voltage characteristic is shown.

The load current can be increased by decreasing the resistance in the load circuit until the point *M* is reached. If the load resistance is further decreased the armature current increases for an instant and then decreases as its m.m.f. wipes out part of

the flux and causes the generated voltage to decrease. Finally, when the load resistance is zero and the generator is short-circuited the terminal voltage and the field current become zero and the residual flux produces a small voltage which is consumed in driving the current *os* through the armature resistance.

The maximum current output of a shunt generator is usually many times full-load current and can be reached only with small machines of poor regulation.

The internal voltage characteristic can be obtained by adding the armature resistance drops to the terminal voltages; that is, by adding the ordinates of curves 1 and 2, Fig. 171, curve 3 is obtained.

151. Effect of Change of Speed on the Voltage of a Shunt Generator.—In Fig. 172(*a*) *oba* is the no-load saturation curve of a shunt generator at normal speed. The point *a* is the normal

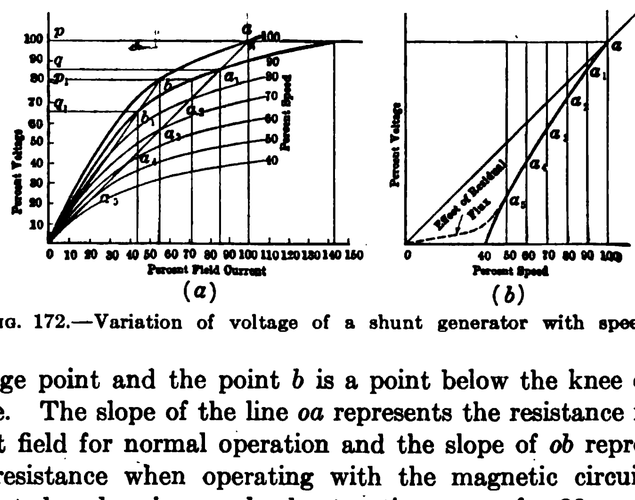


FIG. 172.—Variation of voltage of a shunt generator with speed.

voltage point and the point *b* is a point below the knee of the curve. The slope of the line *oa* represents the resistance in the shunt field for normal operation and the slope of *ob* represents the resistance when operating with the magnetic circuit unsaturated. ob_1a_1 is a no-load saturation curve for 90 per cent. of normal speed, each of its ordinates is 90 per cent. of the corresponding ordinate of *oba*.

From these curves it is seen that a drop of speed of 10 per cent. causes a drop of voltage pq = 14 volts = 14 per cent. when the machine is saturated and a drop p_1q_1 = 16 volts = $\frac{16 \times 100}{82}$ = 19 per cent. when it is unsaturated. Therefore, a saturated shunt generator is more stable than an unsaturated generator, that is, its voltage changes less due to slight changes in speed.

At normal saturation to counteract the 10 per cent. drop in speed the field current must be increased 43 per cent.

The saturation curves for speeds from 80 to 40 per cent. of normal are also shown. They are intersected by the line *oa* at a_2, a_3, etc., and these points show the no-load voltages corresponding to the various speeds.

The voltage speed curve at no load is shown in Fig. 172(*b*). The voltage becomes zero in this case at 40 per cent. of normal speed, since the resistance line *oa* is tangent to the saturation curve at this speed and so does not cut it. The effect of residual magnetism has been neglected; it tends to change the curve as indicated by the dotted lines. The line *oa* in Fig. 172(*b*), shows the voltage speed curve at no load for a separately excited generator.

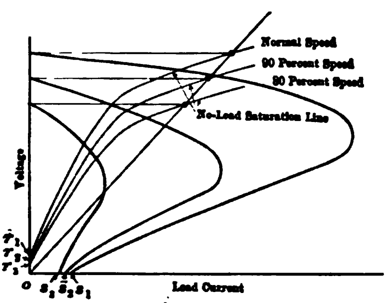


FIG. 173.—Effect of change of speed on the voltage characteristic of a shunt generator.

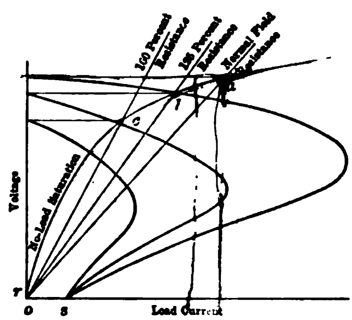


FIG. 174.—Effect of saturation on the voltage characteristic of a shunt generator.

In Fig. 173 are shown the no-load saturation curves for speeds of 100 per cent., 90 per cent. and 80 per cent. of normal and the corresponding voltage characteristics, which may be obtained as in Fig. 171. The curves for the low speeds fall off very quickly since the field m.m.f. is decreased more than the speed; the maximum current outputs are also much reduced. The voltages or_1, or_2 and or_3 produced by the residual magnetism are directly proportional to the speed as are also the currents os_1, os_2 and os_3 produced by them.

152. Effect of Saturation on the Voltage Characteristic of a Shunt Generator.—In Fig. 174 is shown the no-load saturation curve *ra* of a shunt generator intersected by three resistance lines *oa* corresponding to normal resistance, *ob* to 125 per cent. of normal and *oc* to 160 per cent. The characteristic curves for

the lower saturations fall off very quickly since the field m.m.f. is weak but they all pass through a single point s since the voltage produced by the residual flux is the same for all. An unsaturated shunt generator does not tend to maintain a constant terminal voltage under load, that is, its voltage regulation is poor.

153. Compound Generator.—Increase of field m.m.f. under load can be obtained automatically by placing a series winding on the field poles in addition to the shunt winding. The series winding carries the load current.

If the series winding is designed so that the terminal voltage is the same at full load as at no load the generator is flat-compounded, if the terminal voltage at full load is higher than at no load the generator is over-compounded.

In Fig. 179 are shown the voltage characteristics or regulation curves of a generator: (1) self-excited; (2) separately excited; (3) flat-compounded; and (4) over-compounded. If a generator is flat-compounded so that it gives the same voltage at full-load as at no load it will be slightly over-compounded at half load.

Referring to Figs. 167 and 169 for a flat-compound machine the series winding at full load must provide a m.m.f. equal to ad. The number of turns per pole on the series winding may be

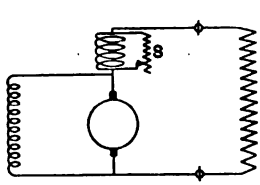


FIG. 175.—Compound generator (short shunt).

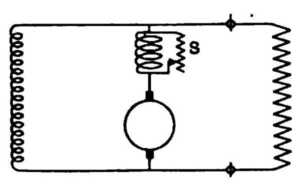


FIG. 176.—Compound generator (long shunt).

found by dividing the required ampere-turns ad by the load current I. In designing a series field winding it is advisable to put on some extra ampere-turns to make up for any decrease in voltage due to decrease in the speed of the prime mover under load. After the machine is completed the series excitation may be corrected by connecting a shunt across the terminals of the series winding (Figs. 175 and 176). By varying the resistance of the shunt any required portion of the current may be taken from the series winding. In the case of a generator supplying a rapidly fluctuating load the shunt to the series winding must be designed with its inductance in the same ratio to the induce

tance of the series winding as its resistance bears to the resist ance of the series winding in order that the variable current ma divide up in the correct proportions to give the require compounding.

154. Voltage Characteristic of a Compound Generator.— The voltage characteristic for a flat-compound generator may b

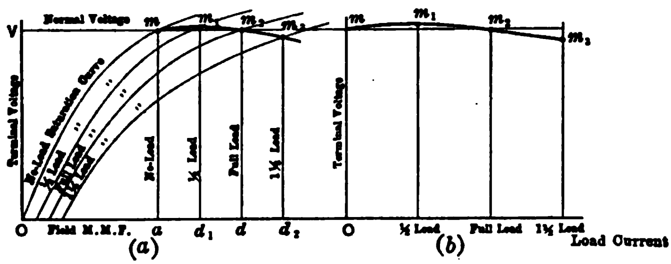


FIG. 177.—Voltage characteristic of a flat compound generator.

obtained from the saturation curves in Fig. 177(a). At fu load the series excitation is ad and the terminal voltage is dm_2 = the no-load voltage am; at half load the series excitation is $ad_1 = \frac{ad}{2}$ and the voltage is d_1m_1 slightly greater than am; at one an one-half load the series excitation is $ad_2 = \frac{3}{2}ad$ and the termina voltage d_2m_3 is less than am. The voltage characteristic is show in Fig. 177(b).

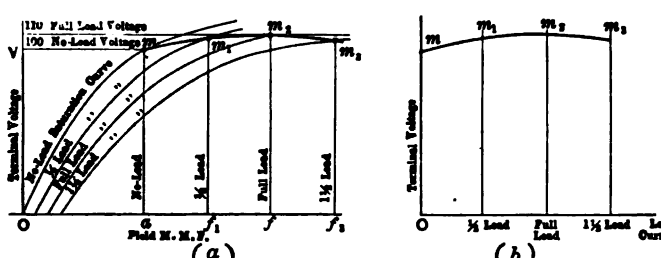


FIG. 178.—Voltage characteristic of an over-compound generator.

The characteristic for a 10 per cent. over-compound generato may be obtained in a similar way from Fig. 178(a). At ful load the series excitation is af and the terminal voltage is fm_2 = 110 per cent. of the no-load voltage am; at half load the serie excitation is $af_1 = \frac{af}{2}$ and the voltage is f_1m_1; m_1 lies slightl

above the straight line joining m and m_2. The voltage characteristic is shown in Fig. 178(b).

The increase of excitation for full load af includes a small component which is supplied by the shunt winding, since as the voltage across it has increased 10 per cent., its current and m.m.f. have increased in the same proportion. Allowance should be made for this in designing series windings.

155. Short-shunt and Long-shunt Connection.—In compound-wound generators the shunt winding may be connected across the armature terminals, Fig. 175, called short-shunt, or outside of the series winding, Fig. 176, called long-shunt. The characteristic curves are not affected to a great extent by the difference in connection since with the short-shunt the voltage across the shunt winding is higher than with the long-shunt by the resistance drop in the series winding and the current in the series winding is less by the amount supplied to the shunt winding.

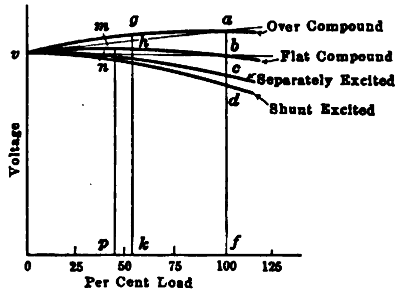


Fig. 179.—Voltage characteristics of generators.

156. Regulation.—The regulation of a separately excited or shunt-excited generator is defined as the rise in voltage when full load is taken off expressed as a per cent. of full-load voltage. Fig. 179 shows the characteristic curves of a generator: (1) separately excited, (2) shunt-excited, (3) flat-compounded and (4) over-compounded. The per cent. voltage regulation of the separately excited generator is $\frac{bc}{cf} \times 100$ per cent.; the regulation of the shunt generator is $\frac{bd}{df} \times 100$ per cent. The regulation of the flat-compounded generator is the per cent. variation of voltage from the normal no-load or full-load value; it is $\frac{mn}{np} \times 100$ per cent. The regulation of the over-compounded generator is $\frac{gh}{hk} \times 100$ per cent., where gh is the maximum intercept between the voltage characteristic and the straight line va.

57. Series Generator.—The series generator, Fig. 180, is excited by a series winding carrying the load current and has no

shunt winding. Its characteristic curves are shown in Fig. 181. The effect of residual magnetism is neglected. Curve 1 is a no-load saturation curve obtained by exciting the generator from a separate source. Curve 2 is the internal voltage characteristic showing the relation between generated voltage and load when the generator is self-excited, and the horizontal distance between curve 1 and curve 2 represents the effect of armature reaction. The armature reaction due to full-load current is represented by mp, and assuming that the armature reaction varies directly as the load current the other points on curve 2 may be found easily. Curve 3, the external characteristic, is obtained by subtracting from the ordinates of curve 2 the corresponding resistance drops in the armature and series field shown in curve 4.

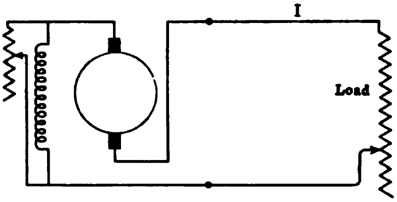


FIG. 180.—Series generator.

The terminal voltage of a series generator can be varied by connecting a variable resistance in shunt to the series winding. The

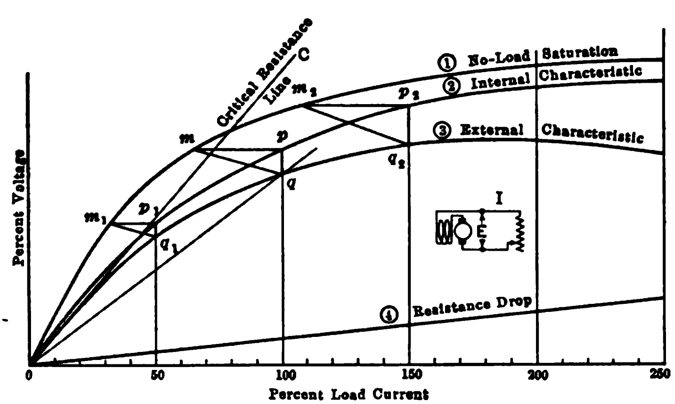


FIG. 181.—Voltage characteristics of a series generator.

slope of the line oq represents the resistance in the load circuit at full load; as this resistance is increased the load current and the terminal voltage decrease. When the resistance line oC is tangent to the voltage characteristic the machine is unstable and is liable to lose its voltage. The load resistance at which this condition occurs is called the critical resistance for the generator.

It is an indefinite quantity on account of the effect of residual magnetism.

158. Electric Motors.—In generators mechanical power is supplied and electrical power is generated. The speed is fixed by the prime mover and is constant. The terminal voltage is approximately constant in the shunt generator and flat-compound generator and increases with load in the over-compound generator and the series generator. The generated voltage is always greater than the terminal voltage by the drop in the armature resistance; it is

$$\mathcal{E} = E + Ir.$$

In motors electrical power is supplied and mechanical power is generated. The impressed voltage is fixed by the supply circuit and is constant. The speed is either approximately constant as in the shunt motor or decreases with load as in the compound motor and series motor. The voltage generated in the armature has the same equation as the voltage in a generator, but it is a back voltage and opposes the current; the impressed voltage E is greater than the back-generated voltage by the armature resistance drop; thus,

$$E = \mathcal{E} + Ir, \tag{246}$$

or Bemf

$$\mathcal{E} = E_T - Ir. \tag{247}$$

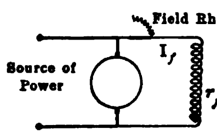


FIG. 182.—Shunt motor.

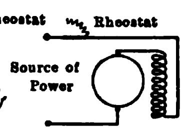


FIG. 183.—Series motor.

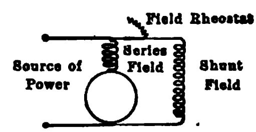


FIG. 184.—Compound motor.

159. Types of Motors.—There are three types of direct-current motors corresponding to the three types of generators, shunt, compound and series. The shunt motor has its field circuit connected across the line in shunt with the armature and therefore has a constant excitation (Fig. 182). The compound motor has a series winding carrying the load current in addition to its shunt winding. The excitation therefore increases with load (Fig. 184).

The series motor has its field circuit in series with the armature and has no shunt winding. Its excitation is zero at no load and increases directly with the load current (Fig. 183).

160. Speed Equation of a Motor.—The impressed voltage is

$$E = \mathcal{E} + Ir,$$

and

$$\mathcal{E} = Zn\Phi \frac{p}{p_1} 10^{-8} = Kn\Phi \text{ volts},$$

where,

$$K = Z\frac{p}{p_1} 10^{-8} \text{ is a constant;}$$

therefore

$$E = Kn\Phi + Ir$$

and

$$n = \frac{E - Ir}{K\Phi}; \tag{248}$$

this is the speed equation.

The term Ir is small in comparison to E and may be neglected except at heavy loads.

The speed equation may therefore be written

$$n = \frac{E}{K\Phi}; \tag{249}$$

thus, the speed of a motor is directly proportional to the impressed voltage and is inversely proportional to the flux crossing the air gap.

161. Methods of Varying Speed.—The speed can be varied in three ways: (1) by varying the impressed voltage E, (2) by varying the flux Φ by reducing the field current, (3) by shifting the brushes.

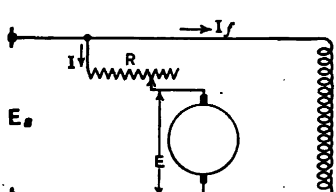


FIG. 185.

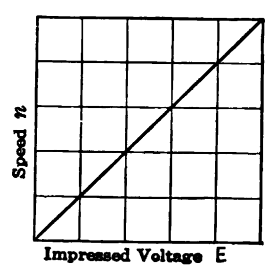


FIG. 186.—Variation of speed with impressed voltage.

1. The voltage impressed on the armature can be varied by introducing a resistance R, Fig. 185, in series with the armature. If E_s is the supply voltage, the voltage impressed on the motor is

$$E = E_s - IR.$$

By increasing R, E can be reduced to any required value and any speed from standstill to normal speed can be obtained but no increase of speed above normal.

Fig. 186 shows n as a function of E; the locus is a straight line passing through the origin.

This method of varying speed is uneconomical as a large amount of power is lost in the control resistance; it is the product of the current input and the voltage consumed in the resistance and is

$$= I \times IR = I^2R \text{ watts.}$$

The resistance R must not be connected in series with the field winding as it would then decrease the field current and therefore the flux and tend to cause an increase in speed.

If the speed is reduced to 50 per cent. of normal in this way the efficiency of the motor is less than 50 per cent.

With a variable voltage supply a wide range of speed can be obtained efficiently (see Arts. 173 and 174).

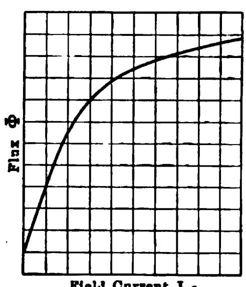


FIG. 187.

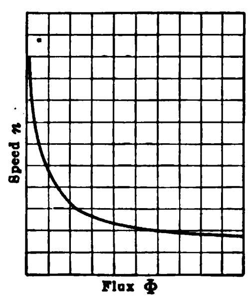


FIG. 188.

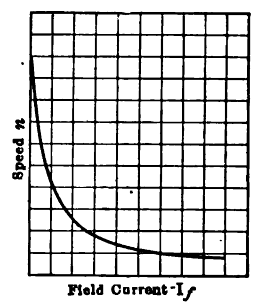


FIG. 189.

2. When varying the speed by field control the full line voltage is impressed on the armature and a resistance R is connected in series with the field winding, Fig. 182. As this resistance is increased the field current I_f decreases according to equation

$$I_f = \frac{E}{R + r_f},$$

and the flux Φ decreases with the field current as shown by the saturation curve of the machine in Fig. 187. Since the speed varies inversely as Φ, Fig. 188, the variation of speed with field current will be represented by a curve of the shape shown in Fig. 189.

By this method the speed can be increased to any required value, and it tends to approach infinity when the field current is zero but is limited by the residual magnetism.

In machines of ordinary design the speed can be increased satisfactorily only about 70 per cent. above normal speed by field weakening. Beyond this point it is not possible to get sparkless commutation of full-load current, since the armature m.m.f. is strong enough to overcome the weak field m.m.f. and wipe out the commutating field and at the same time the time of commutation is decreased.

By using interpoles which neutralize the effect of armature m.m.f. and provide a commutating field, the speed can be increased to four or five times normal by field weakening without injurious sparking.

The loss in power in the resistance controlling the field current is very small since the power used for field excitation is only 1 or 2 per cent. of the rated output.

3. At no load with the brushes on the neutral line, all the conductors on each half of the armature are effective in generating the back e.m.f. and this is therefore the position of minimum speed Fig. 160(*a*).

When the brushes are moved back against the direction of rotation, Fig. 160(*b*), only the belts of conductors under the poles are effective in generating the back e.m.f. and the result is the same as though the flux had been decreased. The speed is therefore increased.

Under load, when current is flowing in the armature, the conductors between the poles exert a demagnetizing m.m.f. and cause a decrease in the flux; the speed therefore rises more than at no load.

This method of speed variation is not used to any extent since it interferes with commutation and causes injurious sparking.

162. Speed Characteristics of Motors.—The speed characteristic is the curve showing the variation of speed with armature current. The speed equation was found to be

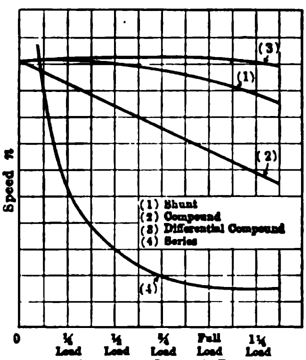
$$n = \frac{E' - Ir}{K\Phi} \text{ r.p.s.} \quad \text{(Art. 160)}$$

Shunt Motor.—As the motor is loaded and I increases the speed is affected in two ways: (1) the resistance drop Ir increases and tends to cause a proportionate decrease in speed; (2) the flux Φ

is decreased by the armature demagnetizing and cross-magnetizing m.m.f.; therefore the armature reaction tends to increase the speed. The voltage drop at the brush contacts (Art. 182) increases with load but not in direct proportion. It acts with the armature resistance to decrease the speed. If the motor is operated at a point just above the knee of the saturation curve the drop in speed due to the armature resistance will be greater than the rise due to armature reaction and the speed characteristic will fall as shown in Fig. 190 curve 1.

The speed regulation of a motor is the rise in speed when full load is thrown off expressed as a per cent. of full-load speed.

The speed regulation of shunt motors of large size is from 2 to 5 per cent.



'Fig. 190.—Motor speed characteristics.

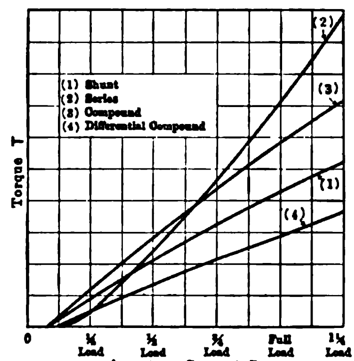


Fig. 191.—Motor torque characteristics.

Compound Motor.—In the compound-wound motor the armature resistance causes a drop in speed and the m.m.f. of the series winding overcomes the effect of the armature m.m.f. and increases the flux and thus decreases the speed more than in the shunt motor. A typical speed characteristic of a compound motor is shown in curve 2, Fig. 190.

If the series winding is reversed, its m.m.f. opposes the field m.m.f. and thus decreases the flux and causes the speed to increase with load as shown in curve 3, Fig. 190.

The motor is then called a "differential compound" motor and may be designed to give constant speed under all loads. If the series-field winding is strong the motor is unstable and tends to run at excessive speed.

Series Motor.—At no load the series motor tends to run at a very high speed limited only by the residual magnetism or the torque required to overcome the losses. As load is applied the current and flux increase and the speed falls rapidly until the magnetic circuit of the machine becomes saturated; the speed characteristic then becomes almost horizontal (Fig. 190, curve 4).

163. Torque Equation.—The torque of a motor is proportional to the product of the flux crossing the air gaps and the current in the armature. Its equation is derived as follows:

The voltage impressed on the armature is

$$E = \mathcal{E} + Ir,$$

where

$$\mathcal{E} = Zn\Phi \frac{p}{p_1} 10^{-8}$$

is the back voltage generated in the armature and Ir is the voltage consumed by the armature resistance.

The power input to the armature is

$$EI = \mathcal{E}I + I^2r \text{ watts.} \qquad (250)$$

The power lost in the armature winding is I^2r watts, and thus the electric power transformed into mechanical power is

$$\mathcal{E}I = Zn\Phi I \frac{p}{p_1} 10^{-8};$$

this is the power output in watts neglecting the friction losses.

The motor output in horsepower is

$$\frac{\mathcal{E}I}{746} = \frac{Zn\Phi I \frac{p}{p_1} 10^{-8}}{746}.$$

If the torque developed is T lb.-ft., then the output in horsepower is

$$\frac{2\pi nT}{550} = \frac{Zn\Phi I \frac{p}{p_1} 10^{-8}}{746}$$

and the torque is

$$T = \frac{550}{2\pi n} \cdot \frac{Zn\Phi I \frac{p}{p_1} 10^{-8}}{746} = 0.1177 Z\Phi I \frac{p}{p_1} 10^{-8} \text{ lb.-ft.} \qquad (251)$$

This is the torque equation of a direct-current motor.

$$T = 0.1177 \times 10^{-8} \times \left(Z\frac{I}{p_1}\right) \times (p\Phi) \text{ lb.-ft.,} \qquad (252)$$

where $Z\frac{I}{p_1}$ is the sum of all the currents in the armature conductors and $p\Phi$ is the sum of the fluxes crossing the air gaps under all the poles.

The torque equation may be written

$$T = K\Phi I \text{ lb.-ft.,} \tag{253}$$

where $K = 0.1177\, Z\frac{p}{p_1}\, 10^{-8}$ is a constant.

The torque of a motor is therefore directly proportional to the armature current and to the flux in the air gap.

This is the torque developed at the shaft of the motor. The torque output is less due to the iron and friction losses.

164. Torque Characteristics of Motors.—The torque characteristic is a curve showing the relation between the torque and the armature current.

The torque equation is

$$T = K\Phi I. \quad \text{(Art. 163.)}$$

Shunt Motor.—In the shunt motor Φ is almost constant under load, decreasing only by a small percentage due to armature reaction. The torque therefore varies almost directly with the current (Curve 1, Fig. 191).

Series Motor.—In the series motor the flux increases almost in direct proportion to the current, while the magnetic circuit is unsaturated, and therefore the torque is proportional to the square of the current; at heavy load, when the magnetic circuit becomes saturated, the flux becomes almost constant and the torque then increases in direct proportion to the current. Curve 2, Fig. 191, is a typical torque-current curve for a series motor.

Compound Motor.—In the compound motor the flux increases with load but not in direct proportion to the current; thus, the torque-current curve (curve 3) lies between those of the shunt and the series motor.

If the series winding is reversed, as in the differential compound motor, the flux decreases with load and the torque-current curve falls below that of the shunt motor (Curve 4).

165. Construction of the Speed Characteristics.—The speed characteristic of any motor can be constructed if the no-load saturation curve is given and the magnitudes of the armature reaction and armature resistance. In Fig. 192 *omn* is the no-

load saturation curve of a motor, flux Φ vs. field current I_f. The field current is maintained constant and the flux at no-load is $\Phi_0 = oa$ and the speed is n_0.

The effect of armature reaction at full load is represented by mp and it reduces the flux to the full-load value oa_1. At no load the impressed voltage and the back voltage are equal and may be represented by oa. The required back voltage at full load is less than at no load by the armature resistance drop Ir and is represented by oa_2. The speed at full load is therefore reduced in the ratio $\frac{oa_2}{oa_1}$ to a value $n = n_0 \frac{oa_2}{oa_1}$.

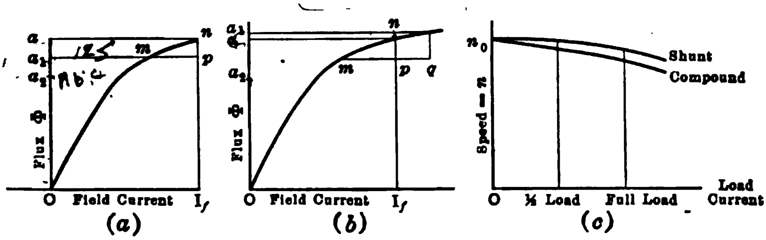


FIG. 192.—Construction of speed characteristics.

If the resistance drop is very small it may happen that the point a_2 comes above a_1, in this case the speed rises above the no-load value. Such a condition often exists at fractional loads. With overloads the demagnetizing effect increases directly with the current and reduces the flux density considerably but on this account the cross-magnetizing effect is relatively smaller and the increased resistance drop always results in a decrease of speed. This is the case of the shunt machine and the resulting speed characteristic is as shown in Fig. 192(c).

In the compound motor the effect of armature reaction will be opposed by the m.m.f. of the series winding and the flux will be reduced less than in the shunt motor or it may be increased. If a series winding is added to the shunt motor above exerting at full load a m.m.f. mq, Fig. 192(b), greater than the armature reaction mp, the flux at full load will be increased to the value oa_1, and the speed will be reduced to $n = n_0 \frac{oa_2}{oa_1}$. Other points on the speed characteristic can be found in the same way, Fig. 192(c).

166. Construction of the Speed Characteristic for a Series Motor.—In Fig. 193(a) om is the no-load saturation curve of a

series motor obtained by separately exciting the fields and driving as a generator at the full-load speed of the motor. If mp represents the effect of armature reaction at full load, oa_1 is the back voltage at full load at a speed n_f and it may be taken to represent also the flux at full load. The impressed voltage is greater than oa_1 by the resistance drop Ir and may be represented by oa. This value is constant.

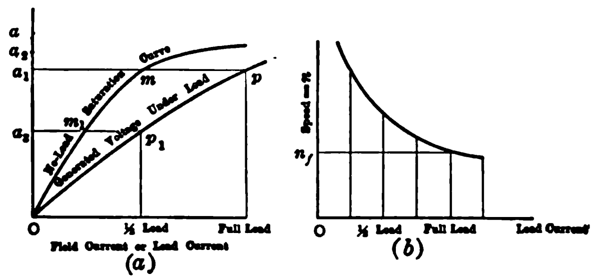


FIG. 193.—Construction of speed characteristics or a series motor.

At one-half load the armature reaction may be taken as $m_1p_1 = \frac{mp}{2}$ and the corresponding flux is oa_3; the resistance drop is only half of its former value $= aa_2 = \frac{aa_1}{2}$ and so the back voltage must be oa_2. The speed is therefore increased to $n = n_f \times \frac{oa_2}{oa_3}$. The complete characteristic is shown in Fig. 193(b).

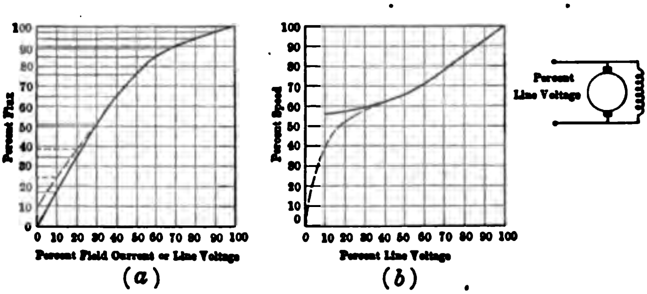


FIG. 194.—Variation of speed with line voltage (shunt motor without load).

167. Variation of Speed of a Shunt Motor with Line Voltage.—If the voltage impressed on the terminals of a shunt motor decreases, the speed decreases but not in direct proportion. Referring to Fig. 194, (a) shows a no-load saturation curve

plotted with flux on a base of field current or line voltage and (*b*) shows motor speed on a voltage base.

When the line voltage is 100 per cent., the field current, the flux and the speed are all 100 per cent. If the line voltage drops to 90 per cent., the field current drops to 90 per cent. and the flux to 97 per cent;. due to the decrease in voltage impressed on the armature the speed tends to drop to 90 per cent. but this is partly counterbalanced by the drop in flux to 97 per cent. The speed is, therefore, $100 \times 0.90 \times \frac{1}{0.97} = 92.9$ per cent. When the voltage falls to 50 per cent. the flux is 75.5 per cent. and the speed is $100 \times 0.5 \times \frac{1}{0.755} = 62.2$ per cent.

If the saturation curve passes through the origin, the speed voltage curve in (*b*) tends to cut the speed axis about 55.5 per cent. speed. This is a theoretical point for a motor without friction and without residual magnetism.

If a residual flux of 10 per cent. is assumed as indicated by the dotted part of the saturation curve the speed voltage curve drops off to the origin along the dotted part of the curve in (*b*). The motor becomes unstable on the lower voltages.

168. Variation of Speed with Temperature of the Field Coils.—If the speed of a motor is measured immediately after being started and again after running under load for some time it will be found to have increased. If the rise in temperature of the field is 40°C., the increase in resistance will be approximately $40 \times 0.4 = 16$ per cent. and the field current will decrease 16 per cent.; this results in a decrease of 7 or 8 per cent. in the flux and an increase of 7 or 8 per cent. in speed. The amount of the speed change depends on the saturation of the machine.

169. Construction of the Torque Characteristic.—In Art. 165 the method of obtaining the flux in the air gap at any given value of load current is indicated. If curves showing the variation of flux with load current are plotted as shown in Fig. 195, the torque characteristics of the motors may be found by multiplying the values of flux by the corresponding values of current. This follows from the torque equation

$$T = K\Phi I.$$

The torque at some particular load must be known in order to fix the scale of the curve.

The values of torque obtained in this way represent the torque developed at the shaft of the motor. The external torque or available torque is less by the torque required to supply the iron and friction losses.

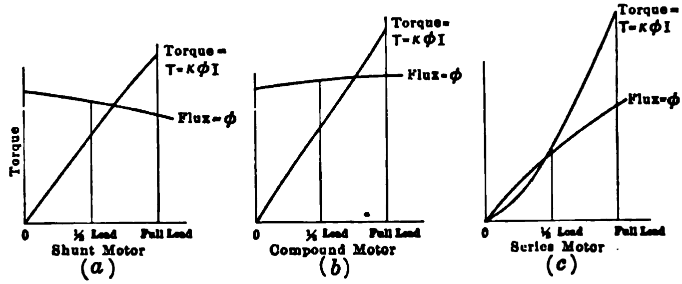


FIG. 195.—Construction of torque characteristics.

170. Starting Torque.—In Fig. 196 are shown typical speed and torque characteristics for shunt, series and compound motors with the same full-load speed and torque.

When starting, a motor must often exert a torque considerably in excess of the full-load value in order to overcome friction and the inertia of the load. At the same time the starting current must be kept within reasonable limits in order to prevent fluctuations of the voltage of the system. If the required starting torque is just equal to full-load torque, each type of motor would draw full-load current. If the starting current is limited to 150 per cent. of full-load current, the starting torque developed by the three motors would be approximately, shunt 140 per cent. of full-load torque, compound 160 per cent., depending on the strength of the series winding and series 200 per cent. Thus for loads requiring starting torques in excess of full-load torque the series and compound motors have the advantage over the shunt motor that they can develop the required torque with smaller currents and thus with smaller demands on the supply system.

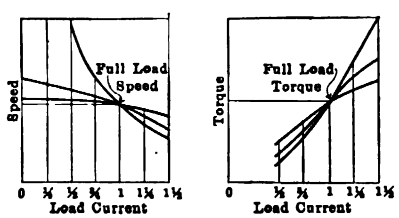


FIG. 196.—Speed and torque characteristics.

When starting shunt or compound motors the field rheostat

should be entirely cut out in order to give as strong a field as possible (Fig. 197).

171. Motor Starter with No-voltage Release.—Fig. 198 shows the connections of a motor starter with a no-voltage release. When the main switch is closed, current cannot flow to the motor until the starting handle is moved to contact *C*, which allows current to flow through the field winding and no-voltage release *B* and through the total starting resistance R_s to the motor armature. The auxiliary contact *A* is connected to contact *C* but is placed in such a position that, when the motor is to be disconnected the starting handle opens the circuit at *A* instead of at *C* and the spark produced by the field discharge burns only the small contact *A* which is easily renewed. In starting up the handle should be held on *C* for a short time to allow the field of the motor to build up. As the motor speeds up and develops its back voltage the handle is gradually moved over to the final position in which all the starting resistance is cut out and the armature is connected directly to the supply. The starting resistance is cut into the field circuit but its effect in reducing the field current is usually negligible.

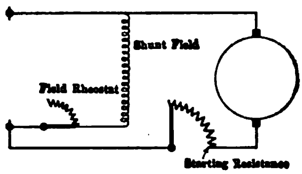


FIG. 197.—Starting a shunt motor.

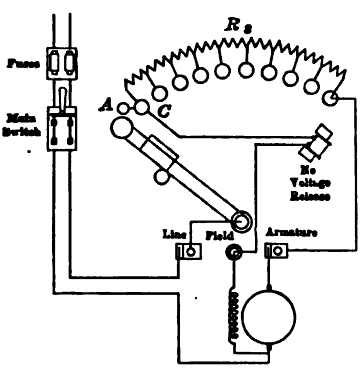


FIG. 198.—Direct-current motor starter.

An iron armature is carried by the starting handle and in the final position it completes the magnetic circuit of the no-voltage release and the handle is held in the running position. If the power is interrupted for any reason, or if the field circuit opens, the no-voltage release becomes demagnetized and a spring pulls the handle back to the starting position.

The full starting resistance R_s must have such a value that the starting current will not be greater than one and one-fourth to one and one-half times full-load current.

The starting resistance is designed only for starting duty and must not be used to operate the motor at low speeds as it would

become overheated and burn out. It is mounted in a ventilated box on the back of the starter.

Example.—A 100-hp., 230-volt, shunt motor has an efficiency of 90 per cent. and its armature resistance is $r = 0.03$ ohm. Find (*a*) the starting resistance to limit the starting current to one and one-fourth times full-load current and (*b*) the armature current if the motor were connected directly to the supply while at standstill.

(*a*) The full-load current is $\frac{100 \times 746}{230 \times 0.90} = 360$ amp.; the allowable starting current is $\frac{5}{4} \times 360 = 450$ amp.

The starting resistance must therefore be

$$R_s = \frac{230}{450} = 0.51 \text{ ohm}.$$

This value includes the small armature resistance.

(*b*) If full voltage were applied to the armature at rest the current would be limited by the armature resistance only and would tend to reach a value $I = \frac{230}{0.03} = 7{,}600$ amp. This would burn out the armature winding unless the circuit was protected by circuit-breakers or fuses.

172. Adjustable Speed Operation.—In Art. 161 three methods of varying motor speeds were discussed: (*a*) field control for increasing the speed; (*b*) armature control for decreasing the speed; and (*c*) shifting of the brushes which is possible only in special cases.

For lathes where the cutting speed must be constant although the diameter of the material changes field control must be used. Armature control is not satisfactory as may be seen from the following example. Referring to Fig. 199, *abc* is a piece of metal to be turned. Assume that the motor driving the lathe is operating on a 115-volt system and that while cutting through the projection between *a* and *b* the armature current is 100 amp., the voltage across the armature terminals is 75 volts and the speed is 300 r.p.m. This would require a resistance in series with the armature $R = \frac{115 - 75}{100} = 0.4$ ohms to take up the extra 40 volts.

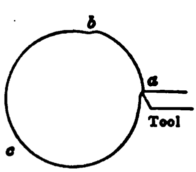


Fig. 199.

After passing the point *b* the cut is light and the current is reduced to 10 amp., the drop across the series resistance is $10 \times 0.4 = 4$ volts and the voltage impressed on the armature is 111; the speed therefore rises to $300 \times \frac{111}{75} = 444$ r.p.m. and the

point of the tool is liable to be broken when it strikes the point *a* again.

Field control alone cannot, however, give a wide enough range for all cases since the speed can be increased only about 70 per cent. above normal without trouble due to commutation. Where speed ranges of 3 to 1 or 4 to 1 are required it is necessary to use interpole motors or to have a multiple-wire system of supply.

173. Multiple-wire Systems of Speed Control.—In Fig. 200 *G* is the main generator giving 250 volts between terminals.

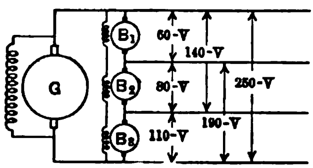


FIG. 200.—Multiple-wire system.

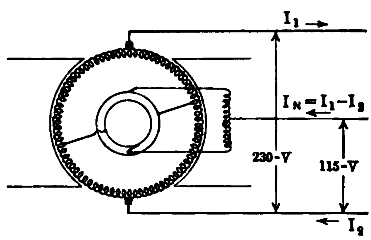


FIG. 201.—Three-wire generator.

$B_1B_2B_3$ is a balancer set with three armatures on the same shaft giving voltages of 60, 80 and 110. With this system a complete range of voltages from 60 to 250 is available for the motor armature and the intermediate speeds can be obtained by varying the field of the motor which is permanently connected across the outer wires at 250 volts. The possible speed range is about 6 to 1, but the system is very complicated and expensive.

Three-wire systems are often used instead of four-wire systems. That shown in Fig. 201 is very common. A three-wire generator (Art. 367) giving 230 volts between outers and 115 volts to neutral is used. A speed range of 4 to 1 is possible but the motors must be specially designed to give a 2 to 1 range by field control.

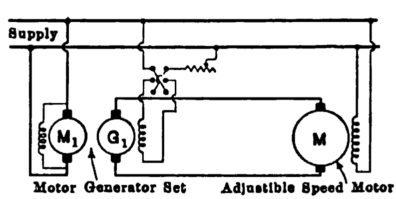


FIG. 202.—Ward Leonard system.

174. Ward Leonard System of Speed Control.—This system is illustrated in Fig. 202. M_1G_1 is a high-speed motor-generator set. The field of the generator is arranged so that it can be varied through a wide range and reversed. Its terminal voltage is impressed on the armature of

the motor M in which speed control is required. A uniform variation of speed from a maximum in one direction through zero to a maximum in the opposite direction may be obtained. The field of the motor M is permanently connected across the supply lines.

175. Speed Control of Series Motor.—The speed of a series motor can be varied by connecting a resistance in series with the armature but in cases where a number of motors are connected to the same load, as in electric-railway operation, series-parallel control may be used resulting in a more efficient speed variation.

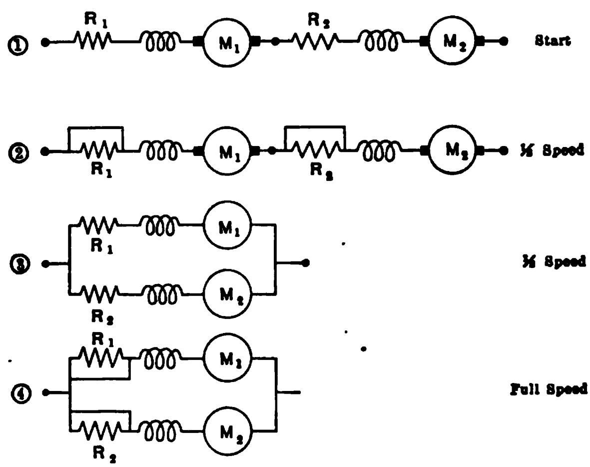


Fig. 203.—Series-parallel control.

In Fig. 203 M_1 and M_2 are two similar motors with their equal starting resistances R_1 and R_2. In position (1) the whole of the line voltage (500 volts) is taken up by the starting resistances R_1 and R_2 and the motors are at rest. In position (2) the starting resistances are short-circuited, the current is the same as before and each motor is running at half speed and develops a back voltage of 250 volts. In (3) the motors are in parallel and the starting resistances are in series again. The current to each motor is the same as before but the current from the line is doubled. The speed is the same as in (2). In (4) the resistances are short-circuited again and the motors are running at full speed with 500 volts impressed on their terminals. The loss of energy in the control resistances is just half what it would have been if the motors had been connected in parallel at the start.

The current per motor is kept constant so that the rate of acceleration of the car may be constant.

If speeds higher than full speed are required they may be obtained by connecting resistances in shunt to the motor fields.

When it is necessary to start a series motor very slowly the armature is shunted by a resistance which reduces the starting torque.

When it is necessary to bring a series motor to rest quickly it may be disconnected from the line and a resistance connected across the armature terminals. The motor acts as a generator and slows up very quickly. The power developed is wasted in heating the resistance. This is one form of dynamic braking.

176. Interpole Motors.—Interpole motors are fitted with small poles, midway between the main poles, excited by the load current (Fig. 224). The interpole m.m.f. increases directly with the load; it is stronger than the armature m.m.f. and neutralizes it in the space directly beneath the interpole and produces a flux in the proper direction to assist commutation. The brushes are placed on the no-load neutral and the motor may be operated in either direction. Since commutation is taken care of by the interpole field speed ranges of 4 to 1 or even 6 to 1 can be obtained by field control without serious sparking.

To prevent saturation of the interpoles at heavy loads they must be designed with very low flux densities at normal load.

If shunts to the interpole windings are used the ratio of their inductance to the inductanee of the interpole windings must be the same as the ratio of their resistances, in order that sudden variations of load may be taken care of.

177. Applications of Motors.—When deciding on the type of motor to be employed in a given case three points are of special importance: (1) speed—whether constant, variable or adjustable; (2) starting torque—whether greater or less than full-load torque; (3) starting current—whether it is liable to disturb the voltage of the system.

The types of motor available are: (1) the shunt motor which runs at approximately constant speed and develops a torque proportional to the current; the speed may be adjusted to any suitable value within limits; (2) the series motor with variable speed and torque proportional to the square of the current below saturation; if twice full-load torque is required at start it draws about one and one-half times full-load current; (3) the compound

motor with variable speed and a torque characteristic between the shunt and the series. The compound motor has the advantage over the series motor that it approaches a limiting speed at light load set by its shunt excitation.

Shunt motors are used for lathes, boring mills, and all constant speed machine tools, for driving line shafting when the starting load is not too heavy, for fans, centrifugal pumps, etc.

Series motors are used in electric-railway operation and for cranes, hoists, etc., where very large starting torque is necessary and where constant speed is not required.

When a load comes on a series motor it responds by decreasing its speed and supplying the increased torque with a small increase of current, thus preventing a sudden shock on the system. A shunt motor under the same conditions would hold its speed nearly constant and would supply the required torque with a large increase of current and would thus make a heavy demand on the system.

Series motors must not be used for belt drives or in any case where the load may be removed suddenly since they run at excessive speed at light loads.

Compound-wound motors are used in classes of work where constant speed is not necessary and where a fairly large starting torque is required, except in those cases in which series motors must be employed on account of their very large starting torque.

For express elevators a compound-wound motor is employed, the series winding is required to give the large torque at start and is cut out after a certain speed is reached. The motor then runs at constant speed as an ordinary shunt machine.

For operating shears, punches, etc., where a high maximum load must be carried for a short period a compound motor, with a flywheel attached, is used. The drooping speed characteristic is necessary to enable the flywheel to give up part of its stored energy to supply the peak of the load and so relieve the supply system. A shunt motor would not drop in speed and the flywheel would not be of any value. A series motor would not be suitable as it would run at excessive speeds before and after the cut.

In rolling mills where the load fluctuates very rapidly a similar compound motor with a heavy flywheel is used.

Compound motors are often installed in positions where the constant speed characteristic of the shunt motor would be more

suitable but where the series winding is required to supply the torque to overcome the inertia of the heavy moving parts, as in the case of heavy planers, etc.

178. Losses in Direct-current Machinery.—The main power losses in direct-current generators and motors may be divided into copper losses, iron losses and friction losses and these may be subdivided as follows:

Copper losses:
- Shunt-field copper loss.
- Series-field copper loss.
- Armature copper loss.

Iron losses:
- Hysteresis loss.
- Eddy-current loss.

Friction losses:
- Brush-friction loss.
- Bearing-friction loss.
- Windage loss.

179. Shunt-field Loss.—The shunt-field copper loss is $I_f^2r_f$ watts, where I_f is the field current and r_f is the resistance of the winding at the operating temperature of the machine. This loss can be expressed as E_fI_f, where $E_f = I_fr_f$ is the voltage impressed on the field winding. All the energy supplied to the field winding is transformed into heat and is wasted, since no energy is required to maintain the magnetic flux after it is once established.

The shunt-field loss is constant under all conditions of load in the shunt motor and the flat-compound generator; it decreases slightly with load in the shunt generator and increases with load in the over-compound generator.

The loss ranges from about 1 per cent. of full-load output in the case of large high-speed machines to 5 per cent. in small low-speed machines.

If there is a rheostat connected in series with the shunt-field winding, the power wasted in it should be included in the shunt-field copper loss.

180. Series-field Loss.—The series-field copper loss is $I_s^2r_s$ watts, where I_s is the current in the series winding and r_s is its resistance. This loss increases as the square of the load current of the machine. In interpole machines the resistance r_s will in-

clude the resistance of the interpole winding. The loss in the shunts to the series winding or in the series-field rheostat must be included in the series-field loss.

181. Armature Copper Loss.—The armature copper loss proper is $I_a{}^2r_a$ watts, where I_a is the armature current and r_a is the resistance of the winding not including the brush contacts. It increases as the square of the load current.

In addition to the loss due to the load current there are extra losses produced by local currents in the armature coils short-circuited by the brushes. If the e.m.fs. induced in the various parts of the armature winding connected in multiple between the terminals are not all equal circulating currents will flow both at no load and under load and cause loss.

182. Loss at the Brush Contacts.—Closely associated with the armature copper loss and usually included with it is the loss at the brush contacts. Between the carbon brush and the copper commutator there is a drop of voltage which is more of the nature of a back voltage than a voltage consumed by resistance. This drop varies with the current density and is also affected by the direction of the current flow. Fig. 204 shows the relation between contact-drop and current density for an ordinary hard carbon brush. The drop is higher at the negative brush, that is, from carbon to copper, than at the positive brush. At low densities it increases with the current but above 35 or 40 amp. per square inch it becomes almost constant.

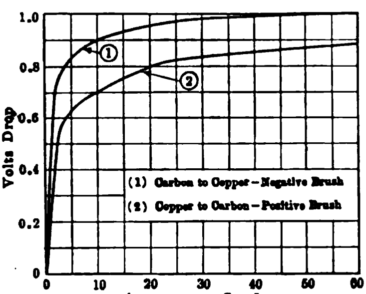


FIG. 204.—Brush contact drop.

The loss of power at the brush contacts is eI_a, where e is the drop of voltage at the positive plus the negative brush and may be taken as approximately 2 volts except for very soft carbon or graphite brushes or for the combined metal and carbon brushes, where it is very much lower.

With a current density of 35 amp. per square inch and a contact drop of 1 volt per brush the power loss is 35 watts per square inch of brush contact. This is a reasonable value. Where the contact drops are smaller higher current densities may be used.

In low-voltage machines this loss of power is an appreciable

per cent. of the output and may be very serious, but for high-voltage machines it is less important.

With copper brushes the contact drop is very small and densities up to 150 amp. per square inch are used.

183. Hysteresis Loss.—The hysteresis loss is due to the reversal of the magnetic flux in the armature iron as it moves across a pair of poles.

The loss per cubic centimeter of iron per cycle is

$$w_h = \eta \mathcal{B}^{1.6} \text{ ergs} \quad \text{(equation 123)},$$

where $\mathcal{B}$ is the maximum induction density in lines per square centimeter and η is the hysteretic constant for the iron. The value of η for ordinary armature iron after assembly may be taken as 0.0027.

The number of cycles of magnetism per second or the frequency is

$$f = \text{r.p.s.} \times \text{number of pairs of poles} = n\,\frac{p}{2}. \tag{254}$$

If the induction density B is expressed in lines per square inch, the hysteresis loss per cubic inch of iron per second is

$$W_h = \eta\left(\frac{B}{2.54^2}\right)^{1.6}(2.54)^3 f \text{ ergs per second}$$
$$= 0.83\eta B^{1.6} f\, 10^{-7} \text{ watts.} \tag{255}$$

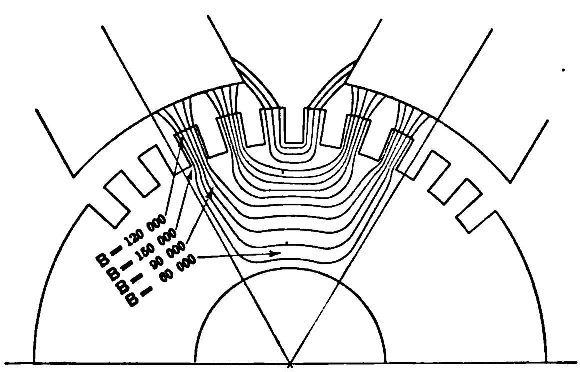


FIG. 205.—Distribution of flux in the armature core.

In transformers where the induction density is very nearly uniform throughout the volume of the iron, this value multiplied by the volume of the iron would give the hysteresis loss very closely; but in the case of dynamos the induction density is not

uniform but varies from a maximum at the roots of the teeth to a very low value at the bottom of the armature core, as indicated in Fig. 205. The loss can only be determined very approximately by making separate calculations for the teeth and the core.

The hysteresis loss varies directly with the frequency, that is, with the speed of the machine; it increases as the 1.6th power of the flux density and thus of the voltage and it increases to some extent under load due to the distortion of the flux. The loss is increased in sections where the density is increased more than it is decreased where the density is decreased.

The hysteresis loss may be very seriously increased by careless handling of the materials during assembly.

In some cases the hysteresis loss has been found to increase after the machine has been in operation for a short time. This is due to aging of the iron and will increase the temperature rise of the machine. Silicon steel in addition to having a low hysteresis loss and high electrical resistance is non-aging.

184. Eddy-current Loss.—The eddy-current loss is due to electric currents set up in the armature iron by the e.m.fs. generated in it as it cuts across the flux.

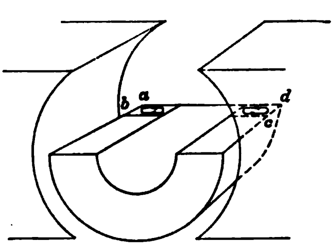


Fig. 206.—Eddy-current loss in the armature core.

In Fig. 206 *abcd* represents a section of an armature punching of thickness t in. If the flux density in the gap is B lines per square inch and the edge ab is moving with a velocity of v in. per second across the flux, then, the e.m.f. generated in the length ab is

$$e = Btv10^{-8} \text{ volts.}$$

This e.m.f. will cause a current to circulate through the iron as indicated by the arrows and its value will be

$$i = k\gamma e = k\gamma Btv10^{-8} \text{ amp.}$$

where γ is the electric conductivity of the iron and k is a constant depending on the dimensions of the section.

The loss in the section will be

$$p = i^2r = \frac{i^2}{k\gamma} = \frac{k^2\gamma^2B^2t^2v^2}{k\gamma}10^{-16} = k_1\gamma B^2t^2v^2 \text{ watts} \quad (256)$$

where $k_1 = k \times 10^{-16}$ is a constant.

The eddy-current loss, therefore, varies as the square of the induction density, that is, the square of the terminal voltage, the square of the thickness of the plates, and the square of the speed; it also depends on the conductivity of the iron used. It cannot be calculated accurately in the case of an armature where the induction density varies throughout the volume.

The loss increases under load due to the distortion of the field but it tends to decrease as the temperature rises and decreases the conductivity.

The eddy-current loss is reduced by building up the armature of thin punchings insulated from one another by varnish which increases the resistance in the path of the eddy currents. If the slots are filed out after assembly to remove rough edges, adjacent plates may be short-circuited and the value of the lamination of the core partly lost. If the punchings are assembled under too great pressure the resistance between them will be reduced and the loss increased.

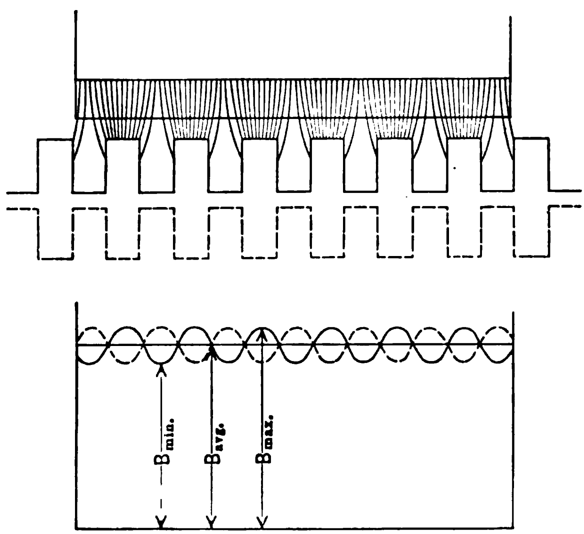


Fig. 207.—Eddy-current loss in the pole face.

Eddy-current losses also occur in the armature end plates, spider and bolts, due to leakage fluxes, and they are comparatively large since these parts are not laminated.

185. Pole-face Loss.—Eddy currents are also produced in the pole faces due to local variations of the induction density as the armature teeth move across them (Fig. 207). The pulsations

of flux do not extend far below the surface of the pole face since they are opposed by the eddy currents. The frequency is very high, being proportional to the product of the revolutions per second and the number of teeth. Formulæ have been derived for the calculation of the pole face losses but they are very complicated and must be applied with great care.

To reduce the loss the pole faces of all direct-current machines should be laminated. Machines with long air gaps have small pole-face losses.

186. Brush-friction Loss.—The brush-friction loss in foot-pounds per second is equal to the product of the total brush pressure in pounds, the peripheral speed of the commutator in feet per second and the coefficient of friction between the brush and the commutator.

With carbon brushes the pressure should be from 1.5 to 2 lb. per square inch; this value multiplied by the area of all the brushes gives the total brush pressure in pounds. In railway motors, where there is a great deal of vibration, pressures up to 5 lb. per square inch are used in order to insure good contact.

The coefficient of friction between an ordinary hard carbon brush and the commutator may be taken as 0.3, for graphite brushes it is about 0.25 and for copper brushes 0.2.

The brush friction loss varies directly with the speed but is independent of the load.

187. Bearing-friction and Windage Losses.—These two losses are difficult to separate and only their combined value can be obtained by test.

The bearing-friction loss is proportional to the $(\text{speed})^{3/2}$ for bearing velocities up to 2,000 ft. per minute and is approximately proportional to the speed for higher values. It can be calculated approximately.

The windage loss varies as the $(\text{speed})^3$ but is usually small up to peripheral speeds of 6,000 ft. per minute. It depends on the construction of the machine and cannot be calculated with any degree of accuracy.

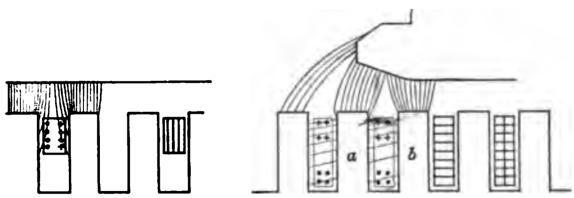
The combined friction and windage losses vary from about ½ per cent. of the rated output in large slow-speed machines to 2 or 3 per cent. in small high-speed machines.

In very high-speed machines the windage loss may become the larger part of the combined loss.

The friction and windage losses are independent of the load.

188. Eddy-current Losses in the Armature Copper.—In addition to the losses discussed above under certain conditions large eddy-current losses may occur in the armature copper both at no load and under load. There are two causes of this loss.

1. Some of the flux from the field poles passes down into the slots and cuts the armature conductors inducing eddy currents in them in the directions indicated in Fig. 208(1). The proportion of the flux entering the slots depends on the length of the air gap, the width of the slots and the saturation of the teeth. In direct-current machines of moderate size the resultant loss is small. In turbo alternators with very wide slots and long air gaps the loss tends to be large and the conductors in the upper part of the slot should be laminated vertically. Under load the loss tends to increase, due to the increased saturation of some of the teeth.



(1) (2)

FIG. 208.—Eddy-current losses in armature conductors.

2. A second and more important cause of eddy currents in the copper is due to the unequal distribution of flux in the armature teeth near the pole tips Fig. 208(2). The flux density in tooth *a* is much lower than that in *b*; the magnetic potential of the roots of the two is the same, but since a larger m.m.f. is required for *b* than for *a* the magnetic potential at the top of *b* must be greater than that at the top of *a* and flux will pass across the slot as indicated. The flux density and the e.m.f. induced in the conductor at the top of the slot will be greater than at the bottom and eddy currents will flow in the direction of the e.m.f. in the upper part of the conductor returning in the lower part. The loss due to these currents is especially large in machines with deep conductors. It is a function of the tooth saturation and therefore increases very rapidly with increase of voltage; it increases with load due to field distortion and also increases with speed. In cases where it is liable to be large conductors should be laminated in a horizontal plane.

Three laminated conductors are shown in Fig. 208.

These eddy-current losses in the copper cannot be calculated but may be kept small by proper design.

189. Constant Losses and Variable Losses.—The losses are sometimes divided into two groups, the constant losses and the variable losses.

The constant losses are those which do not vary to any great extent under load and include the shunt-field copper loss, the iron losses and the friction and windage losses.

The variable losses are those which increase with load, namely, the armature copper loss and the series-field copper loss.

190. Core Loss.—The core loss includes all the losses located in the armature core except the armature copper loss. It may be obtained by running the machine without load, measuring the input and subtracting the shunt-field copper loss and the friction and windage losses. It therefore includes the no-load iron losses and in addition the eddy-current losses in the pole faces and in the armature conductors and any losses which may be produced by currents circulating in the armature windings.

The core loss increases to some extent under load due to field distortion.

In designing machines an approximate value for the core losses may be obtained from curves such as those in Fig. 257, compiled from tests on standard machines.

191. Efficiency.—The efficiency of a machine may be variously expressed, as,

$$\begin{aligned}\eta &= \frac{\text{output}}{\text{input}}\,100 \text{ per cent.}\\ &= \frac{\text{output}}{\text{output} + \text{losses}}\,100 \text{ per cent.}\\ &= \frac{\text{input} - \text{losses}}{\text{input}}\,100 \text{ per cent.} \qquad (257)\end{aligned}$$

The efficiency varies with the output; at light loads it is low on account of the constant losses; between three-fourths load and full load it is maximum and the constant losses and variable losses are nearly equal; above full load it decreases again due to the rapid increase of the variable losses. The limit of the efficiency which can be reached commercially depends on the output, the voltage and the speed. A higher efficiency can be obtained with large machines than with small machines. A higher efficiency

can be obtained with high-voltage or high-speed machines than with low-voltage or low-speed machines.

For 220-volt direct-current motors the full-load efficiency ranges from about 85 per cent. for small sizes to 93 per cent. for large sizes.

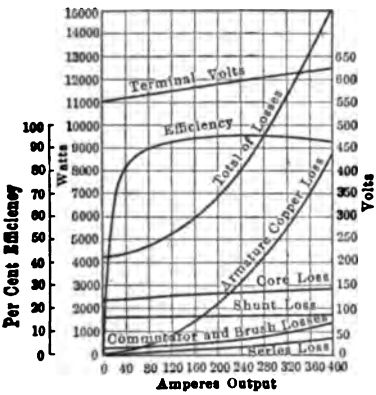


FIG. 209.—Characteristic curves of a 200 kw. compound-wound generator.

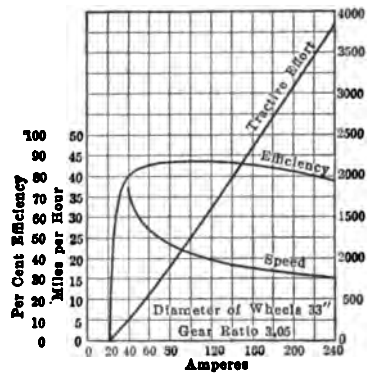


FIG. 210.—Characteristic curves of a 75-h.p. railway motor.

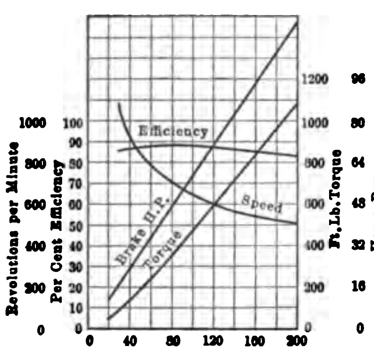


FIG. 211.—Characteristic curves of a 500-volt crane motor with a capacity of 65 h.p. for ½ hour.

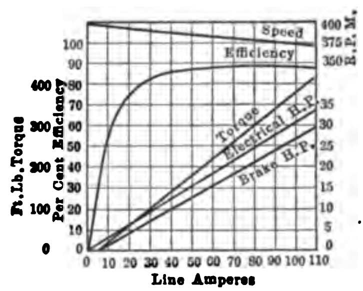


FIG. 212.—Characteristic curves of a 25-h.p. compound-wound motor.

For 550-volt direct-current generators the full-load efficiency ranges from about 90 per cent. for small sizes to 96 per cent. for large sizes.

Fig. 209 shows the characteristic curves of a 200-kw. compound-wound generator, 550 to 625 volts.

Fig. 210 shows the characteristic curves of a 75-h.p. 600-volt railway motor built by the General Electric Co.

Fig. 211 shows the characteristic curves of a 500-volt crane motor built by the Crocker Wheeler Co. Its rated output is 65 hp. for ½ hr. with a temperature rise of 40°C.

Fig. 212 shows the characteristic curves of a 25-hp., 500-volt compound-wound motor of the Westinghouse Co.

192. Commutation.—Commutation is the most serious limitation encountered in direct-current machinery.

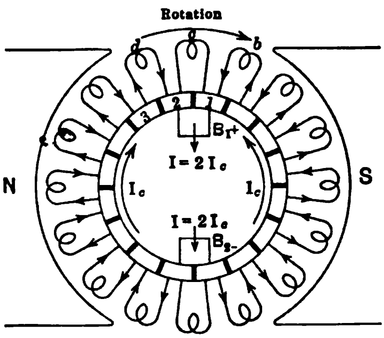


Fig. 213.—Commutation.

Fig. 213 represents the armature winding of a bipolar generator. The current entering by the brush B_2 divides into two equal parts I_c which follow the two paths through the winding and unite again at the brush B_1. Any coil c while moving from B_2 to B_1 carries a current I_c. After passing B_1 it carries an equal current I_c but in the opposite direction and, therefore, while passing under the brush B_1 the current changes from I_c to $-I_c$, that is, it is commutated or reversed.

The factors to be considered in a study of commutation are:

1. I_c, the intensity of the current to be commutated.
2. T, the time of commutation.
3. r_c, the resistance of the contact of the brush with the commutator.
4. r, the resistance of the short-circuited coil.
5. L, the inductance of the coil.
6. The direction and intensity of the flux cut by the coil during commutation.
7. Short-circuit currents in the coil when the brush short-circuits an active e.m.f.
8. Current density and loss of energy at the brush contacts.

The current to be commutated is that carried by each conductor of the armature winding. If I is the load current of the machine and p_1 is the number of paths in parallel through the winding, the current per conductor is

$$I_c = \frac{I}{p_1},$$

and increases directly as the load current.

The time of commutation is the time during which two adjacent commutator bars are short circuited by the brush. In Fig. 214 commutation of the current in coil c begins as soon as the brush touches bar 2 and must be completed when the brush breaks contact with bar 1. If the width of the brush is d in., the thickness of insulation between bars is δ in., and the peripheral speed of the commutator is V in. per second, the time of commutation is

$$T = \frac{d - \delta}{V} \text{ sec.}$$

Since δ is very small, the time of commutation varies directly as the width of the brush and inversely as the speed of the machine. The time of commutation varies from 0.0005 to 0.002 sec.

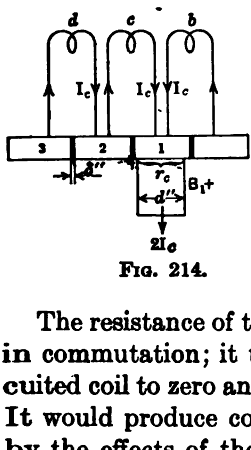


FIG. 214.

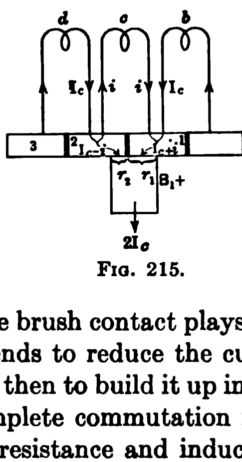


FIG. 215.

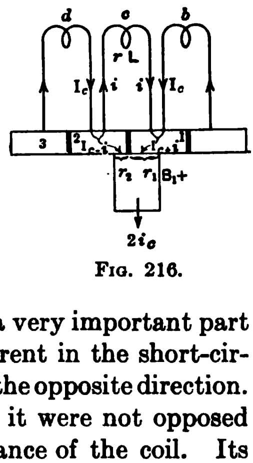


FIG. 216.

The resistance of the brush contact plays a very important part in commutation; it tends to reduce the current in the short-circuited coil to zero and then to build it up in the opposite direction. It would produce complete commutation if it were not opposed by the effects of the resistance and inductance of the coil. Its function is illustrated in Fig. 215. If the resistance of the total brush contact is r_c, then in Fig. 214 the drop of potential between the brush B_1 and bar 1 is $2I_cr_c$. As soon as the brush touches bar 2 commutation begins and the brush-contact resistance must be separated into two parts, r_1 the resistance from the brush to bar 1 and r_2 the resistance from the brush to bar 2. If at the instant represented in Fig. 215 the current in the coil is i, then the current flowing from bar 1 to the brush is $I_c + i$ and the drop of potential is $(I_c + i)r_1$; the current from bar 2 to the brush is $I_c - i$ and the drop of potential is $(I_c - i)r_2$. Since the resistance r_1 is increasing while r_2 is decreasing, the current from bar 2 will increase while that from bar 1 will decrease and the current in the coil will decrease. Neglecting the resistance and inductance of the coil

the current flowing in the coil will be zero when r_1 and r_2 are equal and when therefore half of the time of commutation has passed. Any further increase in the resistance r_1 will cause part of the current from coil b to flow through coil c in order to reach the brush B_1 by the path of least resistance. As the resistance r_1 still increases, more and more current flows through c until r_1 becomes infinite as the brush breaks contact with bar 1 and the total current I_c from b flows through c. Commutation is then complete.

In Fig. 217, curve (1), the current in coil c is plotted on a time base for half of one revolution; it is reversed in the time T, rep-

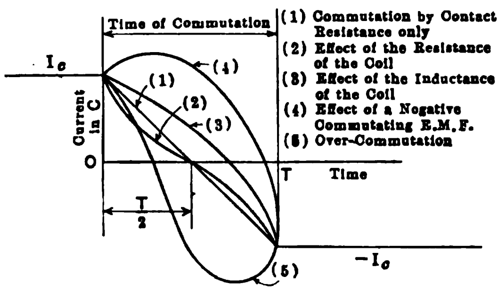


FIG. 217.

resented by OT, during which the coil moves across the brush B_1, and it must vary according to a straight line law. This can be proved as follows:

If Fig. 215 represents the condition t sec. after the beginning of commutation, neglecting the resistance and inductance of the coil, the drop of potential from the commutator to the brush must be the same at both sides, or

$$(I_c + i)r_1 = (I_c - i)r_2;$$

but
$$r_1 = r_c \frac{T}{T - t} \quad \text{and} \quad r_2 = r_c \frac{T}{t},$$

therefore,
$$(I_c + i)\, r_c \frac{T}{T - t} = (I_c - i)\, r_c \frac{T}{t}. \tag{258}$$

Solving for i this gives

$$i = I_c \frac{T - 2t}{T}, \tag{259}$$

which is the equation of a straight line.

When $t = \frac{T}{2}$, $i = 0$, and when $t = T$, $i = -I_c$.

If the resistance of the coil is taken into account, the drop of potential across r_2 (Fig. 216) must be greater than the drop across r_1 by the amount required to maintain the current i through the resistance r; therefore,

$$(I_c + i)\,r_c\,\frac{T}{T - t} + ir = (I_c - i)\,r_c\,\frac{T}{t},$$

and

$$i = I_c\,\frac{r_c\,(T^2 - 2Tt)}{r\,(Tt - t^2) + r_c T^2}. \qquad (260)$$

When $t = \frac{T}{2}$, $i = 0$, and when $t = T$, $i = -I_c$.

The current therefore passes through zero at the same instant as before and is completely reversed in the same time, but the variation does not follow a straight line law but a curve as shown in curve (2), Fig. 217. The effect of the coil resistance is very small and may be neglected.

The effect of the inductance of the armature coil must next be considered. Armature coils are partially surrounded by iron and therefore have a large inductance which is proportional to the square of the number of turns in the coil. With full-pitch

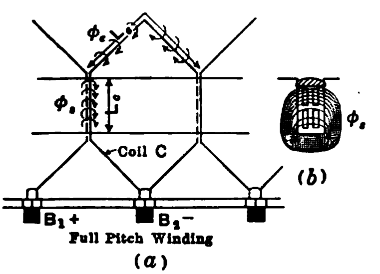


FIG. 218.—Inductive flux in a full-pitch winding.

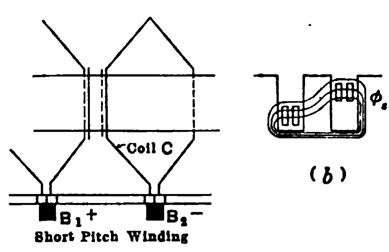


FIG. 219.—Inductive flux in a short-pitch winding.

drum windings, Fig. 218, both the coils in one slot will be short-circuited at one time and the inductive flux linking the slot part of each of them will be almost twice as large as in the case of fractional pitch windings (Fig. 219). The flux around the end connections is approximately the same in the two cases. The inductance of the coil is, therefore, partly self-inductance and partly mutual inductance between adjacent coils but the resultant effect may be considered as due to an inductance L.

When the current in a coil of inductance L henrys is changing at the rate $\frac{di}{dt}$ amp. per second, an e.m.f. $L\frac{di}{dt}$ volts is generated in a direction opposing the change of current.

In Fig. 216 the drop of potential from bar 2 to the brush is the same by the two paths, one through the resistance r_2 and the other through the coil in series with the resistance r_1; and thus

$$(I_c + i)\, r_1 + ri + L\frac{di}{dt} = (I_c - i)\, r_2,$$

or, substituting the values of r_1 and r_2 found above,

$$(I_c + i)\, r_c \frac{T}{T - t} + ri + L\frac{di}{dt} = (I_c - i)\, r_c \frac{T}{t}. \qquad (261)$$

The complete solution of this equation is difficult but a partial solution, for the instant when commutation is complete, may be obtained easily.

Equation (261) may be written,

$$L\frac{di}{dt} = -ri + r_cT\left(\frac{I_c - i}{t}\right) - r_cT\left(\frac{I_c + i}{T - t}\right).$$

when $t = T$ and $i = -I_c$

$$L\frac{di}{dt} = rI_c + 2I_cr_c - r_cT \times \frac{0}{0}; \qquad (262)$$

this last term is indefinite but its value may be found by differentiating the numerator and denominator with respect to the independent variable t.

$$\left[\frac{I_c + i}{T - t}\right]_{t=T} = \frac{\frac{di}{dt}}{-1} = -\frac{di}{dt},$$

and substituting this in equation (262),

$$L\frac{di}{dt} = rI_c + 2I_cr_c + r_cT\frac{di}{dt}$$

or

$$\frac{di}{dt} = \frac{I_c(r + 2r_c)}{L - r_cT}.$$

If $L = r_cT$, $\frac{di}{dt} = \infty$, the reactance voltage $L\frac{di}{dt} = \infty$, and sparking will occur.

If $L > r_cT$, $\frac{di}{dt}$ is positive and the current tends to increase instead of reversing and sparking results.

If $L < r_c T$, $\frac{di}{dt}$ is negative and finite and commutation may be sparkless and satisfactory.

This condition for sparkless commutation may be expressed in another way.

$$L \text{ must be } < r_c T \text{ or } \frac{2I_c}{T} L \text{ must be } < 2I_c r_c;$$

$\frac{2I_c}{T}$ is the average rate of change of current and $\frac{2I_c}{T} L$ is called the average reactance voltage, while $2I_c r_c$ is the voltage drop at one brush contact.

Therefore, the condition for sparkless commutation is that the average reactance voltage must be less than the voltage drop at one brush contact. This, however, applies only to the case in which no flux is cut by the short-circuited coil and where therefore no e.m.f. due to rotation is generated in the coil either assisting or opposing commutation. The brush-contact resistance is the only factor operating to cause the current to reverse.

Due to the effect of inductance, the current does not decrease so quickly as in curve 1, Fig. 217, but follows curve 3, and the rate of change of current becomes very rapid as the brush breaks contact with bar 1 and sparking is liable to occur. The current density in the brush tip also becomes very high, tending to burn both the brush and the commutator.

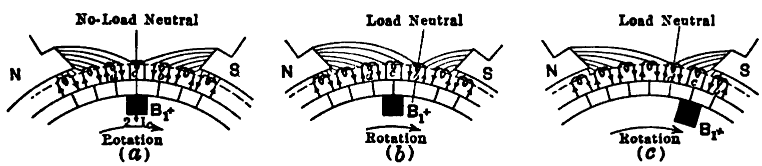


FIG. 220.—Commutating field.

When the inductance and the reactance voltage are large it is necessary to have an e.m.f. generated in the coil to assist commutation. The brushes of a generator are, therefore, moved ahead in the direction of rotation (back in the case of a motor), so that the coil when short-circuited is cutting the fringe of lines from the pole tip (Fig. 220).

If at the time represented in Fig. 216 there is an e.m.f. e generated in the coil assisting commutation, equation (261) can be written

$$e + L\frac{di}{dt} + i\left(r + \frac{r_r T^2}{t(T - t)}\right) + \frac{r_c I_c T(2t - T)}{t(T - t)} = 0. \qquad (263)$$

This equation cannot be solved in general but it is possible to determine the value of e required at any instant to cause the current to vary as a linear function of time from I_c to $-I_c$ in the time T.

On this assumption

$$i = I_c \frac{T - 2t}{T}$$

and

$$\frac{di}{dt} = -\frac{2I_c}{T}.$$

Substituting these values in equation gives

$$e - I_c\left\{\frac{2L}{T} - r\left(1 - 2\frac{t}{T}\right)\right\} = 0,$$

or

$$e = I_c\left\{\frac{2L}{T} - r\left(1 - 2\frac{t}{T}\right)\right\}, \qquad (264)$$

which gives at the beginning of commutation $t = 0$,

$$e_0 = I_c\left(\frac{2L}{T} - r\right), \qquad (265)$$

and at the end of commutation $t = T$,

$$e_T = I_c\left(\frac{2L}{T} + r\right). \qquad (266)$$

This e.m.f. is proportional to the current I_c but is independent of the brush resistance r_c. The average value of this e.m.f. is called the commutating e.m.f. in the coil and is represented by E_c.

If the commutating e.m.f. is less than that required to reverse the current completely in time T, commutation is imperfect and there is a tendency to spark, and if the e.m.f. is so large that the current is more than reversed there is a tendency to spark due to over-commutation.

The contact resistance helps to prevent sparking when the e.m.f. generated in the coil by rotation is either too great or too small to produce perfect commutation.

When commutation is produced by the high-resistance brush contact without the aid of any e.m.f. generated in the coil, it is called "natural" or "resistance" commutation; when it is assisted by an e.m.f. generated in the coil, it is called "forced" or "voltage" commutation.

Resistance commutation can never be perfect unless the inductance of the coil is negligible, but at light loads it will reverse

the current without injurious sparking. Assume that the brushes of a generator delivering half load are set on the corresponding neutral line and that commutation is satisfactory. If the load is increased the increased m.m.f. of the armature causes the neutral line to move ahead so that the coil short-circuited is cutting a field of such a direction as to tend to maintain the current or even to increase it. The reversal of the current is therefore retarded and there is a greater tendency to spark than before. If the load is reduced the neutral line falls behind the brushes and a voltage assisting commutation is generated in the coil.

Voltage commutation is also limited in its application and as the current in the armature is increased a point is reached (usually about 25 per cent. overload) beyond which sparkless commutation is impossible, since when the current is increased a stronger field is required to reverse it, but the stronger current in the armature increases the m.m.f. of the armature and moves the neutral line ahead of the brushes and at the same time decreases the flux. The brushes have to be advanced further and the demagnetizing effect is increased. When the armature m.m.f. is large enough to overbalance the field m.m.f. the flux at the pole tip is wiped out and voltage commutation is impossible. Moving the brushes further ahead only decreases the flux.

To take full advantage of voltage commutation it would be necessary to vary the position of the brushes with varying load, but this is not practicable, and therefore the brushes must be set to give good commutation at some intermediate load and the resistance of the brush contact must be relied on to prevent sparking above and below this point. Modern machines are designed to give good commutation at all loads from no load to 25 per cent. overload with fixed brush position.

193. Commutating Electromotive Force.—The commutating e.m.f. is the e.m.f. generated in the short-circuited coil due to cutting flux and depends on a number of factors.

There are two fluxes to be considered: (1) that produced by the field m.m.f.; and (2) that produced by the armature m.m.f. These two fluxes are represented in Figs. 161 to 164 and their resultant in the interpolar space is shown in Fig. 220. The e.m.f. generated in the coil due to cutting the field flux tends to assist commutation, while that generated by cutting the armature flux opposes commutation. The resultant of the two is called the commutating e.m.f., E_c. This e.m.f., therefore, depends on the

field excitation, the position of the brushes, and the load, that is, the armature current.

When the brushes are on the "no-load" neutral there is no e.m.f. assisting commutation; if the machine is carrying load at the time there will be an armature flux both in the interpolar space and around the end connections and this flux being stationary is cut by the short-circuited coil and generates in it an e.m.f. opposing commutation. E_c is therefore negative.

When the brushes are moved ahead to the position where the e.m.f. due to cutting the field flux is equal to the e.m.f. due to cutting the armature flux, the commutating e.m.f. is zero. This position is called the load neutral. In this case commutation is carried out by the brush-contact resistance and will be satisfactory only if the reactance voltage is very low.

When the brushes are moved ahead of this load neutral to a position where the commutating e.m.f. is positive and equal to the reactance voltage commutation is perfect.

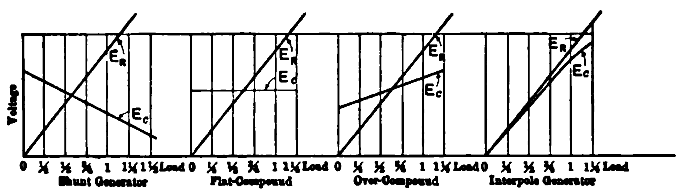


FIG. 221.—Reactance voltage.

In Fig. 221 the reactance voltage E_R and the commutating voltage E_c are shown plotted on a load base for different types of generators.

The reactance voltage is directly proportional to the current and is represented by a straight line passing through the origin. For the shunt generator the commutating e.m.f. varies from a maximum at no load to zero at about 50 per cent. overload. The brushes are set to give perfect commutation at 75 per cent. load. At 25 per cent. overload the armature m.m.f. pretty well wipes out the main field near the pole tip and only a very small commutating e.m.f. is left. Sparking will not be serious so long as the difference between E_R and E_c is not greater than the voltage drop at one brush contact. The reactance voltage at full load should be limited to this or a lower value.

For a flat-compounded machine the conditions are much better, the commutating e.m.f. does not decrease to any great extent

with load and the difference between E_R and E_c is very much less; larger values of reactance voltage may, therefore, be allowed.

For an over-compounded machine conditions are still further improved; E_c increases with load and satisfactory commutation may be obtained with reactance voltages more than double the drop at the brush contact.

194. Conditions Essential to Sparkless Commutation.—It is necessary to limit the armature strength at full load to such a value that it will not interfere too greatly with the flux produced by the field winding. For shunt machines the field ampere-turns required to drive the flux through the gap and teeth should be = 1.2 (armature ampere-turns per pole) + the demagnetizing ampere-turns per pole. This insures good commutation from no load to 25 per cent. overload with fixed brush position provided the reactance voltage does not exceed the limits specified below. Lower values of armature strength tend to give better commutating conditions.

For flat-compounded machines the field ampere-turns for the gap and teeth should be = 1.2 (armature ampere-turns per pole).

The reactance voltage must also be limited approximately to the following values.

In shunt machines from 1 to 1.25 volts except with high-resistance brushes.

In 10 per cent. over-compounded generators from 2.5 to 3.0 volts.

In shunt motors where any large variation of speed by field control is to be obtained values of reactance voltage below 1 volt should be used or a very weak armature.

In interpole machines satisfactory commutation may be obtained with reactance voltages up to 15 volts.

195. Calculation of the Reactance Voltage for a Full-pitch Multiple Winding.—In Fig. 218(*a*) is shown one coil, *c*, of a full-pitch double-layer winding, short-circuited by the brush and at (*b*) is shown a section through a slot. If the number of turns per coil is n, the number of conductors per slot is $2n$. Here $n = 2$. The flux of self and mutual inductance is shown in the figure and may be divided into two parts: (1) that surrounding the slot part of the coil; and (2) that around the end connections.

Let ϕ_S be the number of lines of force which link 1-in. length of the slot part of the coil for each ampere conductor in the group of conductors simultaneously short-circuited; and let ϕ_e be the

number of lines of force which link 1-in. length of the end connections for each ampere conductor in the corresponding group of end connections simultaneously short-circuited.

Then, with a full-pitch winding and a brush covering only one bar the total inductive flux linking the coil, c, when carrying a current i amp., is

$$\phi_c = 2(2ni\phi_s L_c) + 2(ni\phi_e L_e) = 2ni(2\phi_s L_c + \phi_e L_e),$$

where $L_c =$ length of the imbedded part of one conductor in inches, and $L_e =$ length of one end connection.

The inductance of the coil is

$$L = \frac{n\phi_c}{i}10^{-8} = 2n^2(2\phi_s L_c + \phi_e L_e)10^{-8} \text{ henrys}$$

and the average reactance voltage is

$$E_R = \frac{2I_c}{T}L \text{ (page 229).}$$

The time of commutation is

$$\begin{aligned} T &= \frac{\text{width of brush or bar}}{\text{peripheral velocity of commutator}} \\ &= \frac{\text{width of bar}}{\text{r.p.s.} \times \text{no. of bars} \times \text{width of bar}} \\ &= \frac{1}{\text{r.p.s.} \times \text{no. of bars}} = \frac{1}{\text{r.p.s.} \times \text{no. of coils}} \\ &= \frac{1}{\text{r.p.s.} \times \dfrac{\text{armature conductors}}{2n}} = \frac{120n}{\text{r.p.m.} \times Z} \end{aligned}$$

From tests made on a large number of machines Hobart found that ϕ_s has a value of approximately 10 and ϕ_e a value 2.

Therefore, the average reactance voltage is

$$\begin{aligned} E_R &= \frac{2I_c Z \times \text{r.p.m.}}{120n}\left\{2n^2(20L_c + 2L_e)\right\}10^{-8} \\ &= \frac{10L_c + L_e}{15}10^{-8} \times I_c Z \times \text{r.p.m.} \times n \\ &= \frac{10L_c + L_e}{15}10^{-8} \times \text{ampere conductors on the armature} \times \text{r.p.m.} \times \text{turns per coil.} \end{aligned}$$

For the ordinary type of coil L_e may be taken as $= 2L_c$ and the average reactance voltage may be expressed by

$$\begin{aligned} E_R &= 0.8 \times 10^{-8} \times L_c \times ZI_c \times \text{r.p.m.} \times n \qquad (267) \\ &= 0.8 \times 10^{-8} \times \text{gross length of core} \times \text{ampere conductors on the armature} \times \text{r.p.m.} \times \text{turns per coil.} \end{aligned}$$

An increase in the brush width does not change the reactance voltage because although it increases the number of coils short-circuited at any instant it also increases the time of commutation in the same proportion.

If there are six coil sides per slot, Fig. 222, and the brush covers three segments then the flux surrounding the slot part of the coil will be three times as great as before since the m.m.f. is increased to three times, while any increase in the width of the path across the slot will be counterbalanced by a proportional increase in the depth. Similarly, the flux about the end connections will be about three times as great; therefore, the inductance is increased three times but the time of commutation is also increased three times and the reactance voltage remains as before.

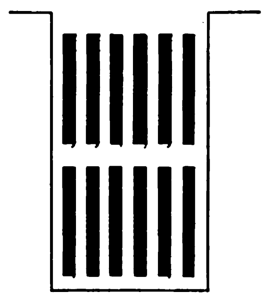


FIG. 222.

With a short-pitch winding (Fig. 219) the slot flux is reduced to about half the value with a full-pitch winding. This is due to the increased reluctance of the path. The flux about the end connections is decreased only slightly, due to their shorter length.

The reactance voltage of a short-pitch winding is

$$E_R = \frac{5L_c + L_e}{15} 10^{-8} \times ZI_c \times \text{r.p.m.} \times n$$

or again substituting $L_e = 2L_c$

$$E_R = 0.46 \times 10^{-8} \times L_c \times ZI_c \times \text{r.p.m.} \times n. \qquad (268)$$

The ratio $\frac{\text{slot depth}}{\text{slot width}}$ was assumed to be approximately 3.5 in the derivation of the formulæ above. If very deep and narrow slots are used the flux ϕ_s will be increased and therefore also the reactance voltage.

In the case of a series or two-circuit winding there are $p/2$ coils short-circuited in series and the reactance of this set of coils is $p/2$ times the reactance of one coil but the commutating e.m.f. is also increased in the proportion $p/2$. Since both these voltages are greater than before it is difficult to keep their difference within the value which can be taken care of by the brush-contact resistance. On the other hand, if the commutating field is too strong, the reactance of $p/2$ coils in series tends to prevent the

current from growing to a large value. Values of reactance voltage calculated as for full-pitch windings, equation (267), should be kept about 50 per cent. lower for series windings.

196. Current Density at the Brush Contacts.—There are two currents in the coil short-circuited by the brush: (1) the working current or load current; and (2) the short-circuit current produced by an active e.m.f. in the coil.

When the e.m.f. generated in the coil is equal and opposite to the reactance voltage there is no circulating current produced in the coil and the load current reverses at a uniform rate due to the action of the contact resistance (page 226); the current density is then uniform over the brush face and the loss of energy is a minimum.

When the two e.m.fs. in the coil do not balance their resultant tends to cause a current to circulate through the coil and brush, and the current density is no longer uniform and the loss at the contacts is increased above normal.

This circulating current crosses the brush contact twice and the contact resistance tends to keep the current down to a reasonable value. When the unbalanced e.m.f. is not greater than 2 to 2.5 volts the current will not be large.

Reducing the width of the brush may in some cases so cut down the short-circuit current that the maximum current density is decreased although the average is increased.

The average current density in machines without interpoles is from 35 to 40 amp. per square inch but higher values are sometimes used with low-contact resistance brushes.

Unequal distribution of current between brushes on the same arm or between arms of the same polarity causes the density to vary from the average. This is particularly true of machines with series windings and gives rise to selective commutation.

In the alternating-current series commutator motor in which the circulating currents tend to become very large the contact resistance is reinforced by the addition of high-resistance leads between the coils and the commutator bars.

197. Burning of the Brush and Commutator.—An apparent electrolytic action takes place under the brushes in the direction of the current flow, carbon is deposited on the commutator under the negative brush and the brush is burned, and copper is deposited on the positive brush and the commutator is burned. The burning is dependent on the energy consumed at the par-

ticular point on the brush contact and this is proportional to the product of the current density at the point and the voltage drop at the contact.

Severe burning of the commutator results in high mica which spoils the contact between the brush and the commutator and increases the liability to spark. To prevent the commutator burning in grooves the brushes should be staggered in sets as shown in Fig. 223.

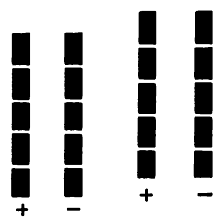


FIG. 223.

Even a very slight deposit of copper on the brush face reduces the brush-contact resistance very materially. On the other hand, the darkening or polish on the commutator surface increases the contact resistance.

198. Poor Commutation Resulting from Too Many Coil Sides per Slot.—When a number of coil sides are placed in one slot, they are not all short-circuited at once and therefore they will be commutated in fields of different strengths and the commutating e.m.fs. will be different since the intensity of the field varies very rapidly as the pole tip is approached. The first coil to be commutated will be in the weakest field and may not have its current completely reversed, while the last may be overcommutated. If there are six coil sides per slot every third commutator bar is liable to be burned due to improper commutation.

It is therefore not advisable to reduce the number of slots too far. Small machines should have at least 12 slots per pole and large machines at least 14 slots per pole.

199. Interpoles.—Interpoles are small poles placed midway between the main poles of either motors or generators. They are magnetized by a winding connected in series with the armature and carrying the load current. Fig. 224 shows an interpole motor or generator. The brushes are fixed on the no-load neutral points. The interpoles have the same effect as moving the brushes since they move the poles magnetically.

The m.m.f. of the interpole winding must oppose the m.m.f. of the armature and must be strong enough to overbalance it and produce a field under the interpole of the proper intensity to reverse the current in the short-circuited coil. Since the interpole winding is in series with the armature the commutating field increases with load and satisfactory commutation up to and

beyond the overload limits of output set by armature heating can be obtained.

The interpole provides a much more definite means of controlling the commutating e.m.f. than is the case where reliance is placed on shifting the brushes. As the output, the speed or the voltage of direct-current machines is increased the reactance voltage is increased and a limit is reached beyond which satisfactory commutation can no longer be obtained economically by shifting the brushes and interpoles must be supplied. They are required for all large reversible motors, where the brushes must be fixed on the no-load neutral and where therefore no commutating e.m.f. can be obtained without them. They are also required

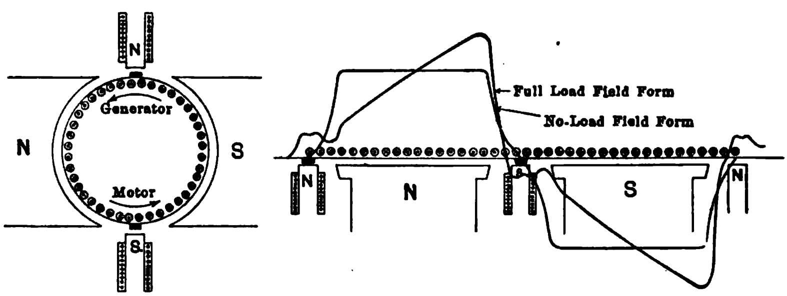


FIG. 224.—Interpole generator or motor.

for adjustable-speed shunt motors where a large speed range is to be obtained by field control. With interpoles a speed range of 3 to 1 or even 4 or 5 to 1 is possible.

Interpoles are not so necessary on generators as on motors but they are useful where very wide variations of load are met with and especially if the variations are rapid. The interpole m.m.f. follows exactly the fluctuations of the armature m.m.f. but if the changes of load are very rapid, it is difficult for the interpole flux to build up rapidly enough to give perfect commutation, since this flux has to pass through the solid yoke and eddy currents are induced which tend to retard it.

If the interpole is provided with a shunt, it is necessary to have the resistance of the shunt so adjusted that the interpole winding will receive its proper proportion of the current, but in addition the shunt must be designed with an inductance proportional to the inductance of the interpole winding to insure the proper division of the current when the load is fluctuating rapidly.

Interpoles must be designed with very low flux densities at normal load, so that, under heavy overloads, they will not become saturated and will be able to carry the flux required to produce the increased commutating e.m.f. Interpoles should be provided on all machines where the reactance voltage is over 3.5 or 4 volts and with them satisfactory commutation can be obtained with reactance voltages up to 15 volts. The interpole shunt must, however, be very carefully adjusted or the difference between the commutating voltage and the reactance voltage may be so large that serious circulating currents will be produced.

When interpoles are provided the ratio of the field ampere-turns per pole for the gap and teeth to the armature ampere-turns per pole may be as low as 0.8 and it is no longer necessary to limit the armature strength to such a low value as that given for non-interpole machines on page 261.

The interpole m.m.f. wipes out the armature flux in the commutating zone but does not prevent the distortion of the flux under the poles (Fig. 224). This distortion is more serious because of the increased armature strength and is liable to produce flashing due to the generation of excessive voltages between adjacent commutator bars (see Art. 200).

200. Flashing.—When an arc passes over the surface of the commutator from a positive to a negative brush, the machine is said to "flashover." A severe flash is equivalent to a dead short-circuit and may have serious effects.

Flashing is usually due to a combination of a high voltage between adjacent commutator bars with a partial short-circuit between bars due to the collection of carbon or graphite dust on the mica insulation.

The causes of flashing are distinct from the causes producing sparking at the brushes but a flash may sometimes be started by a spark originating under the brush due to poor commutation.

Fig. 225 shows the flux distribution in the air gap at no load and at full load. At no load the flux density under the pole face is uniform and the voltage generated between bars is comparatively low; under load the armature m.m.f. causes the flux density to increase at one pole tip t_1 and to decrease at the other tip t. The voltage between bars at t_1 will be very much increased and, if the commutator is dirty, it may cause a flash locally between bars and this flash may be carried around in the direction

of rotation and result in a complete short-circuit between brushes either momentary or permanent depending on conditions.

When the load comes on a motor very suddenly the armature m.m.f. causes a part of the flux to swing quickly across from t to t_1 and, as this movement is opposite to the direction of rotation, the flux is cut more rapidly than under ordinary conditions and the tendency to flash is increased. In the case of a generator the swing of flux is in the direction of rotation and the voltages are not so high.

If the load is suddenly removed from a generator the swing of flux is opposite to the direction of rotation and the voltage between bars may be very high.

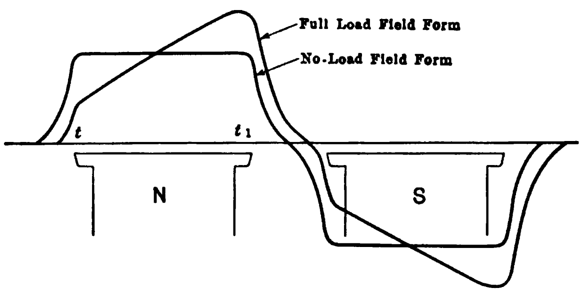


FIG. 225.

When the magnetic circuit is highly saturated and the machine has what is called a stiff field, the flux peak at t_1 is not so high and there is less danger of flashing.

A motor operating at high speed due to field weakening is peculiarly liable to flash; the reduced saturation allows the armature m.m.f. to produce a high flux peak while the increased speed counterbalances the decreased main flux; and at the same time commutating conditions are bad.

Interpole machines are more likely to flash than those without interpoles because of their increased armature strength. In non-interpole machines the field m.m.f. per pole is from 30 to 40 per cent. greater than the armature m.m.f. per pole while in interpole machines the field m.m.f. per pole is approximately equal to the armature m.m.f. and may even be less. Commutation is taken care of by the interpole flux but the armature m.m.f. which is all cross-magnetizing, since the brushes are on the no-load neutral, produces a large distortion of the flux under the poles due to the relatively great armature strength.

If load is suddenly applied to an interpole motor the flux swings rapidly across the pole and piles up at t_1 and induces a high voltage between bars; at the same time sparking at the brushes may occur due to the fact that the interpole commutating flux cannot build up as fast as the armature current since the interpole field winding has a large inductance and the solid yoke opposes the required change of flux by the production of eddy currents. A flash may therefore result. Further, if the interpole shunt is not properly designed it may happen that the rapidly changing current will follow the path of low inductance through the shunt rather than the proper path through the winding. Interpoles should be designed with inductive shunts to take care of such currents.

In the case of an interpole generator flashing is more severe when load is removed suddenly as the swing of flux is then in the direction to produce the greatest voltage between bars; at the same time the interpole commutating field cannot disappear immediately and sparking occurs at the brushes due to overcommutation of the current.

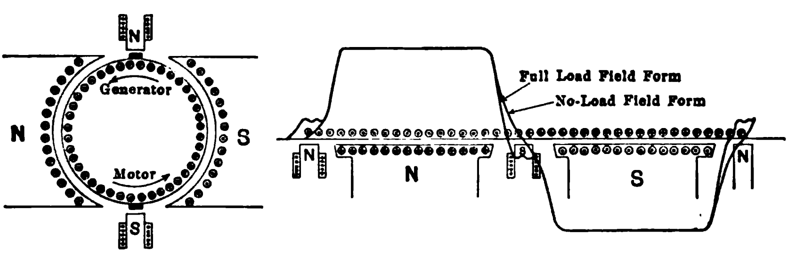


FIG. 226.—Compensating winding.

Where extreme variations of load are liable to occur with great rapidity it is advisable to supply the machines with compensating windings placed in slots in the pole faces, Fig. 226, to counteract the armature m.m.f. over the whole armature surface and so prevent any distortion of the flux.

The maximum value of volts per bar under normal operating conditions should not exceed 30 volts for moderate capacities or 28 volts for large capacities. The average volts per bar will be about half of the maximum value and should not exceed about 15 volts. For small machines operating under steady load higher values may be used but the smaller values are preferable if they

can be obtained without too great a sacrifice, especially when the machine may experience sudden changes of load.

201. Compensating Windings.—In cases where the average voltage between bars is already so high that it is necessary to limit the field distortion to prevent flashing, compensating windings should be provided. They are windings placed in slots in the pole faces, which exert a m.m.f. equal and opposite to the armature m.m.f. and so prevent any distortion of the field. The compensating winding carries the load current and may therefore be designed with a smaller number of turns than the armature. Interpoles are required in addition to provide the commutating e.m.f.; but since the armature m.m.f. is already neutralized by the compensating winding, the m.m.f. of the interpole winding is only that required to produce the flux necessary for commutation, through the magnetic circuit of the interpole. Fig. 226 shows a machine provided with interpoles and a compensating winding.

Generators with compensating windings require only weak series fields to maintain constant terminal voltage since the armature reaction is neutralized. The resistance drop is increased to a slight extent.

202. Parallel Operation.—In power houses in which the load varies at different hours of the day a number of generators are usually installed. When the load is light one generator is operated and supplies the demand and when the load increases a second machine is started up and connected in parallel with the first and its excitation is adjusted until it takes its proper share of the load. Fig. 227 shows two shunt generators (1) supplying power and (2) ready to be connected in parallel with it. Before closing the switches S_1 and S_2 which connect the second machine to the load it is necessary that its polarity be correct and that its terminal voltage be the same or a little higher than that of (1).

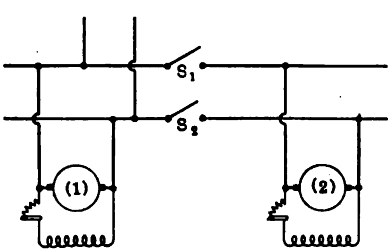


FIG. 227.—Parallel operation of shunt generators.

If the field rheostat of (2) is so adjusted that the voltage of (2) is the same as the voltage of (1) and switch S_1 is closed, then, if there is no voltage across S_2, it may be closed. But if the voltage

across S_2 is found to be about double the terminal voltage, the polarity of (2) must be reversed before closing switch S_2. After closing S_2 the field rheostat of (2) must be adjusted until (2) takes its proper share of the load.

If the voltage of (2) is the same as the voltage of (1) when the switch is closed, (2) will not take any load but will run idle. If the voltage of (2) is less than the voltage of (1), machine (2) will run as a motor driving its prime mover and will draw power from (1). If, however, the terminal voltage of (2) is higher than that of (1), machine (2) will supply part of the load and will relieve (1) until the voltages of the two become the same. Fig. 228 represents the voltage characteristics of the two machines plotted on the same base. If the terminal voltage is E, (1) supplies a current I_1 and (2) a current I_2 and the total current

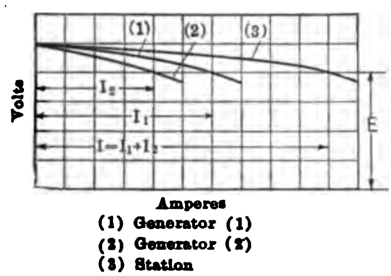


FIG. 228.

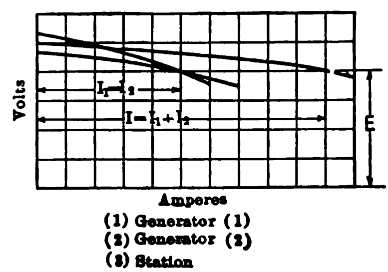


FIG. 229.

supplied by the station is $I = I_1 + I_2$. The machine with the flatter characteristic will supply the greater amount of power. If the two machines are rated at the same current output, (2) can be made to take its share of the load by cutting out resistance from its field rheostat and so raising its voltage characteristic and inserting resistance in the field circuit of (1) and lowering its characteristic as shown in Fig. 229.

Shunt generators will operate in parallel and divide up the load in proportion to their capacities if their voltage characteristics are similar, that is, if their terminal voltage falls from no load to full load by the same amount and in the same manner. If the characteristics are different a proper division of load can be obtained by regulating the field rheostats.

203. Parallel Operation of Compound Generators.—Fig. 230 shows two compound-wound generators connected in parallel and

supplying power to a load circuit. Their voltage characteristics are shown in Fig. 231. Assume that the prime mover of (1) runs for an instant at a slightly increased speed; the voltage of (1) rises and it takes more than its share of the load; the voltage of (2) falls because its load is decreased and its series excitation is decreased. Machine (1) therefore takes more of the total load and its voltage rises higher until it supplies all the load and in

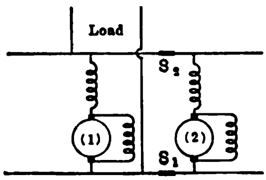


FIG. 230.

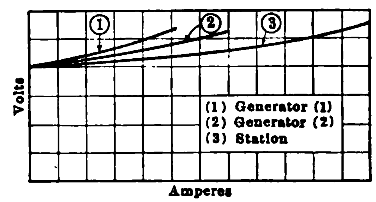


FIG. 231.

addition drives (2) as a motor. Since the current in (2) is reversed the m.m.f. of its series winding is also reversed and it runs as a differential motor driving its prime mover at a high speed until the load on (1) becomes so great that the protective apparatus opens the circuit and shuts down the system.

To get over this difficulty the equalizer connection *ee*, Fig. 232, is used. It is a conductor of low resistance connecting the series windings of the two machines in multiple. Now if the prime mover of (1) runs above normal speed the voltage of (1) rises and it takes an increased load. The increase of current does not all go through the series winding of (1) but divides between the windings of (1) and (2) in inverse proportion to their resistances and so prevents any decrease of the voltage of (2). Thus with an equalizer connection (2) will still hold its load. The resistances of the series windings must be adjusted so that the load current will divide between them in such proportion that each machine will supply its proper share of the load.

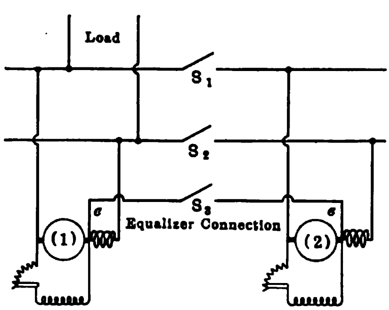


FIG. 232.—Parallel operation of compound generators.

Before connecting machine (2) in parallel with (1) which is delivering power, first close switches S_2 and S_3, Fig. 232, and

adjust the shunt field of (2) until its terminal voltage is the same as that of (1). The excitation of (2) is now provided partly by its shunt field and partly by its series field carrying part of the load current. After checking the polarity to see that it is correct close switch S_1 and adjust the shunt field of (2) until the machines divide the load in proportion to their capacities.

From the above discussion it is seen that two compound-wound generators connected in parallel form an unstable system unless an equalizer connection is placed between their series windings.

204. Storage Batteries.—A storage battery is an apparatus in which electrical energy can be stored to be used at some later time.

Batteries are made up of a number of cells connected in series multiple according to the voltage and current required.

Each cell is composed of two plates or electrodes of suitable materials immersed in an electrolyte. The most commonly used storage battery has a positive plate of lead peroxide PbO_2 and a negative plate of sponge lead Pb immersed in dilute sulphuric acid H_2SO_4.

When the battery is discharging the electrolyte combines with the active materials of the electrodes and when it is being charged the electrodes are reduced to their original condition and the materials taken from the electrolyte are returned to it.

The main chemical changes taking place are represented by the following formula:

$$PbO_2 + Pb + 2H_2SO_4 \underset{\xrightarrow{\text{discharge}}}{\xleftarrow{\text{charge}}} 2PbSO_4 + 2H_2O. \qquad (269)$$

Capacity.—The unit of capacity of a storage cell is the ampere-hour and it is generally based on the 8-hr. discharge rate. An 800-amp.-hr. battery will give a continuous discharge of 100 amp. for 8 hr. If, however, the rate of discharge is increased the ampere-hour capacity of the battery decreases. At a 6-hr. discharge rate the capacity is only about 95 per cent., at a 4-hr. discharge rate it is about 80 per cent. and at a 1-hr. rate it is only 50 per cent. of its 8-hr. rating. Thus the battery mentioned above would give a continuous discharge of 400 amp. for only 1 hr.

The capacity of a cell is proportional to the area of the plates exposed to the electrolyte and for an 8-hr. discharge rate a current density of from 40 to 60 amp. per square foot of positive plate is common practice.

Voltage.—The voltage of a cell depends on the character of the electrodes, the density of the electrolyte and the condition of the cell but is independent of the size. The variation of the terminal voltage of a cell during charge and discharge is shown by the curves in Fig. 233. On charge the voltage begins about 2 volts and rises to 2.5 volts or a little above. When the charging circuit is opened the voltage falls to 2.1 volts and during discharge falls off gradually to about 1.9 volts. Beyond this point the fall of voltage is very rapid and discharge should not be continued after the voltage has fallen to 1.7 volts.

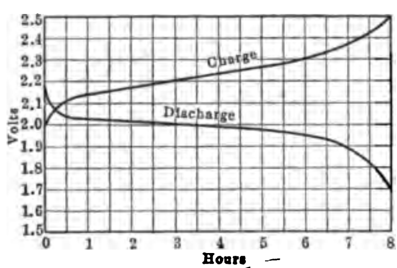


FIG. 233.—Voltage characteristics of a storage cell.

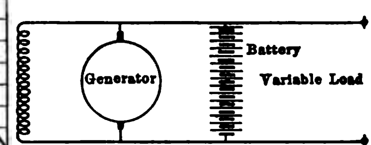


FIG. 234.—Battery with a shunt-generator.

The required battery voltage is obtained by connecting a number of cells in series and the required current is obtained by connecting a number of plates or cells in multiple.

205. Applications.—Batteries are installed in direct-current power stations to store energy during periods of light load and to deliver energy in parallel with the generators during periods of heavy load. When the load is light the generators charge the battery and when the load is heavy the charge is given up and so the load on the generators is maintained nearly constant and they can be operated at maximum efficiency. The result is that the voltage regulation of the system is improved.

Batteries are also installed in electric-railway substations to prevent large variations of the load on the feeders supplying them and so regulate the substation voltage.

A third very important application of storage batteries is in la-

ternating-current power stations where they provide an auxiliary supply of direct current in case of a breakdown of the exciters and may thus prevent a shutdown of the whole system.

Fig. 234 shows a battery connected across the terminals of a shunt generator. At normal load the battery voltage and the generator voltage are equal and the battery floats on the line neither giving nor receiving power. If, however, the load increases the generator voltage falls and the battery discharges and supplies part of the extra load and so relieves the generator and prevents any large drop in its terminal voltage. The battery in this way takes care both of sudden overloads and continuous overloads. During periods of light load the generator voltage rises above the battery voltage and the battery charges.

206. Boosters.—Boosters are direct-current generators connected in series with the line to raise or lower the voltage. They are low-voltage machines of large current capacity and are usually driven at constant speed by shunt motors.

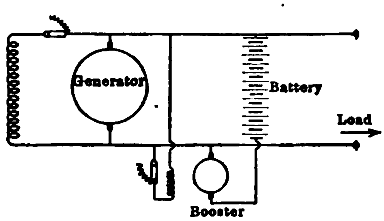


FIG. 235.—Shunt booster.

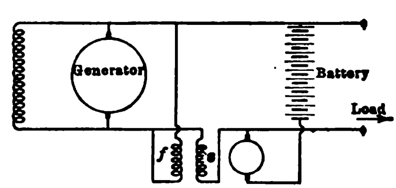


FIG. 236.—Compound booster.

Boosters are used to compensate for line drop in distributing systems by adding an equal voltage to the circuit and they are also used very extensively to regulate the charge and discharge of storage batteries in parallel with the generators in constant-voltage systems. They may be either shunt, series, or compound-wound.

The shunt booster, Fig. 235, is an ordinary generator with its field connected across the station busbars and its armature in series with the generator armature. Its function is to raise the voltage impressed on the battery in order to send current into it to charge it. The booster voltage is controlled by a field rheostat.

Compound boosters are automatic in their action and are divided into two classes, non-reversible and reversible, depending on the relative strengths of their shunt and series windings.

Fig. 236 shows a non-reversible automatic booster. The shunt field f is connected across the station busbars; the series field s carries the load current of the generator and it opposes the shunt field but has at all times a smaller m.m.f. and thus the booster voltage is always in the direction of the generator voltage.

When the load current increases, the increase of current through the series field decreases the booster voltage and allows the battery to discharge; when the load decreases, the booster voltage rises and causes the battery to charge. The current from the generator remains practically constant regardless of the fluctuations of the load on the system.

The reversible booster is similar in construction to the non-reversible booster but has a stronger series field. At normal load the shunt and series fields are of equal strength and the booster voltage is zero. The battery then floats on the line and neither charges nor discharges. An increase of load above normal increases the strength of s and overpowers f and so discharges the battery. When the load decreases s becomes weaker than f and the booster causes the battery to charge.

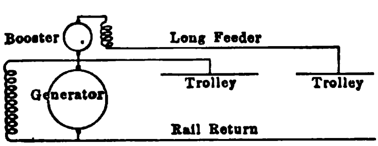


Fig. 237.—Series booster.

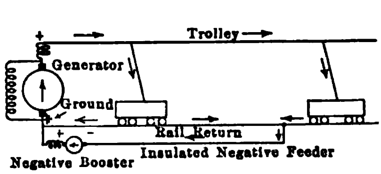


Fig. 238.—Negative booster.

Thus by installing an automatic booster the battery is made more sensitive to variations of load and a better regulation of the generator load and voltage is obtained.

If the load is a combined lighting and power load the lighting circuit can be supplied at constant voltage from the generator terminals and the power circuit connected outside the battery. Sudden variations of load will be cared for by the battery and the voltage on the lighting circuit will not be affected.

Series boosters are used in electric-railway engineering and in general power distribution to raise the voltage on certain sections of the line as in the case of a long feeder supplying power to an outlying section, as shown in Fig. 237. The booster field is

connected in series with the line and its voltage increases with the load and so neutralizes the line drop.

In cases where insulated negative feeders are connected to the track return at various points to carry the current back to the substations and so reduce the danger of electrolysis, a booster connected in series with the negative feeder can be made to reduce its effective resistance to a very low value and so increase its capacity. The use of such negative boosters, Fig. 238, is very common and results in a large saving in conductor material in the return feeders.

207. Balancers.—A direct-current compensator or balancer comprises two or more similar direct-current machines directly coupled to each other and connected in series across the outer conductors of a multiple-wire system of distribution for the purpose of maintaining the potentials of the intermediate wires. They may be either shunt-wound or compound-wound.

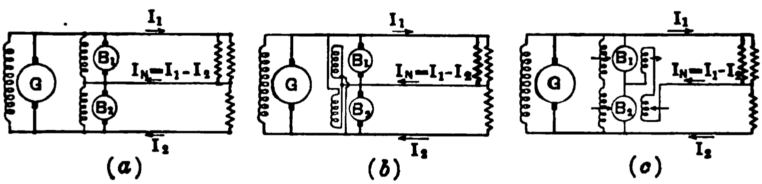


FIG. 239.—Balancers.

The generator G, in Fig. 239(a), develops a voltage of 230 volts between the outer wires and the function of the balancer set B_1B_2 is to maintain the potential of the neutral wire midway between the outers. When the loads on the two sides are balanced, no current flows in the neutral wire and the two machines run light as motors; when the load on one side is heavier than on the other, as shown, the voltage across B_1 is lower than that across B_2 and the neutral point is shifted; the machine B_2 runs as a motor driving B_1 as a generator and partially restores the balance.

The connection shown in Fig. 239(b) gives better results. The field of the motor B_2 is excited from the heavily loaded side and it therefore tends to run at an increased speed, while the field of B_1 is excited from the lightly loaded side and its voltage therefore rises due to increase in speed and increase in excitation. The balance cannot, however, be perfect since a slight inequality of voltages is necessary to make the balancer act.

Where perfect balance is required the machines B_1 and B_2 must be compound-wound and connected as shown in Fig. 239(*c*). The current in the neutral wire flows through the series windings of the two machines, strengthening the field of B_1 and raising the voltage and reducing the field of B_2 and increasing the speed of the set. The automatic balancing action in this case depends on an unbalance of current in the two outers and not on an unbalance of voltage between the outers and neutral as in (*a*) and (*b*).

208. Rosenberg Generator for Train Lighting.—For train lighting it is necessary to have a generator which will give approximately constant current or voltage independent of the speed. The Rosenberg generator shown diagrammatically in Fig. 240 gives a constant current at all speeds above a certain minimum.

AA is a battery which supplies the lighting system when the train is at rest or running at slow speeds. The field winding *ff* is excited by the battery and its m.m.f. M_f remains constant. The main brushes which supply the load are placed under the centers of the poles and are connected to the battery terminals and to the load, brush B_2 directly and brush B_1 through the aluminum cell *C*, which has the property of allowing current to flow from the generator to the battery or load but offers a high resistance to the flow of current in the opposite direction. The auxiliary brushes *bb* are placed on the commutator at right angles to the main brushes and are short-circuited.

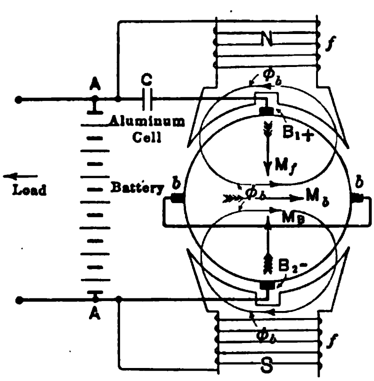


FIG. 240.—Rosenberg train-lighting generator.

As the armature rotates its conductors cut the flux produced by the field m.m.f. M_f and a voltage is generated between the brushes *bb* but none between the main brushes B_1B_2. A short-circuit current flows through the armature winding and the brushes *bb* and exerts a m.m.f. M_b at right angles to the field m.m.f. This m.m.f. produces a comparatively large flux. ϕ_b through the armature and pole faces. This cross-flux is

cut by the armature conductors and a voltage is generated between the brushes B_1B_2 and a current I flows to the load. Current I flowing in the armature conductors exerts a m.m.f. M_B opposing the field m.m.f. M_f and reducing it to a comparatively small value.

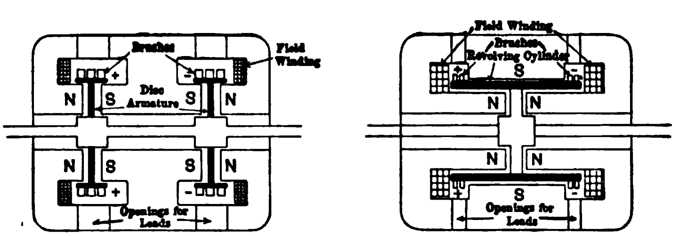
Above a certain speed, which depends on the field excitation, the current remains approximately constant independent of the speed.

The load current is limited by the fact that the m.m.f. M_B must always be less than M_f in order that the m.m.f. M_b and the cross-flux ϕ_b may exist.

If the direction of rotation changes, the direction of the e.m.f. between bb changes, the directions of M_b and ϕ_b change and therefore the direction of the voltage from B_2 to B_1 remains as before.

Notches are cut in the pole faces above the brushes to prevent large voltages being generated in the coils which are being commutated.

The current output may be varied by varying the excitation.



(*a*) Radial Type (*b*) Axial Type

FIG. 241.—Homopolar generators.

209. Homopolar Generators.—Homopolar or acyclic generators are designed with the armature conductors revolving in a unidirectional field and thus the e.m.fs. generated in them do not alternate during the revolution. Two types have been developed, the radial and the axial, so-called from the direction of flow of the armature currents.

In the radial type, Fig. 241(*a*), the armature consists of one or more discs rotating in the magnetic field and an e.m.f. is generated between the center and the edge of the disc. Current is collected by brushes rubbing on the external periphery of the discs.

The e.m.fs. generated are very low and a number of discs, each supplied with brushes, must be connected in series making a very complex and expensive construction.

In the axial type, Fig. 241(*b*), the discs are replaced by one or more copper cylinders with brushes at each end. The direction of the armature current is along the axis of the machine. Here again the e.m.f. is low and it is difficult to arrange enough cylinders in series to give the required e.m.f.

Such machines can be used where very large currents at low voltage are required as in electrolytic processes.

210. Limits of Output of Electric Machines.—The factors which limit the output of electric machines are:

1. Regulation.
2. Efficiency.
3. Heating.
4. Commutation.

1. In motors the regulation is a speed regulation. With increase of load the speed falls off and the increased torque is obtained at a decreased speed. In constant-speed work the shunt motor is used but if it is overloaded its speed falls to a value too low for satisfactory operation.

In generators the regulation is a voltage regulation. As the load is increased the voltage falls off and a point is finally reached where the voltage is so low that the power supplied is unsatisfactory.

2. The efficiency of a machine increases with increasing load to the point where the variable copper losses are equal to the constant losses. Above this point the efficiency decreases due to the rapid increase of the variable losses.

With properly designed machines the output is limited by either heating or commutation beiore the regulation or efficiency becomes too poor.

3. All the losses of power in a machine are converted into heat and raise the temperature of the various parts until the point is reached where the rate at which heat is being radiated or carried off by the ventilating apparatus is equal to the rate at which heat is being generated. The temperature will then remain constant. When a machine is overloaded its losses increase and consequently its temperature rises above normal.

If a machine operates at a high temperature for any length of time permanent injury to the insulating materials will result.

4. Sparking will occur in a machine when the field cut by the coil which is being commutated is not strong enough to reverse the current in the time of commutation. Sparking will therefore occur in generators or motors at heavy load when the armature m.m.f. is so great that it wipes out the field under the pole tip or weakens it to such an extent that it cannot produce the required commutating e.m.f. Motors will also spark at high speed since the time of commutation is reduced, especially when the high speed is produced by field weakening.

Take for example a shunt motor rated at normal speed as 10 hp., 110 volts, 80 amp., 400 r.p.m. and suppose the temperature rise to be 50°C. If the motor is operated at half speed of 200 r.p.m. by reducing the impressed voltage to half, the rating may be taken as 5 hp., 55 volts, 80 amp., but the temperature rise will be greater than before because the armature copper loss is the same, the field copper loss is the same and the iron and friction losses are less due to the low speed, but the ventilation is only about half as good as before.

When operated at twice full speed, produced by field weakening, the rating may be taken as 10 hp., 110 volts, 80 amp., but the temperature rise will be less than before, because the armature copper loss is the same, the field copper loss is reduced to about one-quarter of its normal value and the iron and friction losses are increased, but the ventilation is very much improved. The rated output of the machine for normal temperature rise might be increased but due to the higher speed and consequent reduced time of commutation sparking would occur.

211. Temperature Limits (Standardization Rules A. I. E. E. 1914).—The capacity of a machine is limited by the maximum temperature at which the materials in the machine, especially those employed for insulation, may be operated for long periods without deterioration. When the safe limits are exceeded, deterioration is rapid. The insulating materials become permanently damaged by excessive temperature, the damage increasing with the length of time that the excessive temperature is maintained and with the amount of excess temperature until finally the insulation breaks down.

The actual temperature attained in the different parts of a

machine and not the rises in temperature affect the life of the insulation of the machine. The safe operating temperatures of the various parts of a machine are often expressed in terms of the allowable temperature rise above the temperature of the surrounding air. The temperature of this air is called the ambient temperature and 40°C. is taken as its standard value. The allowable temperature rise plus 40°C. gives the maximum allowable temperature.

A machine may be tested at any convenient ambient temperature but the permissible rises of temperature must not exceed those given in column 2 of the table on page 255.

As it is usually impossible to determine the maximum temperature attained in insulated windings, it is convenient to apply a correction to the measured temperature to cover the errors due to the fallibility in the location of the measuring devices as well as inherent inaccuracies in measurement and method.

The two most usual methods of measuring temperature rises are: (1) by thermometer, and (2) by the increase of resistance.

1. Thermometers are applied to the hottest accessible parts of the machine. In this case the hottest spot temperature for windings should be estimated by adding 15°C. to the highest temperature observed.

2. The measurement of the increase of temperature by increase of resistance is only applicable to windings of comparatively high resistance. The resistance is measured before operation at the ambient temperature and again after operation. The rise of temperature can then be calculated as shown in Art. 84. A hot-spot correction of 10°C. should be added.

The following table gives the limits for the hottest spot temperatures of insulations. The permissible limits are indicated in column 1 of the table. The limits of temperature rise permitted under rated load conditions are given in column 2 and are found by subtracting 40°C. from the figures in column 1. Whatever be the ambient temperature at the time of the test, the rise of temperature observed must never exceed the limits in column 2 and the highest temperature must never exceed the limits given in column 1.

Table of hottest spot temperatures and corresponding permissible temperature rises.

Class	Description of insulation	Column 1, highest permissible temperature for the hot spot, degrees C.	Column 2, highest permissible temperature rise for the hottest spot above 40°C.
A_1	Cotton, silk, paper and other fibrous materials not so treated as to increase the thermal limit..................................	95	55
A_2	Similar to A_1 but treated or impregnated or permanently immersed in oil and including enameled wire..........................	105	65
B	Mica, asbestos or other material capable of resisting high temperatures in which any class *A* material or binder, if used, is for structural purposes only and may be destroyed without impairing the insulating or mechanical qualities..................................	125	85
C	Fireproof and refractory materials as mica, porcelain, etc............................	No limit is specified	

212. Temperature of Commutators.—The observable temperature shall in no case be permitted to exceed the values given in the table above for the insulation employed, either in the commutator or in any insulation whose temperature would be affected by the heat of the commutator. For commutators so constructed that no difficulties from expansion can occur, the following temperature limits have been suggested.

Current per brush arm	Maximum permissible temperature
200 amp. or less	130°C.
200 to 900 amp.	130°C. less 5° for each 100 amp. increase above 200
900 amp. and over	95°C.

213. Temperature of Cores.—The temperature of those parts of the iron core in contact with insulating materials must not exceed the limits of temperature and temperature rise permitted for those materials.

214. Temperature of Other Parts.—Other parts (such as brush holders, brushes, bearings, pole tips, cores, etc.), whose temperature does not affect the temperature of the insulating material, may be operated at such temperatures as shall not be injurious in any respect. But no part of continuous duty machin-

ery subject to handling in operation, such as brush rigging, shall have a temperature in excess of 100°C.

215. Ventilation.—The increase in output per pound of active material in modern machines has been largely due to improved methods of ventilation.

Inlets of sufficient size and proper location are provided; the cool air is drawn in either by natural suction or by fans placed on the rotating member; and the heated air is expelled through outlets in such a direction that it will be thrown completely away from the machine.

The air ducts through the active material are of two kinds, radial ducts, Fig. 242, and axial ducts, Fig. 243. They must be of sufficient section to carry the required amount of air and must have sufficient surface to allow the heat to pass from the copper and iron to the air.

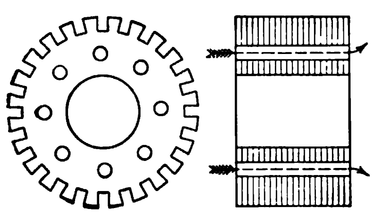


FIG. 242 —Axial ducts.

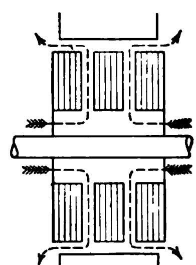


FIG 243.—Radial ducts.

About 100 cu. ft. of air per minute per kilowatt lost should be provided.

Care must be taken that the end bells or overhanging frame do not deflect the hot air back into the machine.

A generator when driven by a belt may run cool due to windage from the belt, whereas, if it is direct-connected to a motor or steam engine it may run hot due to poor ventilation or due to the transfer of hot air from the driving machine.

Machines of short length and large diameter are easy to cool.

216. Semi-enclosed and Totally Enclosed Machines.—When a machine is partially enclosed it is more difficult to get rid of the heat due to the losses and unless a more efficient system of ventilation is provided the output must be decreased 15 or 20 per cent. to keep within the allowable limits of temperature rise.

When totally enclosed the rating must be still further reduced. For the same temperature rise a totally enclosed machine can give about 70 per cent. of the output of an open machine if the speed is increased about 20 per cent. The reduction of output reduces the copper losses and the increase of speed decreases the flux per pole and so decreases the iron losses but not in direct proportion.

CHAPTER VIII

DESIGN OF A DIRECT-CURRENT GENERATOR

217. Symbols.—The subscripts used are c for core, g for air gap, t for teeth, p for pole and y for yoke.

A = sectional area in square inches.
AT = ampere-turns.
AT_g = ampere-turns per pole for the gap.
B = flux density in lines per square inch.
B_{ag} = actual gap density.
B_{at} = actual tooth density.
B_g = apparent gap density.
B_t = apparent tooth density.
D_a = external armature diameter.
D_c = commutator diameter.
I = load current.
I_c = current per conductor.
L_c = axial length of core.
L_g = gross iron in the frame length.
L_n = net iron in the frame length.
S = number of commutator segments.
T_f = field turns per pole.
Z = total conductors on the armature.
d = slot depth.
p = number of poles.
p_1 = number of paths through armature.
q = ampere conductors per inch.
s = slot width.
t = tooth width at top.
δ = air-gap clearance.
η = efficiency.
λ = slot pitch.
ν = leakage factor.
τ = pole pitch.
ϕ = magnetic flux.
ϕ_g = useful flux per pole.
ψ = per cent. pole enclosure.

The meaning of other symbols used will be explained in the context.

Figs. 244, 245, 246, 247, 252, 257 are reproduced from the "Standard Handbook," Section 8, by A. M. GRAY.

218. Design of Direct-current Machinery.—The design of electrical machinery is based on a number of formulæ or equations which can be very easily derived and on a number of empirical relations obtained from experience, which are usually expressed in the form of curves or limiting values.

Following are the most important of these fundamental relationships and limits. They are indicated by Roman numerals for purposes of reference.

I. The e.m.f. equation:

$$\text{generated e.m.f.} = \mathcal{E} = Zn\phi_g \frac{p}{p_1} 10^{-8} \text{ volts (Art. 142),}$$

or

$$\text{useful flux} = \phi_g = \frac{\mathcal{E}}{Zn \frac{p}{p_1} 10^{-8}} \text{ lines.} \tag{270}$$

II. The output equation:

$$\text{output in watts} = \mathcal{E}I = \left(\frac{ZI}{p_1}\right)(\phi_g p)n \times 10^{-8}$$

$$= \frac{10^{-8}}{60}(ZI_c)(\phi_g p)(\text{r.p.m.}) \tag{271}$$

$= \text{a constant} \times \text{electric loading} \times \text{magnetic loading} \times \text{r.p.m.}$

ZI_c = total ampere conductors on the armature is called the electric loading.

$\phi_g p$ = total flux crossing the gaps under the poles is called the magnetic loading.

A large electric loading requires a large amount of copper in the machine, while a large magnetic loading requires a large amount of iron.

The output equation can be written in a more easily applicable form:

$$\text{watts} = \frac{10^{-8}}{60}(\pi D_a q)(B_g \psi \tau L_c p)(\text{r.p.m.})$$

$$= \frac{10^{-8}}{60}\pi D_a q \; B_g \psi \frac{\pi D_a}{p} L_c p \times \text{r.p.m.}$$

$$= \frac{\pi^2}{60 \times 10^8} D_a{}^2 L_c \, q B_g \psi \times \text{r.p.m.}$$

or

$$D_a{}^2 L_c = \frac{\text{watts}}{\text{r.p.m.}} \frac{60.8 \times 10^7}{B_g \psi q}. \tag{272}$$

where,

$D_a{}^2L_c$ = volume of the active material in the armature.

B_g = apparent density in the air gap.

ψ = per cent. pole enclosure.

$q = \frac{ZI_c}{\pi D_a}$ = ampere conductors per inch of the armature periphery.

III. Flux Densities.—The flux density in the air gap is limited by the maximum allowable flux density in the roots of the teeth, which is dependent on the frequency of the reversals of the magnetism.

The approximate limits of the maximum tooth density are 150,000 lines per square inch for 30 cycles per second and 125,000 lines for 60 cycles. The frequency is given as

$$f = \frac{pn}{2} \text{ cycles per second.}$$

Having fixed the density at the roots of the teeth the air-gap density will depend on the diameter of the armature.

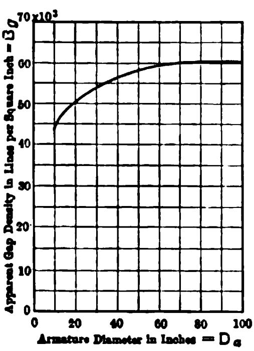


FIG. 244.

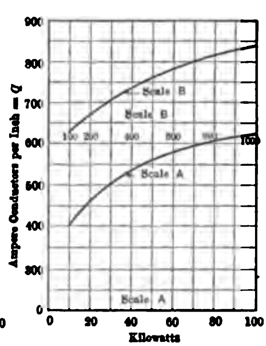


FIG. 245.

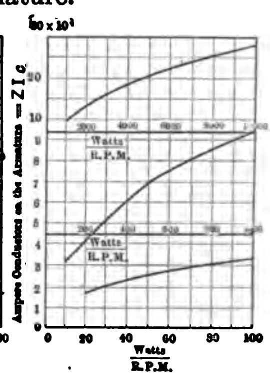


FIG. 246

Fig. 244 shows the relation between B_g and D_a for a frequency of 30 cycles per second. Lower values of B_g should be used for higher frequencies.

The ordinary flux densities in other parts of the magnetic circuit are:

	Flux density, lines per square inch, depending on the frequency
Armature core	70,000–100,000
Pole (cast steel)	90,000– 95,000
Yoke (cast iron)	35,000– 40,000
Yoke (cast steel)	70,000– 80,000

IV. Ampere Conductors per Inch.—The value of q the ampere conductors per inch of armature periphery is limited partly by heating and partly by commutation and it depends on the output of the machine. Fig. 245 shows the relation between q and kilowatt output for machines in satisfactory operation.

V. Electric Loading.—For the most economical construction there is a more or less fixed relation between the electric and the magnetic loading.

Therefore, referring to equation (271), it is seen that the electric loading, or the total ampere conductors on the armature, depends on the ratio $\dfrac{\text{watts}}{\text{r.p.m.}}$.

The relation between ampere conductors, ZI_c and watts per revolution is shown in Fig. 246.

VI. Current Density.—The current density used in the armatures of direct-current generators varies from 2,000 to 3,000 amp. per square inch, that is, 600 to 400 circ. mils per ampere. It is limited by the allowable temperature rise. The value to be used on a given machine depends on the specific electric loading q and on the peripheral speed. Fig. 247 shows the relation between the ratio $\dfrac{\text{ampere conductors per inch}}{\text{circular mils per ampere}}$ and the peripheral speed in feet per minute for a temperature rise of 40°C.

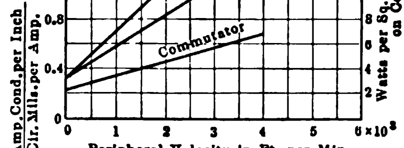


FIG. 247.

VII. Choice of Number of Poles.—The number of field ampere-turns per pole is usually about 50 per cent. greater than the armature ampere-turns per pole. If the number of poles is too small the required ampere-turns per pole will be large and the poles must be made long to carry the windings. The number of poles may be fixed approximately by the two relations:

(*a*) Ratio $\dfrac{\text{pole pitch}}{\text{frame length}}$ lies between 1.1 and 1.7 usually.

(*b*) Armature ampere-turns per pole should not exceed 7,500.

VIII. Commutation.—To provide a commutating field, the ratio $\frac{\text{field ampere-turns per pole for the gap and teeth}}{\text{armature ampere-turns per pole}}$ should not be less than 1.2. If the brushes are advanced so that part of the armature m.m.f. is demagnetizing the field ampere-turns must be increased by the amount of the armature demagnetizing ampere-turns.

IX. Reactance Voltage.—The formulæ for the reactance voltage were derived in Art. 195.

$$E_R = \text{K} \times L_c \times ZI_c \times \text{r.p.m.} \times n \times 10^{-8} \text{ volts}$$

where K is a constant.

For a full-pitch multiple winding $K = 0.8$, and for a short-pitch multiple winding $K = 0.46$.

With the armature strength limited as indicated in VIII above and the brushes advanced into a suitable commutating field, the reactance voltage with a full-pitch winding should not exceed 1.5 to 2 volts for shunt machines. In compound-wound machines values up to 3.5 volts may be taken care of in certain cases. However, when the reactance voltage is much above 2.5 volts it may be advisable to use interpoles.

With short-pitch windings the two sides of the short-circuited coil are not in commutating fields of equal strength and slightly lower limits should be used.

With series windings since a number of coils are short-circuited in series the reactance voltage calculated by the formula for the full-pitch winding should be kept about 50 per cent. lower than for the multiple winding.

X. Slots.—The ratio $\frac{\text{slot depth}}{\text{slot width}}$ varies from 2.5 to 3.5.

The ratio $\frac{\text{slot width}}{\text{maximum tooth width}} = 1$ approximately.

Slots per pole should be greater than 12 for small machines and 14 for large machines.

XI. Commutator.—The commutator diameter is from 60 to 75 per cent. of the armature diameter. If possible the peripheral speed of the commutator should not exceed 3,500 ft. per minute but values up to 5,000 may be used in special cases.

XII. Brushes.—The brush arc should not cover more than three segments and should not subtend more than one-twelfth of the pole pitch.

The current density in the brushes depends on the kind of brush used. For ordinary brushes from 35 to 40 amp. per square inch may be used and the contact resistance drop may be taken as 1 volt per brush. This gives a loss of 35 to 40 watts per square inch of brush contact, which is satisfactory. Where brushes with lower contact resistances are used the current density may be higher.

The brush pressure is from 1½ to 2½ lb. per square inch depending on the service.

The coefficient of friction between the brush and the commutator may be taken as 0.3 for ordinary hard carbon brushes.

219. Magnetic Leakage.—Since there is no material through which magnetic flux cannot pass, it is not possible to confine all the flux produced in a generator to the magnetic circuit. In practice the main circuit is made of such low reluctance that only a small portion of the flux leaves it.

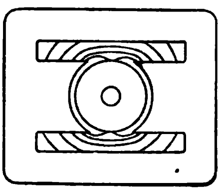


FIG. 248.—Leakage flux about a bipolar dynamo.

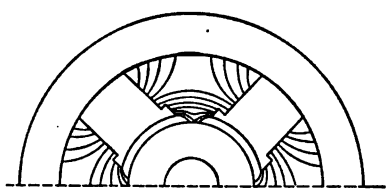


FIG. 249.—Leakage flux about a multipolar dynamo.

Figs. 248 and 249 show the leakage fluxes about the magnetic circuits of a bipolar and multipolar generator.

The principal part of the leakage occurs between the pole tips because the m.m.f. consumed between these points is from 60 to 80 per cent. of the total m.m.f.; it includes the m.m.f. required to drive the flux across the two gaps and through the teeth and armature core. As a result the flux passing through the field poles and yokes is greater than the flux crossing the gap into the armature by an amount depending both on the mechanical construction of the machine and on the load.

The dispersion coefficient or leakage factor is the ratio of the flux through the field poles to the flux crossing the gap into the armature, that is, the ratio of the total flux to the useful flux; thus the leakage factor is

$$\nu = \frac{\phi \text{ pole}}{\phi \text{ gap}}.$$

In preliminary calculations the following values of leakage factor may be used:

Four-pole machines up to 10 in. armature diameter........	1.25
Multipolar machines from 10 to 30 in. armature diameter...	1.2
Multipolar machines from 30 to 60 in. armature diameter...	1.18
Multipolar machines over 60 in. armature diameter........	1.15

Under load the armature exerts a m.m.f. which in part is demagnetizing and opposes the field m.m.f. and in part is cross-magnetizing and increases the reluctance of the magnetic circuit (Art. 145). Thus under load a greater proportion of the field m.m.f. is required for the air gaps, teeth and armature than at no load and the leakage flux is increased in proportion. The leakage factor is therefore greater under load than at no load.

In the case of the flat-compound generator, designed in this chapter armature reaction consumes a component of the full-load field m.m.f. = 2,117 ampere-turns per pole. At no load the m.m.f. consumed in the gap, the teeth and the armature core is 5,928 ampere-turns per pole and the leakage factor is taken as 1.18. At full load the leakage factor is increased to

$$\nu = 1 + 0.18 \times \frac{5{,}928 + 2{,}117}{5{,}928} = 1.25$$

220. Design of a Direct-current Generator.—Determine the dimensions, and characteristics of a generator of the following rating: 250 kw., 250 volts, 400 r.p.m. (Fig. 250).

Armature design.

$\frac{\text{Watts}}{\text{R.p.m.}} = \frac{250{,}000}{400} = 625.$

Ampere conductors $= ZI_c = 78{,}000$, Fig. 246.

Ampere conductors per inch $= q = 690$, Fig. 245.

Armature circumference $= \pi D_a = \frac{ZI_c}{q} = \frac{78{,}000}{690} = 113$ in.

Armature diameter $= D_a = \frac{113}{\pi} = 36$ in.

Apparent gap density $= B_g = 55{,}000$, Fig. 244.

Pole enclosure $= \psi = 0.7$, assumed.

$D_a{}^2L_c = \frac{\text{watts}}{\text{r.p.m.}} \times \frac{60.8 \times 10^7}{B_g\psi q}$ equation (272).

Frame length $= L_c = \frac{250{,}000}{400} \times \frac{60.8 \times 10^7}{55{,}000 \times 0.7 \times 690} \times \frac{1}{(36)^2} = 11$ in.

Number of poles $= p = 8$, assumed.

Armature ampere-turns per pole $= \frac{ZI_c}{2p} = \frac{78{,}000}{2 \times 8} = 4{,}860.$

$$\text{Pole pitch} = \tau = \frac{\pi D_a}{p} = \frac{\pi \times 36}{8} = 14.13 \text{ in.}$$

$$\frac{\text{Pole pitch}}{\text{Frame length}} = \frac{\tau}{L_e} = \frac{14.13}{11} = 1.3.$$

Area of the air gaps $= A_g = \psi\tau L_e = 0.7 \times 14.13 \times 11 = 109$ sq. in.
Useful flux per pole $= \phi_g = B_g A_g = 55{,}000 \times 109 = 6 \times 10^6$ lines.
Winding assumed to be multiple, short pitch, with one turn per coil.
Reactance voltage $= E_R = 0.46 \times 10^{-8} \times L_e \times ZI_c \times \text{r.p.m.} \times n$, equation (268) $= 0.46 \times 10^{-8} \times 11 \times 78{,}000 \times 400 \times 1 = 1.58$. With a full-pitch winding it would have been 2.7 volts.

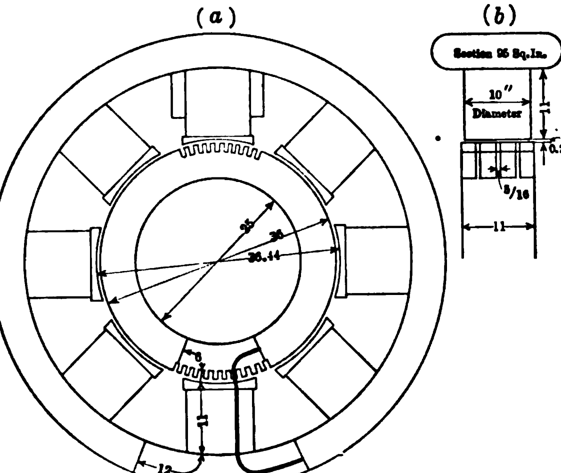


FIG. 250.—Magnetic circuit.

Number of path in multiple between terminals $= p_1 = 8$.
Total conductors on the armature =

$$Z = \frac{\mathcal{E}}{n\phi_g \frac{p}{p_1} 10^{-8}} = \frac{250 \times 10^8}{\frac{400}{60} \times 6 \times 10^6 \times \frac{8}{8}} = 624.$$

Conductors per slot = 4, assumed.

$$\text{Number of slots} = \frac{624}{4} = 156.$$

$$\text{Slots per pole} = \frac{156}{8} = 19.5.$$

Peripheral velocity of the armature in feet per minute =

$$\pi \times \frac{36}{12} \times 400 = 3{,}800.$$

$$\frac{\text{Ampere conductors per inch}}{\text{Circular mils per ampere}} \text{ for 40°C. rise} = 1.4, \text{ Fig. 247.}$$

$$\text{Circular mils per ampere} = \frac{q}{1.4} = \frac{690}{1.4} = 494.$$

Total load current $= I = \frac{\text{watts}}{\text{volts}} = \frac{250{,}000}{250} = 1{,}000$ amp.

Current per conductor $= I_c = \frac{I}{p_1} = \frac{1{,}000}{8} = 125$ amp.

Section of conductor $= 125 \times 494 = 61{,}750$ circ. mils $= 0.0485$ sq. in.

Slot pitch $= \lambda = \frac{\pi D_a}{\text{number of slots}} = \frac{\pi \times 36}{156} = 0.725$ in.

Slot width $= 0.5 \times$ slot pitch, assumed $= 0.5 \times 0.725 = 0.3625$.

Conductor dimensions $=$ width $\times$ depth $= x \times y = 0.0485$ sq. in.

Thickness of insulation in width $= 0.112$ in. (Fig. 251.)

Allowance for clearance $= 0.040$ in.

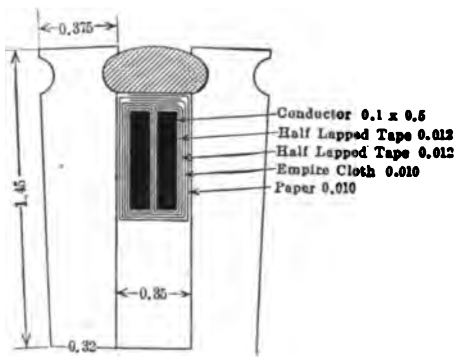


FIG. 251.—Details of slot and insulation.

Width of two conductors $= 2x = 0.3625 - 0.152 = 0.2105$.

Width of conductor $= x = 0.105 = 0.1$ in.

Slot width $= s = 2 \times 0.1 + 0.112 + 0.040 = 0.352 = 0.35$ in.

Depth of conductor $= y = \frac{0.0485}{0.1} = 0.485 = 0.5$ in.

Circular mils per ampere $= 494 \times \frac{0.1 \times 0.5}{0.0485} = 510$.

Amperes per sq. inch $= \frac{4}{\pi} \times \frac{10^6}{510} = 2{,}500$.

Thickness of insulation in slot depth $= 0.216$ in.

Thickness of wedge $= 0.2$.

Depth of two conductors $= 1.0$.

Allowance for clearance $= 0.040$.

Depth of slot $= 1.456 = 1.45$ in.

Maximum tooth width $= t = \lambda - s = 0.725 - 0.35 = 0.375$ in.

Diameter at bottom of slots $= (36 - 2 \times 1.45) = 33.10$ in.

Minimum tooth width $= \frac{\pi \times 33.10}{156} - 0.35 = 0.67 - 0.35 = 0.32$ in.

Tooth taper $= \tau = \frac{t_{\text{max.}}}{t_{\text{min.}}} = \frac{0.375}{0.32} = 1.17$.

Number of vent ducts in the core $= 3 - \frac{5}{16}$ in. wide.

Gross iron in the frame length $= L_g = L_c -$ vent ducts $=$

$(11 - 3 \times 0.3125) = 10.06$ in.

Stacking factor for armature punchings $= 0.9$.

Net iron in the frame length $= L_n = 0.9L_g = 0.9 \times 10.06 = 9.05$ in.

Number of teeth under one pole $= \psi \times \frac{\text{number of teeth}}{p} = 0.7 \times \frac{156}{8} = 13.6.$

Maximum tooth area per pole $= 13.6 \times 0.375 \times 9.05 = 46$ sq. in.

Minimum tooth area per pole $= 13.6 \times 0.32 \times 9.05 = 39.5$ sq. in.

Flux in the teeth $= \phi_t = \phi_g = 6 \times 10^6$ lines.

Apparent minimum flux density in the teeth $= \frac{6 \times 10^6}{46} = 131{,}000$ lines per square inch, Art. 221.

Apparent maximum flux density in the teeth $= \frac{6 \times 10^6}{39.5} = 152{,}000$ lines per square inch.

Frequency of the reversals of magnetism $= f = \frac{p \times n}{2} = \frac{8 \times 400}{2 \times 60} =$ 27 cycles per second.

Flux density in the core $= 85{,}000$ lines per square inch, assumed.

Flux in the core $= \phi_c = \frac{\phi_g}{2} = \frac{6 \times 10^6}{2} = 3 \times 10^6.$

Core area $= \frac{3 \times 10^6}{85{,}000} = 35.4$ sq. in.

Depth of iron below slots $= \frac{\text{core area}}{L_n} = \frac{35.4}{9.05} = 3.92$ in.

Internal diameter of armature $= \{36 - 2(1.45 + 3.92)\} = 25.16 = 25$ in.

Depth of iron below slots $= \frac{36 - 25}{2} - 1.45 = 4.05$ in.

Core area $= A_c = 4.05 \times 9.05 = 36.7$ sq. in.

Flux density in the core $= B_c = \frac{6 \times 10^6}{2 \times 36.7} = 82{,}000$ lines per square inch.

221. Flux in the Air Spaces between the Teeth.—If the flux density in the teeth is above 100,000 lines per square inch it is necessary to take account of the fact that the path through the teeth is paralleled by an air path consisting of the slots, the vent ducts and the insulation between punchings. This path has usually a much larger section than the path through the teeth and consequently at high densities where the permeability of the iron is low it will carry a considerable part of the flux.

In the case of the generator designed:

Maximum tooth area per pole $= 46$ sq. in.

Gap area $= 109$ sq. in.

Area of path through slots and vent ducts $= 63$ sq. in.

Minimum tooth density $= 131{,}000$ lines per square inch.

Ampere-turns per inch for this density $= 800$.

Flux density in the parallel air path $= 800 \times 3.2$.

Flux in the parallel path $= 800 \times 3.2 \times 63 = 160{,}000$.

The average ampere-turns per inch for the teeth will be greater than 800 due to the tooth taper (Art. 222) and it will be better to assume that the

flux carried by the air path is somewhat greater than 160,000, assume 200,000, then flux in the teeth = $6 \times 10^6 - 200{,}000 = 5.8 \times 10^6$.

Maximum flux density in the teeth = 147,000 lines per square inch.

222. Effect of Tooth Taper.—When the diameter of the armature is small or the slots are deep, the flux density at the roots of the teeth is much greater than that at the tops. If the ampere-turns for the teeth are calculated by using the maximum density, the result is too large, while if the minimum density is used the result is much too small. The curves in Fig. 252 have been calculated for various tooth tapers, $k = \frac{t_{max.}}{t_{min.}}$, and give the average ampere-turns per inch. Corresponding to the actual tooth densities at the root.

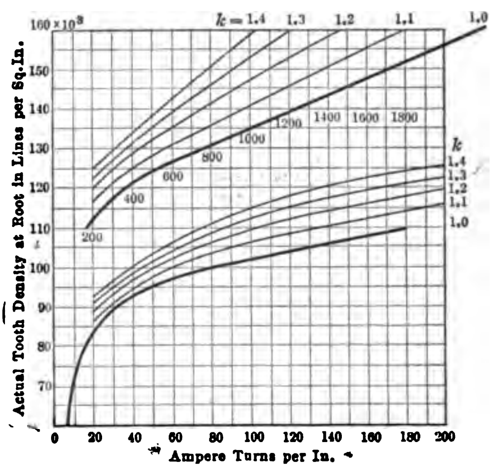


FIG. 252.—Magnetization curves for tapered teeth.

With a flux density of 147,000 lines per square inch at the root and a tooth taper $k = 1.17$ the ampere-turns per inch required = 1,065.

223. Ampere-turns per Inch for an Air Path.—When a flux density B lines per square inch is to be produced in an air path, the ampere-turns per inch required may be found as follows:

$$\phi = \frac{0.4\pi nI}{\frac{l}{A\mu}}$$

Taking a path 1 in. long and 1 sq. in. in section $\phi = B$; $l = 1$

in. = 2.54 cm., A = 1 sq. in. = $(2.54)^2$ sq. cm., $\mu = 1$; and substituting

$$B = \frac{\dfrac{0.4 \times 3.14\,(nI)}{2.54}}{(2.54)^2 \times 1}$$

and the ampere-turns per inch.

$$(nI) \text{ per inch} = B \times \frac{2.54}{0.4 \times 3.14 \times (2.54)^2} = 0.3132B = \frac{B}{3.2}. \quad (273)$$

224. Air Gap.—The air gap is the most important part of the magnetic circuit as it requires the largest proportion of the field m.m.f.

The section of the air gap is $A_g = \psi\tau L_c$ and the apparent flux density is $B_g = \frac{\phi_g}{A_g} = \frac{\phi_g}{\psi\tau L_c}$. This would be the correct density if the flux were uniformly distributed, but due to the presence of the slots and vent ducts the whole space is not utilized and the actual flux density is $B_{ag} = CB_g$, where C is a constant called the Carter coefficient and depends on the tooth width, slot width and gap length. Referring to Fig. 253,

$$C = \frac{t + s}{t + fs} \quad (274)$$

and the values of f which depend principally on the ratio $\frac{\text{slot width}}{\text{gap length}} = \frac{s}{\delta}$ may be obtained from the curve.

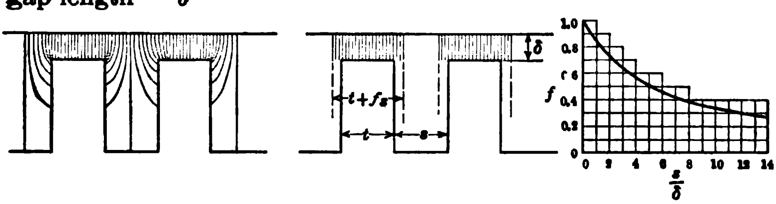


FIG. 253.—Carter's coefficient.

The ampere-turns per inch required for the air gap = 0.3132 × $B_{ag} = \frac{B_{ag}}{3.2} = \frac{CB_g}{3.2}$ (Art. 223); and the ampere-turns per pole for the gap

$$AT_g = \frac{CB_g\delta}{3.2}. \quad (275)$$

225. Length of the Air Gap.—

Armature ampere-turns per pole = 4,860.
Ampere-turns per pole (gap + teeth) = AT_{g+t} = 1.2 × 4,860 = 5,832.

Ampere-turns per pole for the teeth $= AT_t =$ 1,550.
Ampere-turns per pole for the gap $= AT_g =$ 4,282.

$$AT_g = \frac{C \times 55{,}000 \times \delta}{3.2} = 4{,}282.$$

$$C\delta = \frac{4{,}282 \times 3.2}{55{,}000} = 0.25.$$

Gap length $= \delta = 0.22.$

$$\frac{s}{\delta} = \frac{0.35}{0.22} = 1.59.$$

$$f = 0.76, \text{ Fig. } 253.$$

$$C = \frac{t + s}{t + fs} = \frac{0.375 + 0.35}{0.375 + 0.76 \times 0.35} = 1.13.$$

Ampere-turns for the gap

$$= AT_g = \frac{CB_g\delta}{3.2} = \frac{1.13 \times 55{,}000 \times 0.22}{3.2} = 4{,}270.$$

226. Design of Poles and Yoke.—

Density in the pole core $= 95{,}000$ lines per square inch (assumed).
Leakage coefficient $= \nu = 1.18$, Art. 219.
Flux in the pole $= \phi_p = 6 \times 10^6 \times 1.18 = 7.08 \times 10^6$ lines.

Section of the pole core $= \dfrac{7.08 \times 10^6}{95{,}000} = 74.6$ sq. in.

Assume a round pole 10 in. in diameter.

Section of pole $= A_p = \dfrac{3.14}{4} \times 10^2 = 78.5$ sq. in.

Flux density in the pole core $= B_p = \dfrac{7.08 \times 10^6}{78.5} = 90{,}000$ lines per square inch.

The field ampere-turns per pole are approximately 50 per cent. greater than the armature ampere-turns per pole $= 1.5 \times 4{,}860 = 7{,}290$; about 1,000 ampere-turns can be placed on 1 in. of winding space and, therefore, the space for the shunt coil is $\dfrac{7{,}290}{1{,}000} = 7.3$ in.; adding 30 per cent. to this to take care of the series field the total winding space is 10 in. and allowing 1 in. for the pole face, the length of the pole is $l_p = 11$ in.

Yoke material = cast iron.

Yoke density $= 40{,}000$ lines per square inch (assumed).

Flux in yoke $= \dfrac{7.08 \times 10^6}{2} = 3.54 \times 10^6.$

Section of the yoke is $\dfrac{3.54 \times 10^6}{40{,}000} = 88.5$ sq. in., take $A_y = 95$ sq. in.

Density in the yoke $= B_y = \dfrac{3.54 \times 10^6}{95} = 37{,}300.$

227. Determination of the No-load Saturation Curve.—Fig. 250 shows the dimensions of the magnetic circuit of the eight-pole, 250-kw., 250-volt, 400-r.p.m. generator. It is required to obtain the no-load saturation curve for this machine. The armature has a multiple winding with 624 conductors.

The voltage generated in the armature is

$$\mathcal{E} = Zn\phi_g \frac{p}{p_1} 10^{-8} = 624 \times \frac{400}{60} \times \phi_g \times \frac{8}{8} 10^{-8} \text{ volts,}$$

and therefore

$$\phi_g = 24{,}000\mathcal{E}.$$

To produce the rated voltage, 250 volts, a flux is required in the gap, $\phi_g = 24{,}000 \times 250 = 6 \times 10^6$ lines.

The leakage factor may be assumed to be 1.18, Art. 219, so that the flux per pole is

$$\phi_p = 1.18\phi_g = 1.18 \times 6 \times 10^6 = 7.08 \times 10^6.$$

The sections and lengths of the various parts of the magnetic circuit may be obtained from the design sheet or from the sketch.

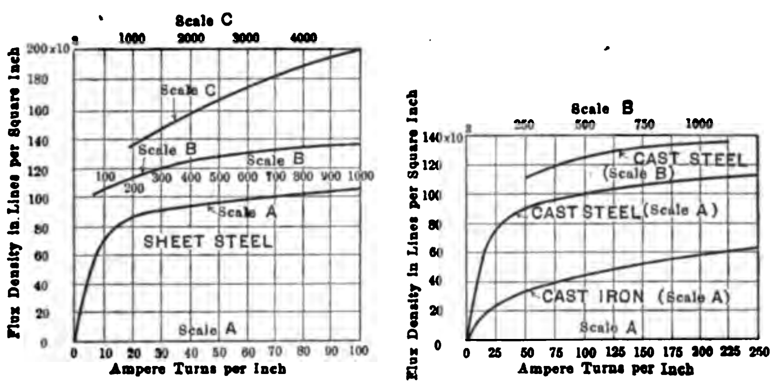


FIG. 254.—Magnetization curves.

Yoke.

Material = cast iron.

Section = A_y = 95 sq. in.

Length of magnetic path = l_y = 12 in., Fig. 250.

Flux in yoke = $\phi_y = \frac{\phi_p}{2} = \frac{7.08 \times 10^6}{2} = 3.54 \times 10^6$ lines.

Flux density = $B_y = \frac{3.54 \times 10^6}{95} = 37{,}300$ lines per square inch.

Ampere-turns per inch = 65, Fig. 254.

Ampere-turns per pole for the yoke = $AT_y = 65 \times 12 = 780$.

Pole.

Material = cast steel.

Section = A_p = 78.5 sq. in.

Length = 11 in.

Flux = ϕ_p = 7.08×10^6 lines.

Flux density = $B_p = \frac{7.08 \times 10^6}{78.5} = 90{,}000$ lines per square inch.

Ampere-turns per inch = 48, Fig. 254.

Ampere-turns per pole for the pole core = $48 \times 11 = 528$.

Air gap.

Section = A_g = 109 sq. in.

Length = δ = 0.22 in.

Flux = $\phi_g = 6 \times 10^6$ lines.

Apparent flux density = $B_g = \dfrac{\phi_g}{A_g} = \dfrac{6 \times 10^6}{109}$ = 55,000 lines per sq. inch.

Carter coefficient $C = 1.13$.

Actual flux density = $B_{ag} = CB_g = 1.13 \times 55{,}000 = 62{,}150$ lines per square inch.

Ampere-turns per inch $= \dfrac{B_{ag}}{3.2} = \dfrac{62{,}150}{3.2} = 19{,}400$.

Ampere-turns per pole for the gap = $19{,}400 \times 0.22 = 4{,}270$.

Teeth.

Material = sheet steel.

Maximum section = 46 sq. in.

Minimum section = A_t = 39.5 sq. in.

Length of magnetic path = l_t = 1.45 in.

Flux = $\phi_g = 6 \times 10^6$ lines.

Apparent maximum flux density $= \dfrac{6 \times 10^6}{39.5}$ = 152,000 lines per square inch.

Apparent minimum flux density $= \dfrac{6 \times 10^6}{46}$ = 131,000 lines per square inch.

Actual maximum flux density = 147,000 lines per square inch, Art. 221.

Tooth taper = k = 1.17.

Ampere-turns per inch = 1,065, Fig. 252.

Ampere-turns per pole for the teeth = $AT_t = 1{,}065 \times 1.45 = 1{,}550$.

Armature core.

Material = sheet steel.

Section = A_c = 36.7 sq. in.

Length = l_c = 6 in., Fig. 250

Flux = $\phi_c = \dfrac{\phi_g}{2} = \dfrac{6 \times 10^6}{2} = 3 \times 10^6$ lines.

Flux density = $B_c = \dfrac{3 \times 10^6}{36.7}$ = 82,000 lines per square inch.

Ampere-turns per inch = 18.

Ampere-turns per pole for the core = $AT_c = 18 \times 6 = 108$.

The ampere-turns per pole required for a voltage of 250 volts at no load is $AT_y + AT_p + AT_g \times AT_t + AT_c = 780 + 528 + 4{,}270 + 1{,}550 + 108 = 7{,}236$.

These results are tabulated below and also the results of similar calculations made for 225 volts and 275 volts.

From these results the saturation curve for the machine is plotted in Fig. 256. Volts vs. field ampere-turns per pole.

No-load voltage = $\mathcal{E}$ Useful flux per pole = ϕ_g Leakage factor			225 5.4×10^6 1.18		250 6×10^6 1.18		275 6.6×10^6 1.18	
	Length	Area	Density	AT	Density	AT	Density	AT
Yoke.........	12.00	95.0	33,600	600	37,300	780	41,000	1,000
Pole..........	11.00	78.5	81,000	280	90,000	528	99,000	1,000
Air gap.......	0.22	109.0	1.13×49,500	3,840	1.13×55,000	4,270	1.13×60,500	4,700
Tooth (min.)..	1.45	39.5	136,800	.....	152,000	.....	167,000	
			133,000	830	147,000	1,550	160,000	2,000
Armature core.	6.00	36.7	73,800	80	82,000	108	90,200	200
Field-ampere turns per pole..			....	5,630	....	7,236	....	8,900

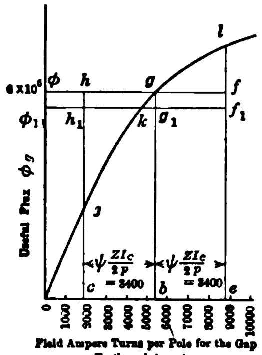


FIG. 255.—Cross-magnetizing effect.

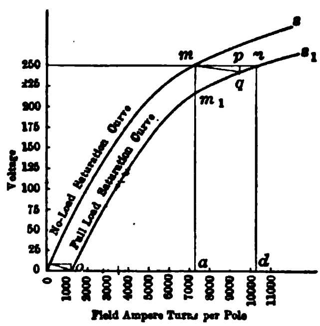


FIG. 256.—Saturation curves.

228. Field Winding.—The field coils must be designed with the required numbers of ampere-turns and with sufficient surface to radiate the heat due to the field copper loss.

If the coils are made thick it will be difficult to find space for them between the poles and they will be liable to get hot in the center layers. A good average thickness is 2 in.; assuming a current density in the field copper of about 800 amp. per square inch or 1,500 cir. mils per ampere and a space factor of 0.6 for the winding, it is possible to place $800 \times 0.6 \times 2 =$ approximately 1,000 ampere-turns on each inch of the coil space.

The radiating surface of the field coil is taken as the external cylindrical surface; this neglects the surface of the ends of the coil and the internal surface from which heat is transferred to the pole. The efficiency of the external radiating surface depends on the speed of the armature and its consequent fanning action. About 0.6 watts per square inch can be radiated from an ordinary field coil for a temperature rise of 40°C. For very short thick

coils a higher value may be used since the neglected surface of the ends is so large.

229. Current Density in Field Windings.—The current densities employed in shunt-field windings vary from 500 amp. per square inch for small slow-speed machines to 1,000 amp. per square inch for large high-speed machines with ventilated field coils.

230. Size of Wire for the Field Winding.—Let E_f = voltage across one coil, I_f = current, T_f = number of turns, l_m = mean turn in inches, A = section of wire in circular mils and R_f = resistance of the coil.

$$\text{At } 60°\text{C. } R_f = 12 \times \frac{\frac{l_m}{12} \times T_f}{A} = \frac{l_m T_f}{A}$$

and

$$E_f = I_f R_f = I_f T_f \frac{l_m}{A}$$

or, the section of the wire is

$$A = \frac{I_f T_f}{E_f} \times l_m; \tag{276}$$

therefore the size of the wire to be used is fixed when the ampere-turns and the voltage per coil are known.

Design of the field winding.

Voltage taken up in the field rheostat = 20 per cent. of 250 = 50 volts.

Voltage across one field coil = $E_f = \frac{250 - 50}{8} = 25$ volts

Ampere-turns per coil = $I_f T_f$ = 7,236.

Thickness of coil = 2 in. (assumed).

Mean turn = 38.3 in.

Section of the wire = $A = \frac{7{,}236}{25} \times 38.3 = 11{,}200$ circ. mils

This is between No. 9 and No. 10. B. & S. wire (Art. 86).

Either use an intermediate size or wind half of each coil with No. 9 and half with No. 10.

Diameter of No. 9 d.c.c. = 0.126.

Diameter of No. 10 d.c.c. = 0.114.

Average = 0.12.

Winding space = 7.2 × 2 sq. in.

Turns per layer = $\frac{7.2}{0.12}$ = 60.

Layers = $\frac{2}{0.12}$ = 16.

Turns per coil = T_f = 60 × 16 = 960

Current = $I_f = \frac{7{,}236}{960}$ = 7.5 amp.

Power lost = $25 \times 7.5 = 188$ watts.
Radiating surface = $7.2 \times 3.14 \times 14.2 = 320$ sq. in.
Watts per square inch = $\frac{188}{320} = 0.59$.
Temperature rise = 40°C. (Art. 228).

231. Resistance of the Armature Winding.—The length of a single armature conductor is given approximately by the empirical formula,

$$l = 1.35\tau + L_c + 3 \text{ in.} \tag{277}$$

If the section of the conductor is A circ. mils, its resistance at 60°C. is approximately

$$r_c = \rho \frac{\text{length in feet}}{\text{section in circ. mils}} = 12 \frac{\frac{l}{12}}{A} = \frac{l}{A} \text{ ohms.}$$

The number of conductors in series between brushes is $\frac{Z}{p_1}$, and the number of paths in multiple is p_1 and therefore, the armature resistance is

$$\begin{aligned} r_a &= \frac{r_c \frac{Z}{p_1}}{p_1} = \frac{Z}{p_1^2} r_c \\ &= \frac{Z}{p_1^2} \times \frac{1.35\tau + L_c + 3}{A} \text{ ohms.} \end{aligned} \tag{278}$$

Armature resistance = $r_a = \frac{624}{8^2} \times \frac{1.35 \times 14.13 + 11 + 3}{617{,}500} = 0.0052$ ohms.
Armature resistance drop at full load $Ir_a = 1{,}000 \times 0.0052 = 5.2$ volts.
Armature copper loss = $I_a^2 r_a = 1{,}000^2 \times 0.0052 = 5{,}200$ watts.

232. Determination of the Magnetomotive Force of the Series Winding.—In the case of a flat-compound generator the series excitation must be great enough to counteract the demagnetizing and cross-magnetizing m.m.f. of the armature and the armature resistance drop.

Demagnetizing Armature Magnetomotive Force.—If the brushes are advanced 10 per cent. of the pole pitch the demagnetizing ampere-turns per pole = $0.2 \frac{ZI_c}{2p} = 0.2 \frac{78{,}000}{2 \times 8} = 972$. The series-field m.m.f. required to overcome this armature m.m.f. = $972 \times$ full-load leakage factor = $972 \times 1.25 = 972 + 243 =$ 1,215 ampere-turns per pole.

Cross-magnetizing Armature Magnetomotive Force.—The effect of the cross-magnetizing armature ampere-turns is difficult to determine accurately but some approximation to it can be made as follows. The cross-magnetizing ampere-turns per pole =

$0.8 \times \frac{ZI_c}{2p} = 0.8 \times \frac{78,000}{2 \times 8} = 3,888$. The turns beyond the pole tips act on a path of large reluctance and may be neglected, while the remaining ampere-turns directly under the poles $= \psi \frac{ZI_c}{2p} = 0.7 \times \frac{78,000}{2 \times 8} = 3,400$ magnetize at one pole tip and increase the flux and demagnetize at the other pole tip and decrease the flux. The magnetic circuit on which they act is that through the armature and teeth and across the air gap, but not through the pole and yoke.

Fig. 255 shows the field ampere-turns per pole required for the armature, the teeth and the gap for various values of flux in the gap; *ob* is the field m.m.f. required to produce normal flux across the gap and through the teeth and armature; $bc = \psi \frac{ZI_c}{2p} = 3,400$ is the demagnetizing m.m.f. at one pole tip and the flux there is decreased from *ch* to *cj*; $be = bc = 3,400$ is the magnetizing m.m.f. at the other pole tip and the flux there is increased to *el*. At the center of the pole the m.m.f. and the flux are not changed. The total flux $o\phi$ in the gap at no load may be taken as proportional to *chfe*; due to cross-magnetizing a flux proportional to area *hgj* is lost and a flux proportional to area *glf* is gained; the result is a loss of flux and voltage proportional to $hgj - glf$. The flux in the gap is now $o\phi_1$ where ϕ_1 is found by making the area h_1hff_1 = area hgj − area glf. The length kg_1 represents the effect of the cross-magnetizing armature m.m.f. as a number of demagnetizing ampere-turns = 600 ampere-turns in this case. The series field m.m.f. required to overcome cross-magnetizing $= 600 \times 1.25 = 750$ ampere-turns per pole.

The loss of flux due to cross-magnetizing does not increase directly as the current but at a faster rate; it also depends very materially on the saturation of the magnetic circuit especially the teeth; with low flux densities the flux added on one side is approximately equal to the flux subtracted on the other side and the cross-magnetizing effect is very small.

To obtain accurate results it is necessary to take account of the increase of the leakage factor under load due to the presence of the armature m.m.f. (Art. 219). In this case the increase is from the assumed no-load value 1.18 to 1.25. The flux densities in the pole and yoke are increased in the same proportion and an increase of 170 ampere-turns per pole is required on the field on this account.

In constructing the full-load saturation curve of a generator as in Art. 147, it is convenient to treat the demagnetizing ampere-turns as a constant and to assume that they can be overcome by an equal number of series-field ampere-turns; the extra turns required on account of the leakage factor = 0.25 × 972 = 243 may be included under cross-magnetizing as also the extra turns for the pole and yoke. With this understanding the series-field turns required to overcome armature demagnetizing = 972 and to overcome armature cross-magnetizing = 750 + 243 + 170 = 1,163. The cross-magnetizing armature ampere-turns per pole = 3,888 and the series-field turns required to overcome them = 1,163 = approximately 30 per cent. of the cross-magnetizing ampere-turns per pole. For points higher up on the saturation curve a larger percentage than 30 would be required. The leakage factor decreases with a decrease of field m.m.f. and so the cross-magnetizing effect may still be assumed to be negligible at the lower points on the full-load saturation curve.

Fig. 256 shows the no-load and full-load saturation curves for the 250-kw., 250-volt generator; *oa* = 7,236 = the field ampere-turns per pole required at no load to produce the rated voltage 250 volts; *mp.* = the field m.m.f. required at full load to overcome armature reaction = 972 + 1,163 = 2,135; *pq* = the voltage consumed in the resistance of the armature and series field = 8 volts; *q* is therefore a point on the full-load saturation curve. The other points may be found as explained in Art. 147. The series-field ampere-turns required to overcome the resistance drop = *pn* = 865 and, therefore, the ampere-turns per pole required in the series-field winding for flat-compounding = *mn* = *ad* = 2,135 + 865 = 3,000.

233. Design of the Series-Field Winding.—

The series-field ampere-turns per pole = 3,000.

The load current = 1,000 amp.

$$\text{Turns per series coil} = \frac{3,000}{1,000} = 3.$$

Use 3½ turns per coil and shunt the part of the current not required, through a diverter.

$$\text{The current in the winding} = \frac{3,000}{3\frac{1}{2}} = 860 \text{ amperes.}$$

Current density in the series winding = 1,000 amp. per square inch (assumed).

$$\text{Size of conductor} = \frac{860}{1,000} = 0.86 \text{ sq. in.}$$

Use three strips 2.5×0.125 in. connected in parallel.
Actual section of conductor $= 3 \times 2.5 \times 0.125 = 0.94$ sq. in.
Length of the mean turn $= 36$ in. $= 3$ ft.

Resistance of one coil at 60°C. $= 12 \times \dfrac{3 \times 3\frac{1}{2}}{0.94 \times \frac{4}{\pi} 10^6} = 1.03 \times 10^{-4}$ ohms.

Voltage drop in one coil $= 860 \times 1.03 \times 10^{-4} = 0.088$ volts.
Power lost $= 0.088 \times 860 = 75$ watts.
The cylindrical radiating surface of the coil $= 2.5 \times 40 = 100$ sq. in.
Watts per square inch of surface $= \dfrac{75}{100} = 0.75$, which is quite satisfactory.
The voltage drop in the series-field winding $= 8 \times 0.088 = 0.7$ volts.
Power lost in the series-field winding $= 8 \times 75 = 600$ watts.

234. Design of the Commutator.—

Winding is one turn multiple, short pitch.
Numbers of commutator segments $= \dfrac{Z}{2} = \dfrac{624}{2} = 312$.
Diameter $= 0.75D_a = 0.75 \times 36 = 27$ in.
Width of segment + mica $= \dfrac{3.14 \times 27}{312} = 0.271$ in.
Width of mica $= \frac{1}{32}$ in. $= 0.031$ in.
Width of segment $= 0.24$ in.
Brush arc should not cover more than three segments $= 3 \times 0.271 = 0.813$ in.
Take brush arc $= 0.75$ in. $= \frac{3}{4}$ in.
Sets of positive brushes $= 4$.
Current per set $= \dfrac{1,000}{4} = 250$ amp.
Current density in the brush $= 35$ amp. per square inch.
Section of brushes per set $= \dfrac{250}{35} = 7.5$ sq. in.
Length of brushes per set $= \dfrac{7.5}{0.75} = 10$ in.
Brushes per set $= 5 - (\frac{3}{4}$ in. $\times 2$ in.).
Length of commutator $= 12$ in.
Peripheral velocity $= 3.14 \times \dfrac{27}{12} \times 400 = 2,825$ ft. per second.
Drop of voltage at each brush contact $= 1$ volt.
Loss of power at the brush contacts $= 1,000 \times 1 \times 2 = 2,000$ watts.
Brush pressure $= 2.5$ lb. per square inch.
Total brush pressure $= 2.5 \times 7.5 \times 8 = 150$ pounds.
Coef. of friction $= 0.3$.
Brush-friction loss $= 150 \times 0.3 \times 2,825 = 127,000$ ft.-lb. per minute.

$$= \frac{127,000}{33,000} \times 746 = 2,900 \text{ watts.}$$

Radiating surface of commutator = cylindrical surface $= 3.14 \times 27 \times 12 = 1,020$ sq. in.
Total watts lost $= 2,000 + 2,900 = 4,900$ watt.
Watts per square inch $= \dfrac{4,900}{1,020} = 4.8$.
Temperature rise will be under 40°C. (Fig. 247).

235. Losses and Efficiency.—The core losses cannot be calculated accurately by any formula but fairly satisfactory values may be obtained by the use of curves such as those in Fig. 257 compiled from the results of tests on completed machines. The loss in the teeth must be calculated separately from that in the core since the densities are different.

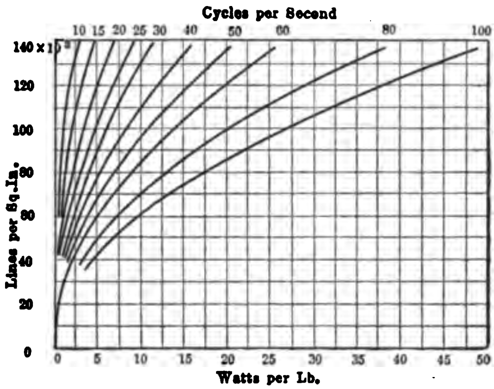


FIG. 257.—Iron-loss curves for direct-current machines.

Weight of iron in the teeth

$$= \left\{ \left(\frac{\pi \times 36^2}{4} - \frac{\pi \times \overline{33.1}^2}{4} \right) - 156 \times 0.35 \times 1.45 \right\} \times 9.05 \times 0.28 = 200 \text{ lb.}$$

where 0.28 is the weight of a cubic inch of iron.

The frequency is $\frac{400}{60} \times \frac{8}{2} = 27$ cycles per second.

Watts lost at 140,000 lines per square inch = 10 watts per pound.

Watts lost in the teeth = 200 × 10 = 2,000 watts.

Weight of iron in the core below the teeth $= \left(\frac{\pi \times \overline{33.1}^2}{4} - \frac{\pi \times 25^2}{4} \right) \times$ $9.05 \times 0.28 = 940$ lb.

Watts lost per pound at density of 80,600 lines per square inch = 3.3.

Loss = 940 × 3.3 = 3,100 watts.

Total core loss = 2,000 + 3,100	= 5,100 watts.
Shunt-field copper loss = EI_f = 250 × 7.5	= 1,875 watts.
Series-field copper loss	= 600 watts.
Armature copper loss	= 5,200 watts.
Brush-contact loss	= 2,000 watts.
Brush-friction loss	= 2,900 watts.
Windage and journal friction loss = 1 per cent.	= 2,500 watts (assumed).
Total losses	= 20,175 watts.

Efficiency at full load $= \frac{\text{output}}{\text{output} + \text{losses}}$ 100 per cent. =

$$\frac{250{,}000}{250{,}000 + 20{,}175} \times 100 \text{ per cent.} = 92.6 \text{ per cent.}$$

CHAPTER IX

SYNCHRONOUS MACHINERY

236. Alternator.—An alternating-current generator or alternator in its simplest form consists of an open coil of wire revolving at uniform speed in the magnetic field between a pair of unlike poles (Fig. 258).

The fields are excited by direct current from a separate machine called an exciter at 125 or 250 volts.

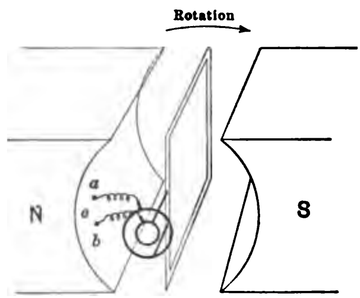


Fig. 258.—Single-phase alternator, revolving-armature type.

Between the slip rings a and b an alternating e.m.f. is generated of instantaneous value

$$e = n \frac{d\phi}{dt} 10^{-8} \text{ volts} \qquad (279)$$

where n is the number of turns in the coil and $\frac{d\phi}{dt}$ is the rate of change of the flux interlinking with the coil or the rate at which the coil is cutting the flux. The result is the same if the armature is stationary and the field revolves.

237. Types of Alternators.—There are three principal types of alternators:

(a) Revolving armature.
(b) Revolving field.
(c) Inductor.

Type (*a*) is illustrated in Fig. 258. The field poles are stationary and the armature revolves between them. The ends of the winding are brought out to two slip rings in single-phase machines and to three or more slip rings in polyphase machines and the current is collected by copper or carbon brushes.

The armature is necessarily of small size since the peripheral speed is limited and there is very little space for insulation. The armature conductors are also acted upon by centrifugal forces which tend to throw them out of the slots. The revolving armature is therefore only suitable for machines of small size and low voltage. It is, however, necessary in the case of rotary converters where the same armature winding carries both alternating and direct currents.

(*b*) The revolving field type illustrated in Fig. 259 and Fig. 260, is almost universal for all sizes and voltages. The armature is the stationary part and the field poles revolve. This type has many advantages over the revolving armature type. (1) It

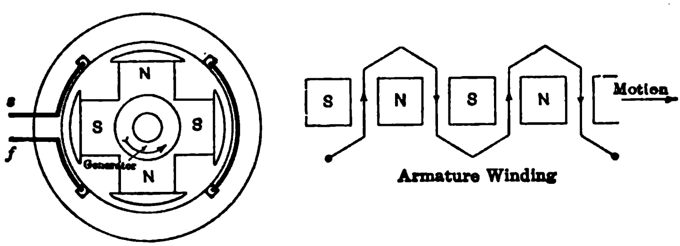


Fig. 259.—Single-phase alternator, revolving-field type.

requires only two slip rings even for polyphase machines and these slip rings carry only the direct current supplied to the field winding, while the load current is taken off from stationary terminals. (2) There is much more space for the armature windings and they are relieved from all centrifugal strains. They can, therefore, be much better insulated and ventilated. The field windings are made of copper strap and the revolving member is very rugged and is not affected by strains due to rotation; thus, much higher peripheral speeds may be used than with type (*a*), with consequent increase in economy of material.

The revolving-field members are made in two forms, the salient pole rotor and the cylindrical rotor. The rotor with salient poles, Figs. 259 and 260, is used almost universally for machines of all but the largest outputs and highest speeds.

Cylindrical rotors, Fig. 335, are employed for turbo-alternators of very large output which must run at extremely high speeds in order to obtain the maximum output per pound of material. The peripheral speeds run up to 25,000 ft. per minute and very large stresses are developed due to centrifugal force. The rotors are made of solid steel castings or of a number of steel discs bolted together and have slots milled out to carry the field windings. The shaft is bolted or dovetailed to the ends of the rotor.

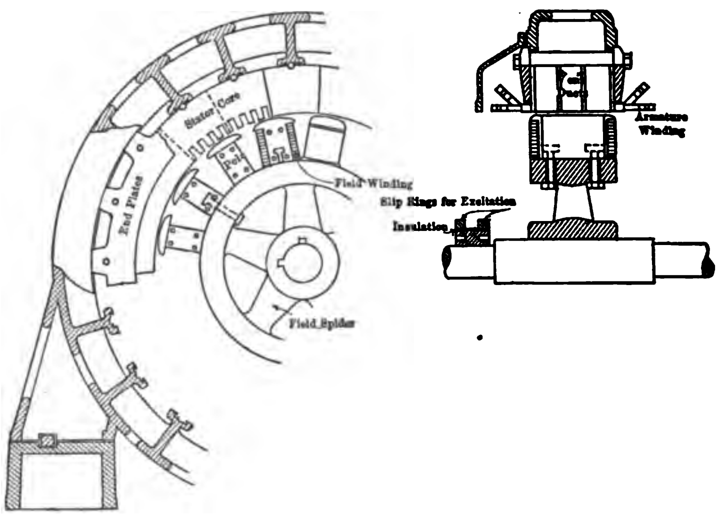


Fig. 260.—Revolving-field alternator.

The cylindrical rotor gives an air gap of practically uniform length and reluctance over the whole periphery and the field m.m.f. is distributed. It lends itself more readily to analytical treatment than the salient pole rotor with its varying gap reluctance and concentrated field m.m.f. The vector diagrams in Figs. 303 to 306 give very satisfactory results for machines with cylindrical rotors but one more approximate for those with salient poles.

(c) The inductor alternator is almost obsolete. In these machines the field and armature windings are both stationary and a part of the iron of the magnetic circuit revolves, producing a periodic pulsation of the reluctance of the magnetic circuit and consequently a variation of the flux linking with the armature winding. Fig. 261 represents one type of inductor alternator:

ff is the stationary field winding which produces the magnetic flux ϕ,ϕ, in the direction indicated. *a,b, a,b,* are the armature coils which may be connected either in series or in multiple. *I* is the revolving part of the magnetic circuit and is called the inductor. The polar projections on it are all north poles but the amount of flux issuing from *I* and linking with the armature coils *a,a, b,b,* depends on the relative position of the inductor projections and the projections carrying the armature coils.

In Fig. 262, curve 1 represents the variation of the flux ϕ_a interlinking with one armature coil *a* starting from the position when this flux is maximum, Fig. 261. Curve 2 represents the e.m.f. generated in coil *a* by the varying flux. As ϕ_a decreases

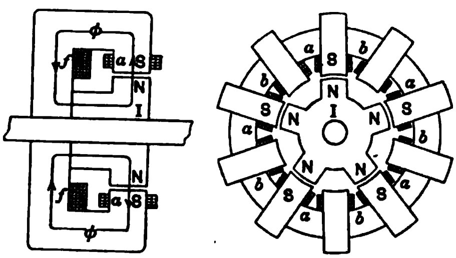


Fig. 261.—Inductor alternator.

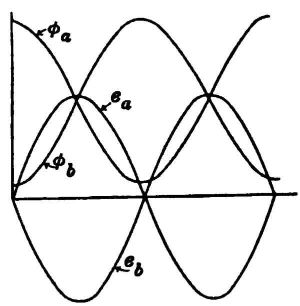


Fig. 262.—Fluxes and e.m.fs. in an inductor alternator.

an e.m.f. is generated in the positive direction and as it increases again an e.m.f. is generated in the negative direction. Thus although the flux does not reverse its direction and never reaches zero, an alternating e.m.f. is generated in the armature coil. If the pole pieces are properly shaped a sine wave of e.m.f. will be produced. Curves 3 and 4 represent the variation of flux and e.m.f. in coil *b*. The e.m.f. in coil *b* is displaced 180 degrees from that in *a* and before the two are connected in series the terminals of one must be reversed.

Inductor alternators were very heavy and expensive and have been superseded by the other types.

Any one of these three types may be wound as single-phase or polyphase machines.

Alternators are divided into classes depending on the type of prime mover employed: (*a*) Engine type, (*b*) waterwheel type, (*c*) steam turbine-driven type.

(*a*) Engine-type alternators are direct-connected to reciprocating steam engines and they require a large flywheel effect to ensure uniform angular velocity.

(*b*) Waterwheel-type alternators range in speed from 50 to 400 r.p.m. depending largely on the available head of water. The angular velocity of water turbines is uniform and flywheels are not required. Waterwheel alternators may be either horizontal or vertical.

(*c*) Steam turbine-driven alternators or turbo-alternators are extra high-speed generators usually 750 or 1,500 r.p.m. for 25 cycles and 1,800 or 3,600 r.p.m. for 60 cycles, corresponding to four-pole and two-pole rotors respectively. Cylindrical rotors are ordinarily employed on account of their greater mechanical strength and smaller windage loss. On account of the large output per pound of material forced ventilation is required to keep the temperature within the required limits and therefore such machines must be entirely enclosed.

238. Electromotive Force Equation.—Fig. 258 represents a two-pole, single-phase alternator. The armature winding is a single coil of n turns revolving at a constant speed. The magnetic field is assumed to be uniform.

The e.m.f. generated in the winding goes through one complete cycle during each revolution, and thus the frequency in cycles per second is equal to the speed in revolutions per second or $f =$ rev. per sec.

The angular velocity of the coil is ω radians per second, and therefore

$$\omega = 2\pi \times \text{rev. per sec.} = 2\pi f \qquad (280)$$

If Φ is the maximum flux inclosed by the coil, that is, the flux inclosed when the coil is vertical, as shown, and time is measured from this instant, then at time t, after the coil has turned through an angle θ, the flux inclosed is $\phi = \Phi \cos \theta$, and the e.m.f. generated in the coil is

$$e = -n\frac{d\phi}{dt}10^{-8}$$

$$= -n\frac{d}{dt}(\Phi \cos \theta)10^{-8},$$

but $\theta = \omega t = 2\pi f t$, and thus

$$e = -n\frac{d}{dt}(\Phi \cos 2\pi f t)10^{-8}$$

$$= 2\pi f n \Phi\, 10^{-8} \sin 2\pi f t \text{ volts} \qquad (281)$$

$$= E_m \sin \theta.$$

This is a sine wave of maximum value

$$E_m = 2\pi f n \Phi \, 10^{-8} \text{ volts} \tag{282}$$

and effective value

$$E = \frac{E_m}{\sqrt{2}} = 4.44 f n \Phi \, 10^{-8} \text{ volts} \tag{283}$$

This is the e.m.f. equation for an alternator which produces a sine wave of e.m.f. and has a concentrated winding, that is, all the turns wound in a single coil.

This result may also be obtained as follows. The flux cut per second by each turn of the coil is $4f\Phi$ lines, and therefore the average e.m.f. generated in the coil is

$$E_{avg} = 4 f n \Phi \, 10^{-8} \text{ volts,} \tag{284}$$

but for a sine wave the ratio of the maximum to the average ordinate is $\frac{\pi}{2}$, and therefore the maximum e.m.f. is

$$E_m = \frac{\pi}{2} E_{avg} = 2\pi f n \Phi \, 10^{-8},$$

and the effective value is as before

$$E = \frac{E_m}{\sqrt{2}} = 4.44 f n \Phi \, 10^{-8}.$$

The e.m.f. generated in an alternator is directly proportional to the frequency f, to the number of turns in series n and to the flux under each pole Φ.

In a two-pole machine one revolution or 360 mechanical degrees corresponds to one cycle or 360 electrical degrees and the frequency is equal to the number of revolutions per second; in a p-pole machine the e.m.f. goes through a complete cycle when the coil moves across a pair of poles and thus through $\frac{p}{2}$ cycles in one revolution. In this case 360 mechanical degrees $= \frac{p}{2} \times$ 360 electrical degrees or one mechanical degree $= \frac{p}{2}$ electrical degrees.

The frequency in a p-pole alternator in cycles per second is

$$f = \frac{p}{2} \times \text{rev. per sec.} \tag{285}$$

239. Form Factor.—If the flux in the air gap is not so distributed as to give a sine wave of e.m.f., the average value of the generated e.m.f. is still given by equation (284),

$$E_{avg} = 4\,fn\Phi\,10^{-8}$$

but the ratio of maximum to average value is not $\frac{\pi}{2}$ and the ratio of effective to maximum value is not $\frac{1}{\sqrt{2}}$.

The effective value of the general alternating wave can be expressed as

$$E = 4\gamma fn\Phi\,10^{-8}\text{ volts,} \qquad (286)$$

where γ is called the form factor of the wave and is defined as the ratio of the effective value to the average value of the ordinate of the wave.

The form factor of a rectangular wave is 1.00 and for all other waves is greater than 1.00. For a sine wave it is

$$\gamma = \frac{E_{eff}}{E_{avg}} = \frac{\frac{E_m}{\sqrt{2}}}{\frac{E_m}{\frac{\pi}{2}}} = \frac{\pi}{2\sqrt{2}} = 1.11.$$

240. Polyphase Alternating-current Generators.—If the armature of an alternator carries two similar windings displaced 90 electrical degrees from one another, Fig. 263, the winding is

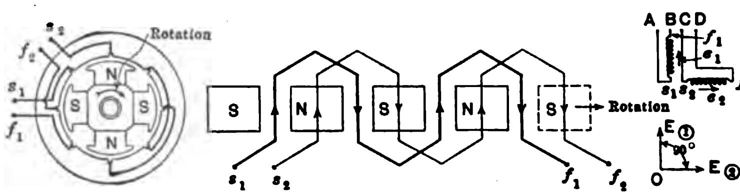


FIG. 263.—Two-phase alternator.

a two-phase winding and the alternator a two-phase alternator. Two e.m.fs. are produced equal in value but displaced 90 degrees in phase.

If the armature carries three similar windings displaced from one another by 120 electrical degrees (Fig. 264), the winding is a three-phase winding and the alternator a three-phase alternator. Three e.m.fs. are produced equal in value but displaced 120 degrees in phase.

If the windings of the three phases start at s_1, s_2 and s_3 and end at f_1, f_2 and f_3, the phases may be interconnected in two ways: (1) join f_1 to s_2, f_2 to s_3 and f_3 to s_1, this is the "delta" connection, Fig. 265; (2) join s_1, s_2 and s_3 together and f_1, f_2 and f_3 to the three terminals, this is the "star" or Y connection (Fig. 266).

For a given number of turns per phase the Y connection gives a higher terminal voltage than the delta connection and a correspondingly smaller current output.

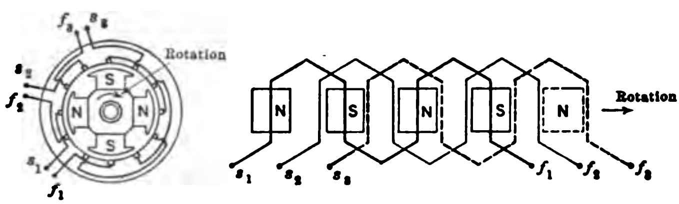


FIG. 264.—Three-phase alternator.

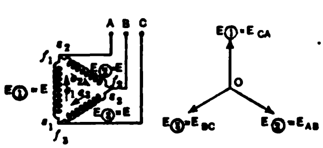


FIG. 265.—Delta connection.

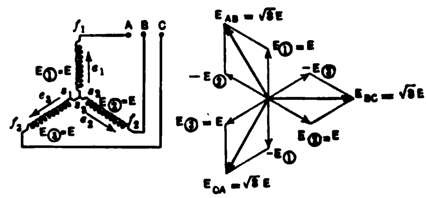


FIG. 266.—Star or "Y" connection.

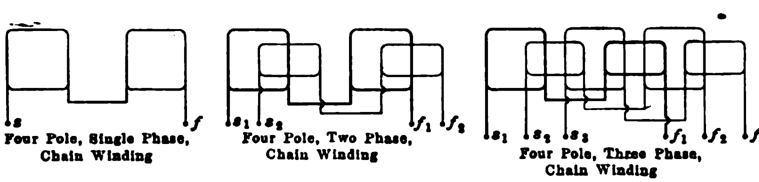


FIG. 267.—Concentrated chain windings.

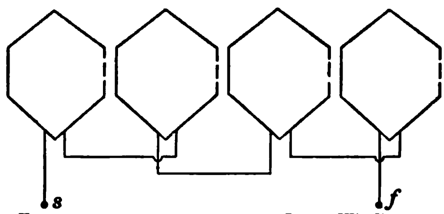
The e.m.fs. currents and power in polyphase circuits are discussed in Art. 124.

The electrical power developed at any instant in a single phase alternator is the product of the instantaneous values of the e.m.f. and current (Fig. 89). It pulsates between a maximum positive value $\frac{E_m I_m}{2}(1 + \cos \phi)$ and a negative value $\frac{E_m I_m}{2}(1 - \cos \phi)$ and its average value is $EI \cos \phi$.

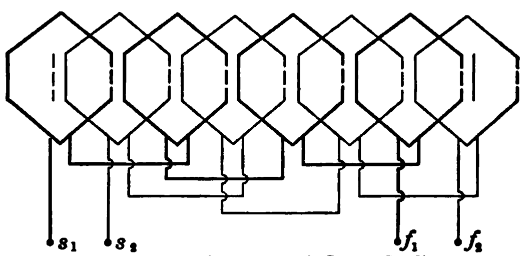
The power developed in a three phase alternator is the sum of the instantaneous powers developed in the three phases; its value

at any instant is $e_1i_1 + e_2i_2 + e_3i_3 = 3EI \cos \phi$ and remains constant.

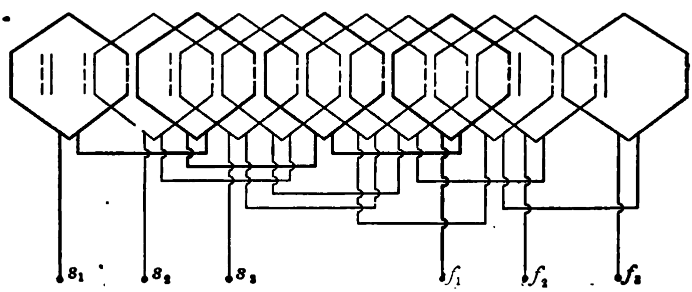
The armature m.m.f. of the single-phase alternator pulsates likewise between nI_m and zero where n is the number of turns on the armature, but it remains fixed in direction relative to the armature and revolves at synchronous speed relative to the poles.



Four Pole, Single Phase, Double Layer Winding



Four Pole, Two Phase, Double Layer Winding



Four Pole, Three Phase, Double Layer Winding

FIG. 268.—Concentrated double-layer windings.

The armature m.m.fs. of the various phases of a polyphase alternator combine to produce a constant m.m.f. of armature reaction fixed in direction relative to the poles and revolving at synchronous speed relative to the armature.

The conditions of operation of a polyphase alternator or synchronous motor are thus much more satisfactory than those of

the single-phase machines. Single-phase machines are built only in the smaller sizes except in special cases as for instance to supply power to a single-phase electric-railway system.

241. Alternator Windings.—There are a great many special alternator windings but the majority of them come under the two classes of chain windings and double-layer windings.

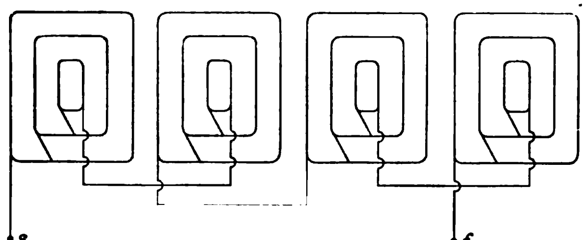


Fig. 269.—Four-pole, single-phase, chain winding distributed in six slots per pole.

Fig. 267 shows four-pole single-, two- and three-phase chain windings for armatures with one slot per phase per pole. These windings are all concentrated windings.

Fig. 268 shows the corresponding double-layer windings.

Fig. 269 shows a four-pole single-phase chain winding distributed in six slots per pole. Fig. 270 shows a winding for the same machine using only four of the six slots per pole as explained in Art. 242.

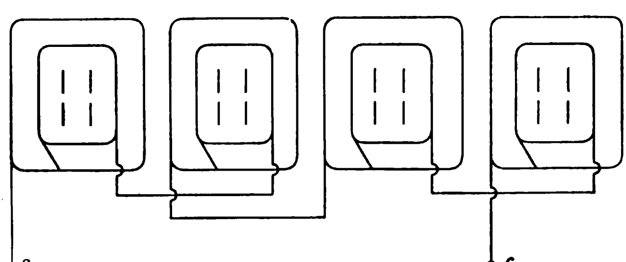


Fig. 270.—Four-pole, single-phase chain winding using only four of the six slots per pole.

Fig. 271 shows a two-phase chain winding for the armature in Fig. 269. The windings are distributed in three slots per phase per pole.

Fig. 272 shows a three-phase chain winding for the same armature, distributed in two slots per phase per pole.

Figs. 273 to 276 show the double-layer windings corresponding to Figs. 269 to 272.

242. Distribution Factors.—The windings in Figs. 267 and 268 are all concentrated windings, that is, they are placed in one slot per phase per pole.

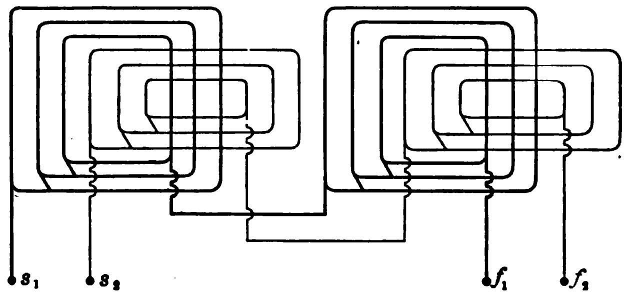


FIG. 271.—Four-pole, two-phase, chain winding distributed in three slots per phase per pole.

When a winding is made up of a number of coils placed in separate slots the e.m.fs. generated in the various coils are displaced in phase and the terminal e.m.f. is less than if the winding had been concentrated. The factor by which the e.m.f. of a concentrated winding must be multiplied to give the e.m.f. of a distrib-

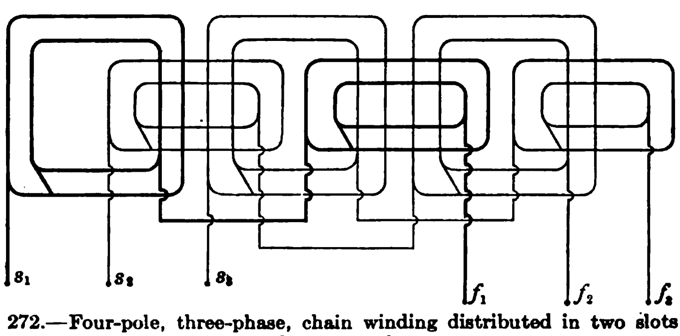


FIG. 272.—Four-pole, three-phase, chain winding distributed in two slots per phase per pole.

uted winding of the same number of turns is called the distribution factor for the winding and it is always less than unity.

When a single-phase winding is distributed in two slots per pole spaced at 90 degrees the e.m.fs. in the two coils are 90 degrees out of phase. If the effective value of the e.m.f. generated

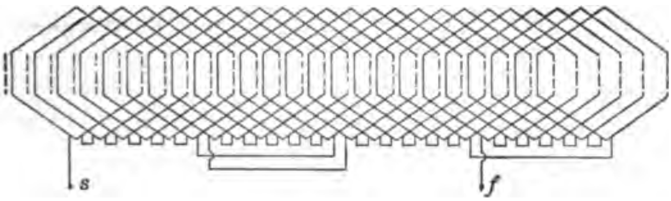


Fig. 273.—Four-pole, single-phase, double-layer winding distributed in six slots per pole.

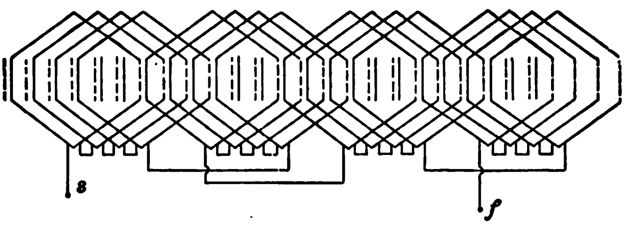


Fig. 274.—Four-pole, single-phase, double-layer winding using only four of the six slots per pole.

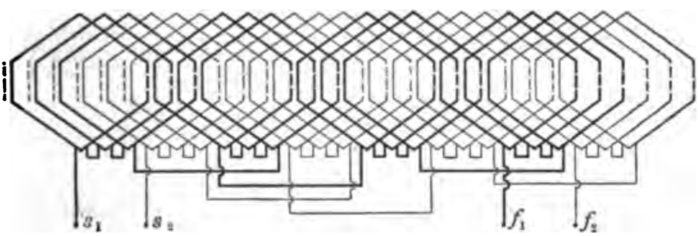


Fig. 275.—Four-pole, two-phase, double-layer winding distributed in three slots per phase per pole.

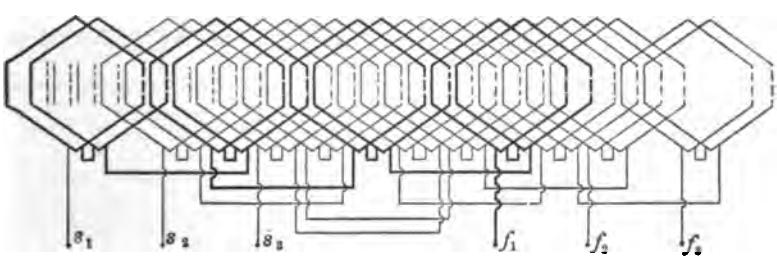


Fig. 276.—Four pole, three-phase, double-layer winding distributed in two slots per phase per pole.

in each coil is e, then the terminal e.m.f. is $e_t = \sqrt{2}e$, Fig. 277, and the distribution factor is

$$\delta = \frac{e_t}{2e} = \frac{\sqrt{2}e}{2e} = 0.707.$$

When a single-phase winding is distributed in three slots per pole spaced at 60 degrees the terminal e.m.f. is the sum of three e.m.fs. e at 60 degrees to one another. It is $e_t = 2e$, Fig. 278, and the distribution factor is

$$\delta = \frac{e_t}{3e} = \frac{2e}{3e} = 0.666.$$

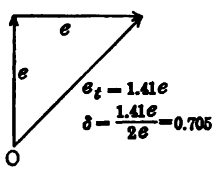


FIG. 277.

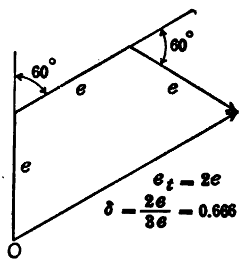


FIG. 278.

When a single-phase winding is distributed in six or more slots per pole the distribution factor may be taken as

$$\delta = \frac{2}{\pi} = 0.64.$$

In Fig. 279 the semi-circumference represents the e.m.f. of the concentrated winding and the diameter represents the e.m.f. of the distributed winding.

The e.m.fs. in the coils from b–c add very little to the terminal e.m.f. and this part of the winding is usually omitted and the terminal e.m.f. is decreased in the ratio $\frac{ob}{oc} = \cos 30° = 0.866$, or is decreased 13.4 per cent. while the resistance and reactance of the winding are decreased $33\frac{1}{3}$ per cent.

The distribution factor for this case is

$$\delta = \frac{ob}{4e} = \frac{3.31e}{4e} = 0.83.$$

Figs. 270 and 274 show four-pole single-phase windings with only four of the six slots per pole used.

The terminal e.m.f. of a two-phase winding distributed in two slots per phase per pole is made up of two e.m.fs. of value e displaced 45 degrees from one another. It is $e_t = 1.848\ e$, Fig. 280, and the distribution factor is

$$\delta = \frac{e_t}{2e} = \frac{1.848e}{2e} = 0.924.$$

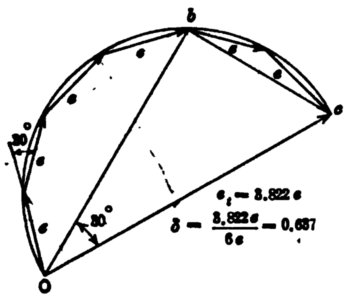


Fig. 279.

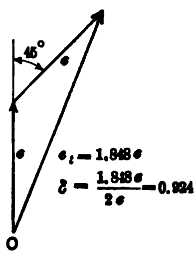


Fig. 280.

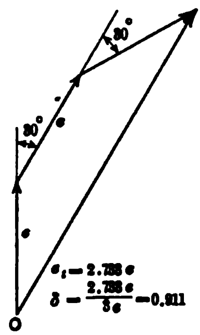


Fig. 281.

For a two-phase winding with three slots per phase per pole, Figs. 271 and 275, the terminal e.m.f. is made up of three e.m.fs. displaced 30 degrees from one another. It is $e_t = 2.733e$, Fig. 281, and the distribution factor is

$$\delta = \frac{e_t}{3e} = \frac{2.733e}{3e} = 0.911.$$

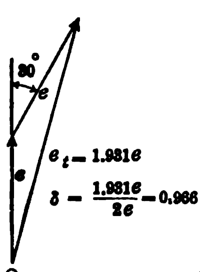


Fig. 282.

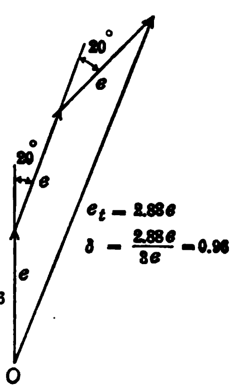


Fig. 283.

The terminal e.m.f. of a three-phase winding distributed in two slots per phase per pole, Figs. 272 and 276, is made up of two

e.m.fs. of value e displaced 30 degrees from one another. It is $e_t = 1.931e$, Fig. 282, and the distribution factor is

$$\delta = \frac{e_t}{2e} = \frac{1.931e}{2e} = 0.966.$$

With three slots per phase per pole the factor is

$$\delta = 0.96, \text{ Fig. } 283.$$

The following table gives the distribution factors for single-, two- and three-phase windings.

Slots per phase per pole	Distribution factor		
	Single-phase	Two-phase	Three-phase
1	1.000	1.000	1.000
2	0.705	0.924	0.966
3	0.666	0.911	0.960
4	0.653	0.906	0.958
6	0.64	0.903	0.956
6	0.83, if only two-thirds of slots are used.		

243. Multiple-circuit Windings.—The windings already discussed are all single circuit, that is, all the turns of one phase are connected in series. In low-voltage machines with a large current output it is necessary to connect the coils forming each phase in multiple circuit. When connected two-circuit the terminal e.m.f. is reduced to one-half and the current output is doubled; the power output, therefore, remains the same.

The e.m.fs. generated in the sections of the windings which are connected in multiple must be of the same value and must be in phase or circulating currents will flow. It is also necessary that the resistances and reactances of the sections be of the same value or one part of the winding will supply more current than the other.

Fig. 284 shows a four-pole three-phase double-layer winding with one slot per phase per pole connected Y and Δ single circuit and two circuit. A winding may be connected with as many circuits in multiple as there are pairs of poles.

244. Short-pitch Windings.—The pitch of a winding is the distance between the two sides of one of the coils forming the winding. When the coil pitch is equal to the pole pitch or the

distance between the centers of adjacent poles, the winding is full pitch. When the coil pitch is less than the pole pitch the winding is fractional pitch or short pitch.

In Fig. 286 *abcdf* shows the distribution of flux under two adjacent poles of an alternator. The area under the section of

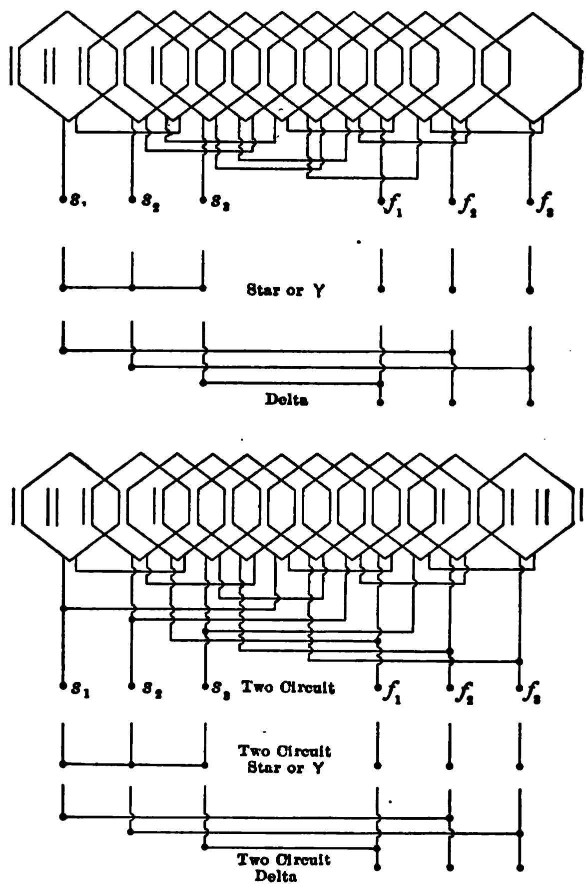


FIG. 284.—Four-pole, three-phase, multiple-circuit windings.

the curve *abc* or *cdf* multiplied by the length of the pole parallel to the shaft gives the flux Φ crossing the air gap under each pole.

As the side g of the coil cuts across the flux in the gap an e.m.f. is generated in it of the same wave shape as the flux distribution. If gh is a full-pitch coil the side h will occupy a position under the adjacent pole similar to that of g and the e.m.fs. generated in the

two sides will be of the same value and wave shape but displaced 180 degrees in phase; they therefore act in the same direction around the coil and add directly to give the terminal e.m.f. If e is the effective value of the e.m.f. generated in one side of the coil the terminal e.m.f. is $E = 2e$. With a full-pitch concentrated winding the wave form of the generated e.m.f. is the same as the wave of flux distribution under the poles.

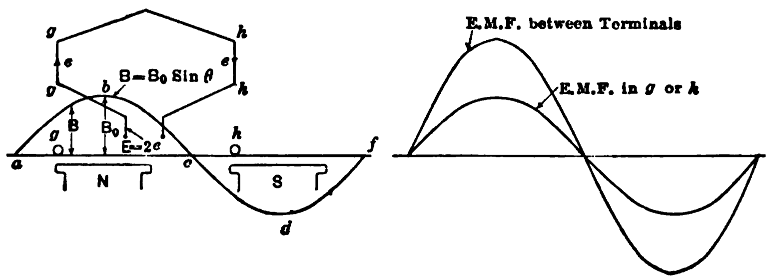


FIG. 285.—Full-pitch coil.

If the coil pitch is less than the pole pitch by an angle α, the e.m.f. wave generated in the side h leads the e.m.f. in g by an angle α, Fig. 285, and the terminal e.m.f. is the vector sum of two e.m.fs. of effective value e displaced in phase by an angle α. It is

$$E = 2e \cos \frac{\alpha}{2}, \tag{287}$$

and is less than the e.m.f. generated in the full-pitch winding in the ratio $\cos \frac{\alpha}{2} : 1$.

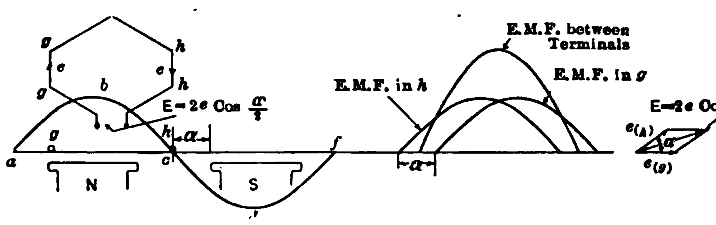


FIG. 286.—Short-pitch coil.

Fractional-pitch windings are sometimes used in order to eliminate certain harmonics from the e.m.f. wave of the generator. Take the case of a machine with the wave of flux distribution, shown in Fig. 287, consisting of a fundamental and a fifth harmonic. With a full-pitch winding the e.m.f. wave would consist

of a fundamental and the prominent fifth harmonic. If, however, the coil pitch is made only 80 per cent. of the pole pitch the e.m.f. in one side of the coil will lead that in the other by 36 degrees and the fifth harmonics in the two sides will be in direct opposition and will disappear (Fig. 288). The terminal e.m.f. will consist only of the fundamental and it will be decreased in ratio cos 18 degrees:1. To eliminate an *n*th harmonic the coil pitch must be either lengthened or shortened by $\frac{1}{n}$th of the pole pitch. Thus the wave form of the e.m.f. generated in a short-pitch winding is not the same as the wave of the flux distribution in the air gap.

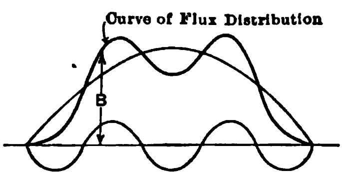


FIG. 287.—Flux wave with fifth harmonic.

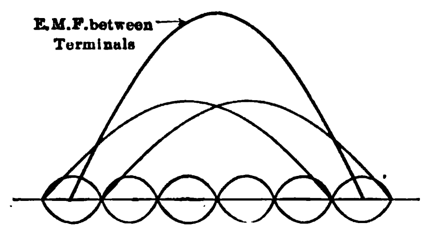


FIG. 288.—Elimination of fifth harmonic.

Similarly the wave form of any distributed winding differs from the wave of flux distribution, since the terminal e.m.f. is the sum of a number of waves displaced from one another. A fully distributed winding gives an e.m.f. wave of approximately sine form at no load regardless of the flux distribution.

245. Effect of Distributing the Winding.—(1) The core is used to better advantage since a number of small slots evenly spaced are used instead of a few large ones. (2) The copper is evenly distributed over the armature surface and thus the copper loss is also distributed and the heat developed by it can more easily be dissipated. A higher current density in the copper can, therefore, be used. (3) The self-inductive reactance is very largely decreased by distributing the winding in a large number of slots, since the coefficient of inductance of a coil is proportional to the square of the number of turns. (4) The terminal e.m.f. is decreased as shown in Art. 242 but the wave form is made more nearly sinusoidal.

246. Harmonics Due to the Teeth.—Fig. 289 shows two positions of the armature of an alternator relative to the field poles. In *A* there are four teeth under the pole and three slots, while in

B there are three teeth and four slots. The reluctance of the magnetic circuit of the generator is a minimum in *A* and the flux in the circuit is a maximum, while in *B* the reluctance is a maximum and the flux is a minimum. If there are *a* teeth or slots per pole the flux per pole pulsates once in the distance of a slot pitch or $2a$ times in two pole pitches. The frequency of the pulsations is $2af$ where f is the normal frequency of the machine.

The flux per pole consists of a constant value Φ and superimposed on it an alternating flux of amplitude Φ_1 and frequency $2af$.

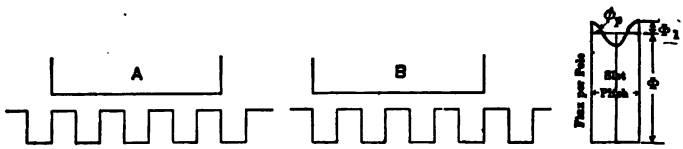


FIG. 289.—Harmonics due to the teeth.

Measuring time from the instant represented in *A* when the flux is maximum, the flux at any time t or angle θ is

$$\phi_p = \Phi + \Phi_1 \cos 2a\theta,$$

and the flux enclosed by the coil at angle θ is

$$\begin{aligned} \phi_p \cos \theta &= \Phi \cos \theta + \Phi_1 \cos 2a\theta \cdot \cos \theta \\ &= \Phi \cos \theta + \frac{\Phi_1}{2}\{\cos (2a + 1)\theta + \cos (2a - 1)\theta\}; \end{aligned}$$

the e.m.f. generated in a coil of N turns concentrated in a single slot is

$$\begin{aligned} e = -N\frac{d}{dt}(\phi_p \cos \theta) &= -N\frac{d}{d\theta}\Big[\Phi \cos \theta + \frac{\Phi_1}{2}\{\cos (2a + 1)\theta \\ &\qquad + \cos (2a - 1)\theta\}\Big]\frac{d\theta}{dt} \\ &= -N\Big[-\Phi \sin \theta + \frac{\Phi_1}{2}\{-(2a + 1) \sin (2a + 1)\theta \\ &\qquad - (2a - 1) \sin (2a - 1)\theta\}\Big]\frac{d\theta}{dt} \end{aligned}$$

but

$$\theta = 2\pi ft \text{ and } \frac{d\theta}{dt} = 2\pi f,$$

and therefore,

$$\begin{aligned} e = 2\pi fN\Big\{\Phi \sin \theta &+ \frac{2a + 1}{2}\Phi_1 \sin (2a + 1)\theta \\ &+ \frac{2a - 1}{2}\Phi_1 \sin (2a - 1)\theta\Big\}, \end{aligned}$$

a fundamental and two harmonics the $(2a + 1)$th and the $(2a - 1)$th. With three slots per pole the tooth harmonics are the seventh and the fifth; and with six slots per pole the harmonics are the thirteenth and the eleventh.

The pulsations of the flux are opposed by the large inductance of the field winding and they are therefore very small and in addition an nth harmonic current is opposed by a reactance in the alternator armature and load circuit which is n times as great as the reactance for the fundamental current.

If the width of the pole face plus a small allowance for fringing is a multiple of the slot pitch the variation of the gap reluctance is negligible and the pulsation of flux does not occur.

247. Effect of Third Harmonics in Three-phase Alternators.—If the e.m.f. per phase in a three-phase delta-connected alternator contains a third harmonic, triple-frequency currents will circulate through the closed delta at all times. Referring to Fig. 129, the resultant e.m.f. around the closed circuit at any instant is $e_1 + e_2 + e_3$ and this was shown to be equal to zero in the case of three similar sine waves displaced at 120 degrees, Art. 123. The third harmonics are, however, not combined at 120 degrees but at $3 \times 120 = 360$ degrees and are in phase and the resultant e.m.f. is three times the magnitude of the third harmonic of one phase and this will cause a third harmonic of current to circulate through the closed winding. This current may be of the order of full-load current in the case of alternators of low reactance. The reactance of the alternator winding to the triple-frequency current is three times that opposed to the current of fundamental frequency but only the true reactance and not the synchronous reactance is effective in limiting the circulating current. The third harmonic of e.m.f. does not appear at the terminals since it is consumed in producing the circulating current.

If the alternator is connected Y, the third harmonic e.m.f. will not appear in the e.m.f. between terminals, since this e.m.f. is the difference of two e.m.fs. at 120 degrees to one another or the sum of two e.m.fs. at 60 degrees, the third harmonics will be combined at $3 \times 60 = 180$ degrees and will therefore neutralize one another. If, however, the neutral is connected to ground at both the generator and receiver ends. A third harmonic of current may flow in the neutral supplied from the three phases.

Alternators should, whenever possible, be connected Y instead of Δ to reduce the danger of circulating currents.

248. General Electromotive Force Equation.—The e.m.f. equation, derived in Art. 239,

$$E = 4\gamma f n \Phi \, 10^{-8} \text{ volts}$$

which applies only to concentrated windings may be extended to include all windings by introducing the distribution factor δ.

Thus the general equation for the effective value of the e.m.f. between terminals of an alternator is

$$E = 4\delta\gamma f n \Phi \, 10^{-8} \text{ volts,} \tag{288}$$

where

f = frequency in cycles per second,
n = number of turns in series between terminals,
Φ = flux from one pole,
γ = form factor of the e.m.f. wave,
δ = distribution factor of the winding.

This equation holds both for the single-phase alternator and for any phase of a polyphase alternator with n turns in series per phase.

If the winding is short pitch the e.m.f. is reduced in the ratio $\cos\frac{\alpha}{2} : 1$ where the coil pitch is $180 - \alpha$ electrical degrees.

249. Rating of Alternators.—Alternators are designed to give a certain terminal voltage and to supply any current up to a certain maximum or full-load current.

The output is

$$P = nEI \cos\theta \text{ watts,}$$

where

E is the voltage per phase,
I is the full-load current per phase,
$\cos\theta$ is the power factor of the load, and
n is the number of phases.

The power output, therefore, depends on the voltage which is a fixed quantity, the current which is variable and is limited by the allowable temperature rise caused by the copper losses and other losses in the machine, and the power factor of the load over which the designer has no control.

Alternators should, therefore, be rated not in watts or kilowatts which depend on the power factor but in volt-amperes or kilovolt-amperes.

A machine rated at 1,000 kva. can supply 1,000 kw. to a non-inductive load at unity power factor or it can supply $1{,}000 \times 0.80 = 800$ kw. to an inductive load of 80 per cent. power factor.

250. Comparative Ratings of an Alternator Wound Single-, Two- and Three-Phase.—Take the case of a machine with six slots per pole. Let e be the effective value of the e.m.f. generated in each coil of the winding and I be the current per conductor. The current will be the same in the three cases for the same temperature rise.

When wound single-phase using all the slots the distribution factor is 0.64 and the terminal e.m.f. is

$$E = 6e \times 0.64$$

and the output is

$$P'_1 = EI \cos \phi = 3.84eI \cos \phi,$$

where $\cos \phi$ is the power factor of the load.

When wound single-phase using only four slots per pole the terminal e.m.f. is

$$E = 4e \times 0.83$$

and the output is

$$P_1 = EI \cos \phi = 3.32eI \cos \phi.$$

When wound two-phase with three slots per phase per pole the distribution factor is 0.91, the e.m.f. per phase is

$$E = 3e \times 0.91$$

and the output is

$$P_2 = 2EI \cos \phi = 5.46eI \cos \phi.$$

When wound three-phase with two slots per phase per pole the distribution factor is 0.96, the e.m.f. per phase is

$$E = 2e \times 0.96$$

and the output is

$$P_3 = 3EI \cos \phi = 5.76eI \cos \phi.$$

Taking the three-phase rating as 100 the comparative ratings are as given below.

Number of phases	Rating
Three-phase	100.0
Two-phase	95.0
Single-phase using all the slots	67.0
Single-phase using only four slots per pole	57.7

In practice an alternator is given the same rating two- and three-phase and 65 per cent. of that rating single-phase.

251. Armature Reaction.—The flux distribution in the air gap of an alternator at no load is symmetrical about the center line of the pole and usually follows approximately a sine wave. The e.m.f. generated in the armature is also a sine wave (see Fig. 285).

When current flows in the armature winding, the m.m.f. of the armature combines with the m.m.f. of the field and changes both

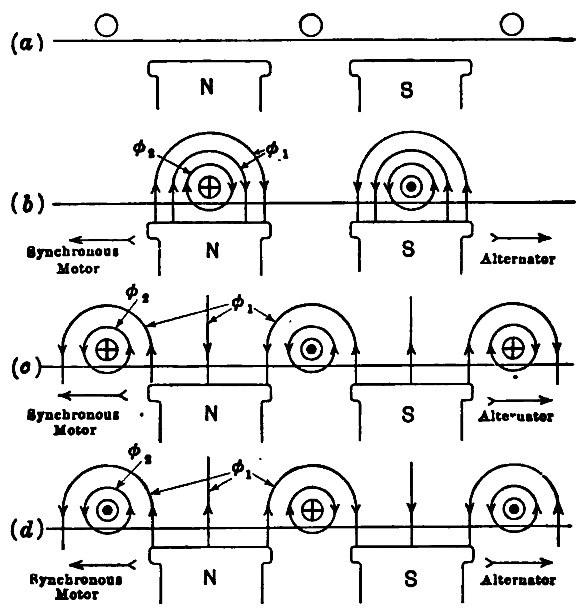


FIG. 290.—Armature reaction and reactance.

the magnitude and distribution of the flux crossing the air gap and cut by the armature conductors. It thus changes both the magnitude and the wave form of the e.m.f. generated. These results are termed the armature reaction.

Armature reaction depends not only on the intensity of the current in the armature but also on its phase relation with the generated e.m.f. Fig. 290 illustrates armature reaction in a machine with a single-phase concentrated winding.

In (*a*) the armature coil is shown in the position of zero e.m.f; if the current is in phase it is also zero.

In (*b*) the e.m.f. is maximum and the current is maximum. The m.m.f. of the armature is cross-magnetizing, that is, it decreases the flux over one-half of the pole and increases it over the other half. The useful flux is only decreased by the small amount lost due to the higher saturation, and, therefore, decreased permeability over the half of the pole where the density is increased. The flux distribution no longer follows a sine wave and the e.m.f. will not be a sine wave.

In (*c*) the current is maximum but lags 90 degrees behind the generated e.m.f. The m.m.f. of the armature acts directly against the m.m.f. of the field. It is, therefore, demagnetizing and decreases the flux but does not distort it.

In (*d*) the current is maximum and leads the e.m.f. by 90 degrees. The m.m.f. of the armature acts directly with the field and magnetizes it. The useful flux is increased and is not distorted.

The following results have been obtained:

1. A current in phase with the generated e.m.f. is cross-magnetizing and only decreases the flux to a very slight extent.

2. A current lagging 90 degrees behind the generated e.m.f. demagnetizes the field and decreases the flux and decreases the generated e.m.f.

3. A current leading the generated e.m.f. by 90 degrees magnetizes the field, increases the flux and increases the generated e.m.f.

If the current lags behind the e.m.f. by angle ϕ, it may be resolved into two components, $I \cos \phi$ in phase with the e.m.f. and, therefore, cross-magnetizing and $I \sin \phi$ lagging 90 degrees behind the e.m.f. and demagnetizing.

The effect of armature reaction increases almost directly with the current until the magnetic circuit becomes saturated after which it increases much faster than the current. Due to saturation the cross-magnetizing effect increases faster than the current; the demagnetizing m.m.f. increases directly with the current and the decrease of flux caused by it would be proportional to the current, if it were not for the change in the leakage factor of the machine. The increase of the leakage factor under load is largely due to the presence of the demagnetizing armature m.m.f. and the resulting decrease of flux must be charged against it.

In salient pole machines the cross-magnetizing m.m.f. acts on a path of much larger reluctance than the demagnetizing m.m.f.

and its effect is correspondingly smaller. In machines with cylindrical rotors the reluctance of the air gap is uniform over the whole periphery and the two m.m.fs. therefore act on similar paths.

252. Armature Reactance.—The flux produced by the current in the armature coil in Fig. 290 may be separated into two parts as shown.

Part (1) is the flux of armature reaction which crosses the gap and interferes with the flux threading the field circuit. Its effect is either cross-magnetizing, demagnetizing or magnetizing.

Part (2) is the flux which only interlinks with the coil itself and does not interfere with the flux produced by the field m.m.f. It is the self-inductive flux of the coil and generates in the coil an e.m.f. of self-inductance, which consumes a component of the e.m.f. generated by rotation. This e.m.f. called the armature reactance drop is equal to the product of the armature current I and the armature reactance x and leads the current by 90 degrees.

The reactance is $x = 2\pi fL$, where L is the inductance of the armature. L and x both decrease as the armature current increases due to the increased saturation and, therefore, decreased permeability of the leakage path surrounding the armature conductors. They also vary as the armature is rotated, when the conductor is under the pole the reluctance of its local leakage

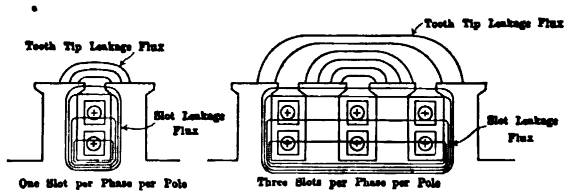


FIG. 291.—Slot leakage flux and tooth-tip leakage flux.

path is minimum and L and x are large, when between the poles the reluctance is maximum and L and x are reduced. An average value of x is chosen to represent the armature reactance. At light loads when the current is small the reactance drop will be greater than the value corresponding to the average reactance and when the current is large it will be smaller.

253. Leakage Fluxes.—The leakage fluxes of armature reactance may be separated into three parts: (*a*) the slot leakage flux, (*b*) the tooth-tip leakage flux, Fig. 291, and (*c*) the end connection leakage flux, Fig. 292.

(a) The slot leakage flux is directly proportional to the number of ampere conductors in the slot, that is, to the total current in the slot; it increases directly with the slot depth and is inversely proportional to the slot width. If the slots are partly closed, the component of the flux across the narrow opening is very large. The slot leakage flux is independent of the position of the slot relative to the pole; due to saturation of the teeth it does not increase directly with the current and therefore the component of the armature reactance due to it decreases with increase of current.

(b) The tooth tip leakage follows paths such as those shown in Fig. 291. It cannot be calculated so easily or so accurately as the slot leakage and it depends to a limited extent on the position of the slot relative to the pole, especially if the air gap is short. This does not apply to machines with cylindrical rotors.

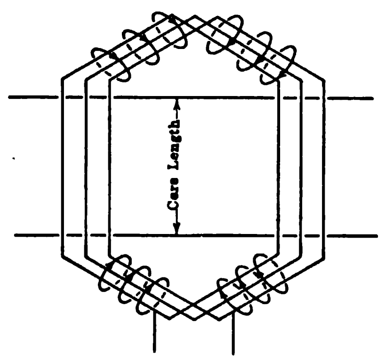


FIG. 292.—End-connection leakage flux.

(c) The end-connection leakage flux follows a path entirely in air. It is proportional to the number of ampere conductors in the phase belt but when the winding is distributed in a number of slots the length of the leakage path is increased. The great length of the end connections makes this component of the leakage flux comparatively large. Since the path never becomes saturated this flux is directly proportional to the current. Distributing the winding in a number of slots per phase per pole decreases all of these leakage fluxes by increasing the length of the leakage paths.

254. Polyphase Armature Reaction.—If n is the number of turns per phase per pair of poles on a two-phase alternator and $i_1 = I_m \cos \theta$ is the current in phase 1 and $i_2 = I_m \cos (\theta - 90°) = I_m \sin \theta$ is the current in phase 2, the m.m.fs. of the two phases are $m_1 = ni_1 = nI_m \cos \theta$ and $m_2 = nI_m \sin \theta$. The two m.m.fs. are in quadrature in time and space but combine to give a constant m.m.f. nI_m fixed in position relative to the field m.m.f. and revolving synchronously backward relative to the armature. This can be seen by reference to Fig. 293. AB is the winding of phase 1 and the current is assumed to lag behind the e.m.f. by

angle ϕ. At the instant represented the current is maximum and the m.m.f. of the coil is maximum and acts in direction OY. The current in phase 2 is now zero. The m.m.f.s. acting are shown in Fig. 294.

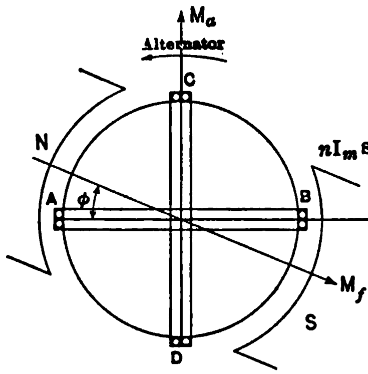


FIG. 293.

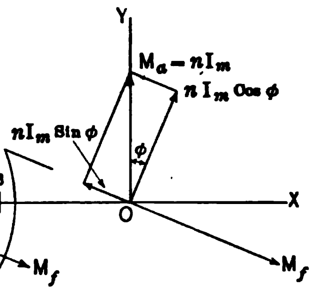


FIG. 294.

In Fig. 295 the coil AB has moved through angle θ and its current has decreased to $I_m \cos\theta$ and its m.m.f. to $nI_m \cos\theta$. The current in coil CD has a value $I_m \sin\theta$ and its m.m.f. is $nI_m \sin\theta$.

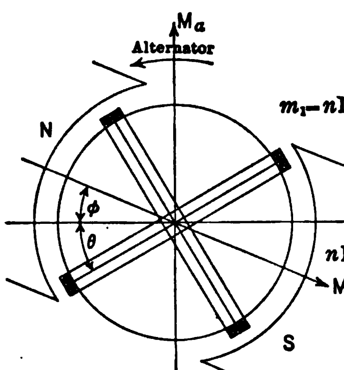


FIG. 295.

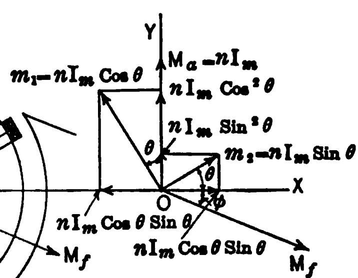


FIG. 296.

The component of the m.m.f. of phase 1 in direction OY is $nI_m \cos\theta \cdot \cos\theta = nI_m \cos^2\theta$ and the component in direction OX is $-\ nI_m \cos\theta \cdot \sin\theta$.

The component of the m.m.f. of phase 2 in direction OY is $nI_m \sin\theta \cdot \sin\theta = nI_m \sin^2\theta$ and the component in direction OX is

$+ nI_m \sin\theta \cdot \cos\theta$. The resultant m.m.f. of the two phases in direction OY is $nI_m \cos^2\theta + nI_m \sin^2\theta = nI_m$ and in the direction OX is $nI_m \cos\theta \cdot \sin\theta - nI_m \cos\theta \cdot \sin\theta = 0$.

Thus the resultant armature m.m.f. is nI_m in fixed direction relative to the field m.m.f. and, therefore, revolving synchronously relative to the armature (Fig. 296).

The direction of the resultant armature m.m.f. relative to the field m.m.f. is determined by the angle of phase difference between the current and the e.m.f. generated at no load.

If the current is in phase with the e.m.f., the armature m.m.f. acts at right angles to the field m.m.f. and is, therefore, cross-magnetizing only; if the current lags by angle ϕ, the armature m.m.f. can be separated into two components $nI_m \sin\phi$ which is demagnetizing and $nI_m \cos\phi$ which is cross-magnetizing (Fig. 294).

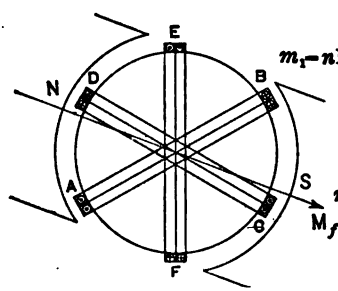


Fig. 297.

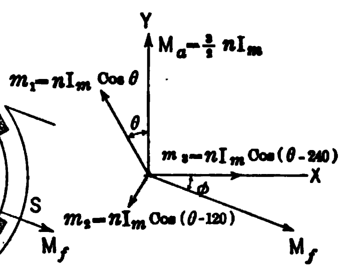


Fig. 298.

If the two-phase winding, Fig. 295, is replaced by a three-phase winding, Fig. 297, with the first phase AB in the same position as before and the other phases CD and EF displaced 120 degrees and 240 degrees from it, the m.m.fs. of the three phases will be respectively $nI_m \cos\theta$, $nI_m \cos(\theta - 120)$ and $nI_m \cos(\theta - 240)$ and will act in the directions represented. As before θ is measured from the instant of maximum current and the currents in the three phases are assumed to lag behind the e.m.fs. by angle ϕ.

The sum of the components of m.m.f. in direction OY is

$$nI_m \cos^2\theta + nI_m \cos^2(\theta - 120) + nI_m \cos^2(\theta - 240) = \tfrac{3}{2} nI_m.$$

and the sum of the components in the direction OX is

$$nI_m \cos\theta \sin\theta + nI_m \cos(\theta - 120)\sin(\theta - 120) + nI_m \cos(\theta - 240)\sin(\theta - 240) = 0.$$

Thus, the resultant m.m.f. of the armature of a three-phase alternator is

$$M_a = \tfrac{3}{2}\, nI_m, \tag{289}$$

where n is the number of turns in series per phase and I_m is the maximum value of the armature current.

The armature m.m.f. is fixed in direction relative to the fields and revolves synchronously relative to the armature (Fig. 298).

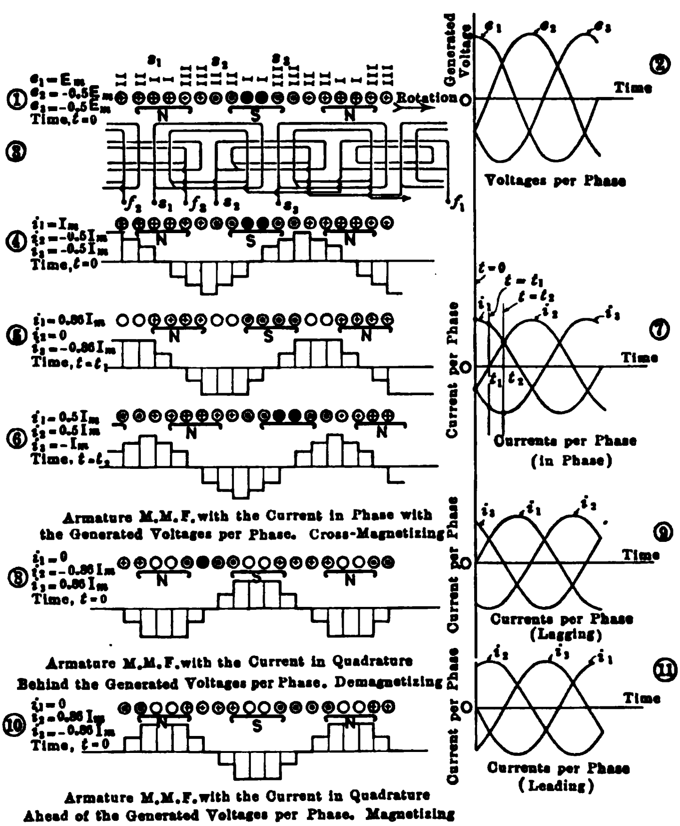


Fig. 299.—Armature m.m.f. of a three-phase alternator.

Fig. 299 shows the same facts in another way in the case of a three-phase alternator with its winding distributed in six slots per pole.

(1) is a section through the conductors indicating the directions and intensities of the e.m.fs. for the time $t = 0$.

(2) shows the e.m.f. waves generated in the three phases.

(3) shows the armature winding.

(4), (5) and (6) represent the armature m.m.fs. corresponding to the three instants, $t = 0$, $t = t_1$ and $t = t_2$ or $\theta = 0$, $\theta = 30$ degrees and $\theta = 60$ degrees, with the currents (7) in phase with the generated e.m.f. The armature m.m.f. is cross-magnetizing; its value represented by the area under the m.m.f. lines is approximately constant and it is fixed in position relative to the poles and therefore revolves relative to the armature.

(8) and (9) show the armature m.m.f. and currents at time $t = 0$, with the currents in quadrature behind the e.m.fs. and therefore demagnetizing.

(10) and (11) show the armature currents leading and the armature m.m.f. magnetizing,

255. Single-phase Armature Reaction.—In a single-phase alternator with n armature turns per pair of poles carrying a current $i = I_m \sin \theta$, the armature m.m.f. varies from a maximum value nI_m to zero; it is fixed in direction relative to the armature and revolves relative to the poles. It produces a double-frequency pulsation of the field and a third harmonic of e.m.f.

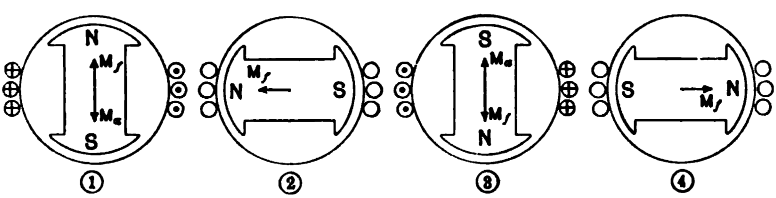


FIG. 300.—Single-phase armature reaction.

Fig. 300 shows four instants during the revolution of a single-phase alternator in which the current is assumed to be in quadrature behind the generated e.m.f. In (1) the e.m.f. is zero and the current is maximum, the armature m.m.f. is demagnetizing and reduces the flux. In (2) the current is zero and the flux has its maximum value. In (3) the current is maximum again and the flux is reduced as in (1). In (4) the current is zero again.

The values of the flux crossing the gap for one revolution or one cycle are shown in Fig. 301. The flux may be separated into two parts, the constant value Φ and the alternating flux of amplitude ϕ_1, which goes through two complete cycles during one cycle of the current or e.m.f. Single-phase armature reaction

thus produces a double-frequency pulsation of the flux crossing the gap.

If time t and angular displacement θ of the rotating field are measured from position (1), the flux inclosed by the coil at this instant may be represented by

$$\phi_0 = \Phi - \phi_1 \cos 2\theta.$$

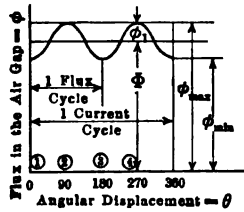


FIG. 301.

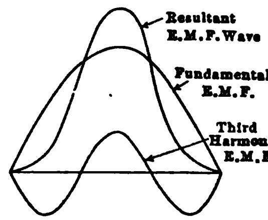


FIG. 302.

At time t and angle θ after position (1) the flux inclosed is

$$\begin{aligned}\phi = \phi_0 \cos\theta &= \Phi\cos\theta - \phi_1 \cos 2\theta \cdot \cos\theta \\ &= \Phi\cos\theta - \frac{\phi_1}{2}(\cos 3\theta + \cos\theta)\end{aligned}$$

and the e.m.f. generated in the coil is

$$\begin{aligned}e &= -n\frac{d\phi}{dt} = -n\frac{d}{dt}\left\{\Phi\cos\theta - \frac{\phi_1}{2}(\cos 3\theta + \cos\theta)\right\} \\ &= n\left\{\Phi\sin\theta - \frac{\phi_1}{2}(3\sin 3\theta + \sin\theta)\right\}\frac{d\theta}{dt},\end{aligned}$$

but $\theta = 2\pi ft$ and $\frac{d\theta}{dt} = 2\pi f$.

and therefore

$$\begin{aligned}e &= 2\pi fn\left\{\left(\Phi - \frac{\phi_1}{2}\right)\sin\theta - \tfrac{3}{2}\phi_1 \sin 3\theta\right\} \\ &= E_m^{\mathrm{I}} \sin\theta - E_m^{\mathrm{III}} \sin 3\theta.\end{aligned}$$

The generated e.m.f. consists of an e.m.f. of fundamental frequency having a maximum value $E_m^{\mathrm{I}} = 2\pi fn\left(\Phi - \frac{\phi_1}{2}\right)$ and a third harmonic of maximum value $E_m^{\mathrm{III}} = 3\pi fn\,\phi_1$; the two e.m.fs. pass through zero together but in opposite directions and the resultant wave is symmetrical with a peak at the center (Fig. 302).

Thus single-phase armature reaction produces a double-frequency pulsation of the field and a third harmonic of e.m.f.

Since the magnetic circuit is surrounded by a field winding, consisting of a large number of turns, the pulsations will be less than the above results indicate. The variation of the flux linking with the field winding induces in it e.m.fs. and currents which oppose the variation and limit it to a small value.

In a machine with a large number of field turns the pulsation produced by armature reaction up to full-load current is very small and the armature reaction may be considered as constant in value with reference to the fields.

In the case of a short-circuit, however, where from three to five times full-load current flows in the armature, a large pulsation of flux is produced in the magnetic circuit and very large e.m.fs. and currents may be induced in the field windings.

If a short-circuit occurs on one phase only of a three-phase alternator, the armature reaction can be separated into the ordinary three-phase armature reaction with equal currents and a single-phase armature reaction due to the excess of the short-circuit current over normal current acting in the turns of one phase. This single-phase armature reaction produces a double-frequency pulsation of the field and a third harmonic of e.m.f. in all the phases.

The effects of single-phase armature reaction are relatively greater in machines with a small number of turns on the fields, as turbo alternators.

256. Electromotive Forces in the Alternator.—In studying the performance of an alternator it is necessary to determine the relation between the terminal e.m.f. E, the e.m.f. E_1 generated by rotation and e.m.f. E_0 generated at no load.

E_1 is the e.m.f. generated in the armature by the rotation of the flux produced in the air gap by the resultant of the m.m.fs. of the field and armature. It is the vector sum of the terminal e.m.f. E and the e.m.f. consumed by the impedance of the armature. The armature impedance is $Z = \sqrt{r^2 + x^2}$, or expressed in rectangular coördinates $\underset{.}{Z} = r + jx$, where r is the resistance of the armature and consumes a component of e.m.f. Ir in phase with the current I, and x is the true self-inductive reactance of the armature and consumes a component of e.m.f. Ix in quadrature ahead of the current.

The generated e.m.f. thus is

$$\begin{aligned} \underset{.}{E}_1 &= \underset{.}{E} + \underset{.}{I}\underset{.}{Z} \\ &= \underset{.}{E} + \underset{.}{I}\,(r + jx), \end{aligned} \tag{290}$$

and the terminal e.m.f. is the vector difference between the e.m.f. generated in the armature by rotation and the impedance drop

$$\dot{E} = \dot{E}_1 - \dot{I}\,(r + jx). \tag{291}$$

E_0 is the e.m.f. generated at no load due to cutting the flux produced by the field m.m.f. M_f acting alone. Under load current flows in the armature and exerts a m.m.f. M_a, which is either cross-magnetizing, demagnetizing or magnetizing depending on the phase relation of the current and the terminal e.m.f. This armature m.m.f. combines with the field m.m.f. and changes both the intensity and the distribution of the flux in the gap, so that under load the e.m.f. generated in the armature is not the same as at no load.

The difference between the two is the e.m.f. consumed or the e.m.f. not generated due to the presence of the armature reaction. This e.m.f. is proportional to the current and can be expressed as the product of the current I and a component of reactance x'. It is Ix' and is in quadrature ahead of the current. Thus

$$\begin{aligned} \dot{E}_0 &= \dot{E}_1 + j\dot{I}x' \\ &= \dot{E} + \dot{I}\,(r + jx) + j\dot{I}x' \\ &= \dot{E} + \dot{I}\,\{r + j\,(x + x')\} \\ &= \dot{E} + \dot{I}\,(r + jx_s). \end{aligned} \tag{292}$$

The total reactance of the armature x_s is called the synchronous reactance and consists of two components, x which represents the effect of the armature leakage flux and which has been called the armature reactance and x' which represents the effect of armature reaction.

The quantity $\dot{Z}_s = r + jx_s$ is the synchronous impedance of the armature and consumes a component of the no-load e.m.f. $I\dot{Z}_s = I\,(r + jx_s)$ which is the synchronous impedance drop in the armature.

Thus the e.m.f. generated at no load is the vector sum of the terminal e.m.f. and the synchronous impedance drop

$$\dot{E}_0 = \dot{E} + \dot{I}\,(r + jx_s).$$

257. Armature Resistance.—The resistance of the armature r, used in calculations for regulation and efficiency, is not the true ohmic resistance as measured by passing direct current through the winding but is from 20 to 100 per cent. greater than this value. This increased resistance is sometimes called the effective resistance of the armature.

The loss of power which is charged against the armature copper is the increase in the total losses in the machine due to the presence of the armature winding carrying current. This loss includes (1) the true I^2r loss due to the passage of the load current, (2) any increase in the iron losses due to the distortion of the flux by the armature currents and (3) eddy-current losses in the armature copper, such as those discussed in Art. 188.

In ordinary cases it is advisable to add about 50 per cent. to the resistance measured by direct current; a slight error will not affect the efficiency of the machine to any great extent since the copper loss is usually considerably less than the iron losses.

258. Vector Diagram of Electromotive Forces and Magnetomotive Forces.—In Fig. 303(a) is shown the vector diagram of an alternator supplying an inductive load.

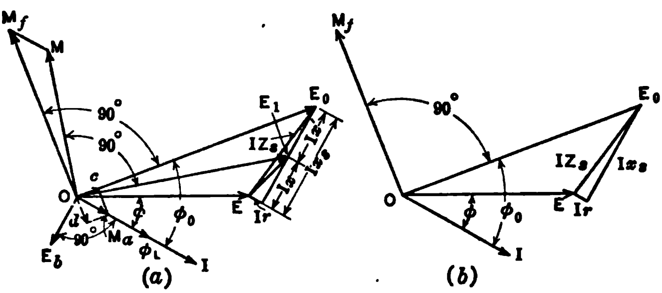


FIG. 303.

E = terminal e.m.f.

$\dot{I}$ = armature current lagging behind E by angle ϕ.

Ir = e.m.f. consumed by armature resistance, in phase with I.

Ix = e.m.f. consumed by armature reactance, in quadrature ahead of the current.

E_b = $-Ix$ = counter e.m.f. of armature reactance due to the armature leakage flux ϕ_L. E_b is in quadrature behind ϕ_L and I.

E_1 = e.m.f. generated by rotation, that is, the e.m.f. generated due to cutting the flux produced in the magnetic circuit by the resultant of the m.m.fs. of field and armature, $E_1 = \dot{E} + \dot{I}\,(r + jx)$.

M_a = armature m.m.f., in phase with I.

M = resultant of the field m.m.f. and armature m.m.f., in quadrature ahead of the e.m.f. E_1 due to it.

M_f = field m.m.f., $\dot{M}_f + \dot{M}_a = \dot{M}$.

E_0 = e.m.f. generated at no load by the field m.m.f. acting alone, in quadrature behind M_f.

$Ix' = \overline{E_1E_0}$ = component of the no-load e.m.f. E_0 consumed by armature reaction.

$Ix_s = Ix + Ix' = I(x + x')$ = e.m.f. consumed by armature synchronous reactance.

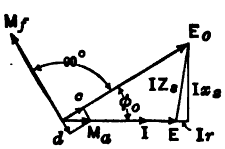
$IZ_s = \overline{EE_0}$ = e.m.f. consumed by synchronous impedance, $\dot{I}\dot{Z}_s = \dot{I}(r + jx_s)$.

ϕ_0 = angle of lag of the current I behind the no-load generated e.m.f. E_0.

$M_a \sin \phi_0$ = demagnetizing component of the armature m.m.f.

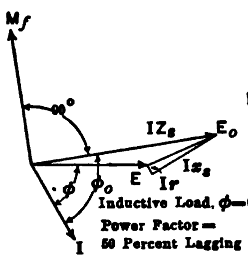
$M_a \cos \phi_0$ = cross-magnetizing component of the armature m.m.f.

This diagram may be simplified to that shown in (*b*).



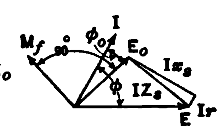
Non-Inductive Load, $\phi = 0$

FIG. 304.



Inductive Load, $\phi = 60°$
Power Factor = 50 Percent Lagging

FIG. 305.



Capacity Load, $\phi = 60°$
Power Factor = 50 Percent Leading

FIG. 306.

In Figs. 304, 305 and 306 are drawn the diagrams for the three cases: (1) $\phi = 0$, or non-inductive load; (2) $\phi = 60$ degrees lag, or inductive load with a power factor of 50 per cent.; and (3) $\phi = 60$ degrees lead, or a capacity load with a power factor of 50 per cent. The same value of armature current is taken in the three cases.

Referring to these diagrams it may be seen that for the same terminal voltage and the same armature current much larger values of E_0 and M_f are required for inductive loads than for non-inductive loads and much smaller values for capacity loads than for non-inductive loads.

While the magnetic circuit of the machine is unsaturated the field excitation required to produce the e.m.f. E_0 is approximately

proportional to it, but higher up on the saturation curve the required excitation increases faster than the e.m.f. The relation between the field current or field m.m.f. and the no-load e.m.f. can be obtained by reference to the no-load saturation curve of the alternator in Fig. 310.

The relation between E_0, E and I can be expressed algebraically as indicated in Fig. 307 which is a reproduction of Fig. 303(b).

$$E_0 = od = \sqrt{\overline{ob}^2 + \overline{bd}^2} = \sqrt{\overline{oa + ab}^2 + \overline{bc + cd}^2}$$
$$= \sqrt{(E\cos\phi + Ir)^2 + (E\sin\phi + Ix_s)^2}.$$

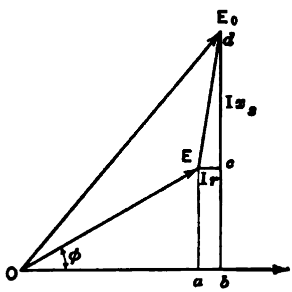


FIG. 307.

The same result may be obtained by expressing the e.m.fs. in rectangular coördinates taking the current I as real axis.

The terminal e.m.f. is

$$\underset{\cdot}{E} = E\cos\phi + jE\sin\phi;$$

the synchronous impedance drop is

$$\underset{\cdot}{I}\underset{\cdot}{Z}_s = Ir + jIx_s;$$

and the no-load e.m.f. is

$$\underset{\cdot}{E} = \underset{\cdot}{E} + \underset{\cdot}{I}\underset{\cdot}{Z}_s = (E\cos\phi + Ir) + j\,(E\sin\phi + 1x_s);$$

or taking the absolute value

$$E_0 = \sqrt{(E\cos\phi + Ir)^2 + (E\sin\phi + Ix_s)^2}. \qquad (293)$$

259. Voltage Characteristics.—The relation between the terminal e.m.f. and armature current of an alternator, with a fixed value of field current and a given load power factor, is called the "regulation curve" or "voltage characteristic" for the given power factor.

When the power factor is unity and the current is in phase with the terminal e.m.f. E, it lags by an angle ϕ_0 (Fig. 304) behind the

no-load e.m.f. E_0 and there is thus a small demagnetizing effect proportional to $I \sin \phi_0$, which decreases the flux and a large cross-magnetizing effect proportional to $I \cos \phi_0$ which changes the distribution of the flux but only decreases it slightly due to saturation. Thus even with non-inductive load the armature reaction causes a decrease in the flux crossing the air gap, and the e.m.f. E_1 generated by rotation is less than the no-load e.m.f. E_0. In addition, the armature reactance x and the resistance r both consume components of e.m.f. proportional to the current.

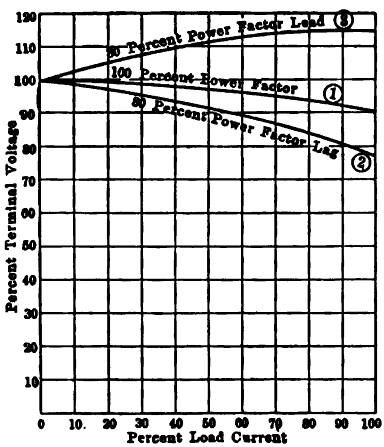


FIG. 308.—Voltage characteristics of an alternator.

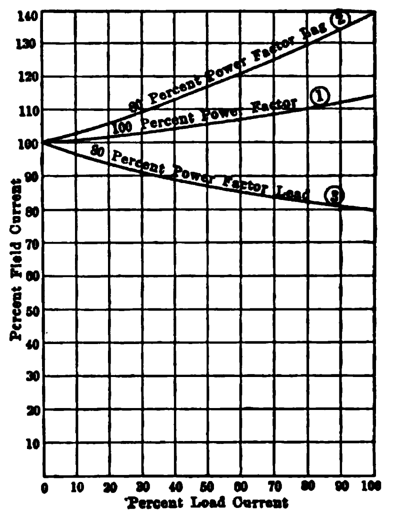


FIG. 309.—Compounding curves of an alternator.

Therefore, at non-inductive load the terminal e.m.f. falls with increasing current as shown in curve (1), Fig. 308, which is the voltage characteristic for unity power factor.

With inductive load the demagnetizing effect is increased and the terminal e.m.f. falls off more, curve (2). With a capacity load in which the current leads the terminal e.m.f. the armature m.m.f. is magnetizing and so raises the terminal e.m.f., curve (3).

These voltage characteristics are calculated from equation (293) on page 315. The value of field current I_f is chosen and the corresponding value of E_0 obtained from Fig. 310. Any required power factor $\cos \phi$ is taken, the current I is varied and the values of E obtained and plotted as ordinates.

260. Compounding Curves.—The "compounding curves" or "field characteristics" show the relation between the field current and armature current for a constant terminal e.m.f. at any required power factor.

Fig. 309 shows the compounding curves for unity power factor, curve (1), 80 per cent. power factor lagging, curve (2), and 80 per cent. power factor leading, curve (3).

At non-inductive load an increase of field current is required as the load current increases to maintain a constant terminal e.m.f.

With inductive load a much larger increase of field current is required to counteract the effect of the lagging current.

With capacity load the field current must be decreased in order to maintain a constant terminal e.m.f.

The same results are shown in the diagrams, Figs. 304, 305, and 306.

These compounding curves can also be calculated from equation

$$E_0 = \sqrt{(E \cos \phi + Ir)^2 + (E \sin \phi + Ix_s)^2}.$$

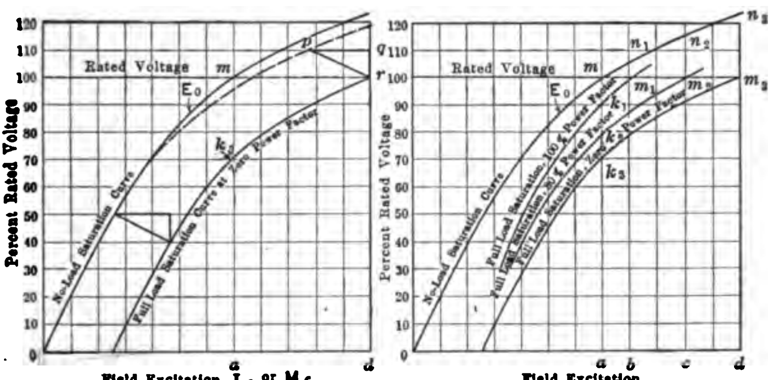


FIG. 310.—Saturation curves.

E remains constant, a certain value of cos ϕ is chosen, I is varied and the value of E_0 is calculated. The corresponding value of I_f is obtained from the saturation curve, Fig. 310, and is plotted on the armature-current base.

261. Tests for the Determination of the Regulation of Alternating-current Generators.—The standards committee of the American Institute of Electrical Engineers suggests the three following methods of determining the regulation of alternating-current generators. They are given in the order of preference.

Method (*a*).—The regulation may be measured directly by loading the generator with the specified load and power factor and then reducing the load to zero. The difference between the voltage readings at no load and full load expressed as a per cent. of the full-load voltage is the regulation under the specified conditions. The two voltage readings must be taken under the same conditions of speed and excitation.

This method cannot be applied generally for shop tests on large generators and either method (*b*) or (*c*) must be used.

Method (*b*).—The regulation is determined from data obtained from the no-load saturation curve and the full-load saturation curve for zero power factor (Fig. 310). The latter curve may be obtained by loading the generator with under-excited synchronous motors, which can be made to give a very low power factor.

From these two curves points on the load saturation curves for any other power factor can be obtained as follows: With

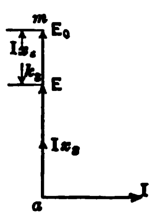


FIG. 311.

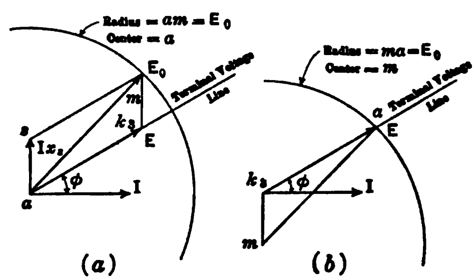


FIG. 312.

excitation *oa* the open-circuit voltage is $am = E_0$ and the terminal voltage with full-load current at zero power factor is ak_3. The drop of voltage is mk_3 and this is the voltage consumed by the synchronous reactance of the armature, since under the condition of zero power factor it is directly subtracted from the open-circuit voltage if the resistance drop is neglected.

Fig. 311 shows the vector diagram for this case.

$E_0 = am =$ no-load or open-circuit voltage.

$I =$ full-load current in quadrature behind E_0.

$Ix_s =$ voltage consumed by the synchronous reactance in quadrature ahead of I.

$E = ak_3 =$ terminal voltage; it is obtained by subtracting $k_3m = Ix_s$ from E_0.

To obtain the corresponding point on the load saturation curve for any other power factor the vector $k_3m = Ix_s$ must be subtracted from E_0 in its proper phase relation.

The diagram in Fig. 312(*a*) may be applied for any power factor. The horizontal line through *a* is taken as the direction of the constant current vector *I* and a circle of radius $am = E_0$ is described about *a*. The terminal voltage line for any power factor cos ϕ makes an angle ϕ with the current line; the voltage consumed by synchronous reactance $as = Ix_s$ is set up in quadrature ahead of *I* and through its extremity *s* a line is drawn parallel to the terminal voltage to cut the circle of radius E_0 at *m*. From *m* the line mk_3 is drawn parallel to *as* to cut the terminal voltage line at k_3. The vector ak_3 then represents the terminal voltage *E* under the given conditions.

This diagram may be replaced by the simplified diagram in Fig. 312(*b*) which gives the same results. The circle of radius E_0 is drawn about *m* as center instead of *a*.

If it is desired to take account of the resistance drop the diagrams in Fig. 312 may be replaced by those in Fig. 313.

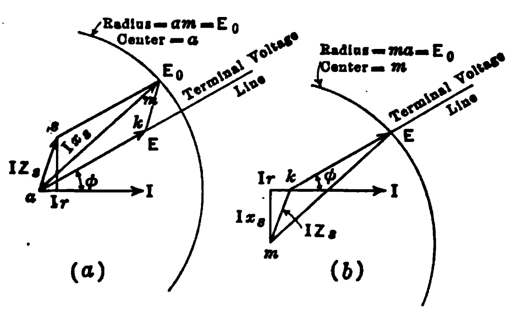


FIG. 313.

In Fig. 310 load saturation curves for unity power factor ($\phi = 0$) and 80 per cent. power factor ($\phi = 36.8$ degrees) are shown.

The values of regulation corresponding to the various power factors and with a fixed field excitation *oa* are as follows:

Regulation at unity power factor $= \frac{mk_1}{k_1a}100$ per cent. $= \frac{8}{92}100$ per cent. $= 8.7$ per cent.

Regulation at 80 per cent. power factor $= \frac{mk_2}{k_2a}100$ per cent. $= \frac{21}{79}100$ per cent. $= 26.5$ per cent.

Regulation at zero power factor $= \frac{mk_3}{k_3a}100$ per cent. $= \frac{30}{70}100$ per cent. $= 43.0$ per cent.

When expressing the regulation of a generator for various power factors it is usual to make the comparison on a basis of equal terminal voltages = rated voltage and not under the condition of fixed excitation.

To obtain these values of regulation the various saturation curves must be extended to cut the rated voltage line.

The regulation at unity power factor $= \frac{n_1m_1}{m_1b} 100$ per cent. = 7 per cent.

The regulation at 80 per cent. power factor $= \frac{n_2m_2}{m_2c} 100$ per cent. = 16 per cent.

The regulation at zero power factor $= \frac{n_3m_3}{m_3d} 100$ per cent. = 25 per cent.

These values could not be obtained from a single vector diagram since the synchronous reactance drop is not constant but decreases as the saturation increases.

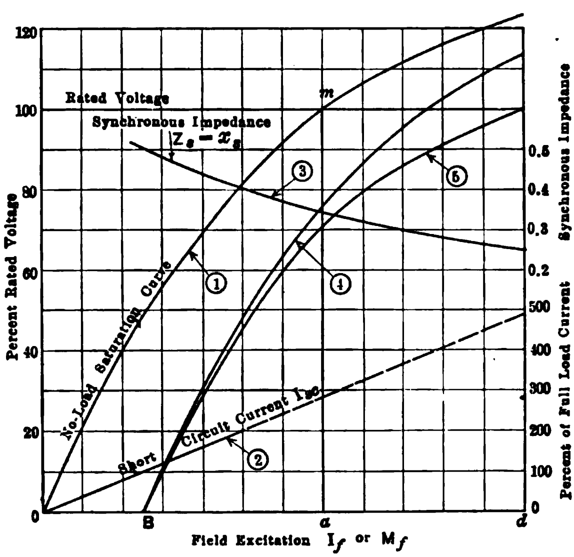


Fig. 314.

Method (*c*).—If it is not possible to obtain the full-load saturation curve at zero power factor from test, it may be constructed from the no-load saturation curve and the short-circuit curve (Fig. 314). The short-circuit curve is obtained by short-circuiting the generator through an ammeter. A very low value of field excitation must be taken at start and gradually increased until the armature current is about twice full-load current. The

curve showing the relation between the armature current and the field current is a straight line passing through the origin and it may be extended to any desired length.

Since the armature is short-circuited and there is therefore no terminal voltage the no-load voltages E_0 (curve (1)) must be consumed as synchronous impedance drops by the armature currents (curve (2)). The ordinates of curve (1) divided by the corresponding ordinates of curve (2) give the values of the synchronous impedance corresponding to the various values of exciting current (curve (3)).

If these values of Z_s or x_s are multiplied by full-load current and subtracted from the ordinates of curve (1), the result is curve (5) which may approximate very closely to the full-load, zero-power-factor, saturation curve. Using this curve the regulation can be obtained as in method (*b*).

In machines of low saturation, low reactance and small magnetic leakage, a curve such as (4) obtained by moving curve (1) to the right through a constant distance OB will be fairly close to the required zero-power-factor saturation curve, but in machines of high saturation large reactance and large magnetic leakage the curve obtained in this way is much too high. This latter is the method suggested by the standards committee.

In Fig. 310 the broken line shows the upper part of a no-load saturation curve calculated using the full-load leakage factor. When this correction is made the horizontal distance between the no-load and the full-load zero-power-factor saturation curves is more nearly constant but is still considerably increased on the upper part of the curve.

The triangle pqr, Fig. 310, has as its horizontal side the line pq which represents the effect of armature reaction expressed in ampere-turns and as its vertical side the line qr representing a voltage drop, which is the true reactance drop Ix. The distance between the two curves measured in the direction pr should be very nearly constant.

262. Regulation.—The voltage regulation of an alternator is expressed as the per cent. rise of the terminal voltage when full load is removed without changing the speed or field excitation. The regulation is usually expressed for 100 per cent. power-factor and 80 per cent. power-factor loads. Usual values are 6 to 9 per cent. at 100 per cent. power factor and 15 to 25 per cent. for 80 per cent. power factor. Where automatic voltage regulators are

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used to maintain constant terminal voltage close regulation is not necessary. In modern machines of large output values of regulation as low as 14 or 15 per cent. at 100 per cent. power factor are considered satisfactory. Such poor regulation indicates a large synchronous reactance which limits the short-circuit current to one and one-half or two times the full-load current and the instantaneous short-circuit current to six or eight times full-load current. This minimizes the stresses on the end turns which are proportional to the square of the current. In many cases reactance coils are connected in series with alternators for the same purpose.

The voltage regulation depends mainly on two factors: (*a*) the ratio of field strength to armature strength, and (*b*) the degree of saturation of the iron parts of the magnetic circuit.

The excitation regulation of an alternator is the per cent. increase of field excitation required to maintain constant terminal voltage as the load increases from zero to full load. In Fig. 310 the excitation regulation at 100 per cent. power factor is $\frac{ab}{oa}$ 100 per cent. = 14 per cent. and at 80 per cent. power factor it is $\frac{ac}{oa} \times 100$ per cent. = 43 per cent.

The extra excitation has three components:

1. That required to counteract the effect of armature reaction especially its demagnetizing component.
2. That required to make up for the voltage drop due to the reactance and resistance of the armature winding.
3. That required to provide for the increase in the magnetic leakage.

The excitation regulation of an alternator under different conditions of load and power factor may be obtained from the compounding curves, Fig. 309, or from to no-load and full-load saturation curves, Fig. 310.

(*a*) Fig. 315(*a*) shows four no-load saturation curves drawn for a machine with its magnetic circuit unsaturated but with the length of the air gap increased by equal amounts from 100 per cent. in (1) to 400 per cent. in (4), the armature strength remaining constant. The field strength increases directly as the air-gap length and the regulation as represented by $\frac{bc_1}{OV}$, $\frac{bc_2}{OV}$ $\frac{bc_3}{OV}$ and $\frac{bc_4}{OV}$ is very much improved. In alternators of ordinary design

constants the field strength is two to three times the armature strength and the air gaps are much longer than in direct-current generators.

Fig. 315(*b*) shows four saturation curves for a machine with a fixed gap but with increasing saturation of the iron parts of the magnetic circuit. The voltage regulation is very much improved by saturation.

Such machines require a minimum of iron in the magnetic circuit and are light in weight but it is difficult to obtain any considerable increase of voltage without overheating the fields.

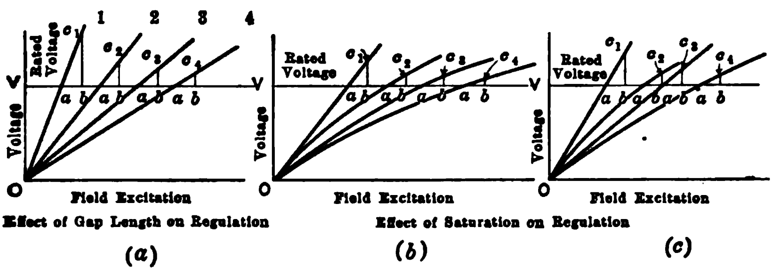


FIG. 315.—Regulation of alternators.

In Fig. 315(*c*), Oc_1 and Oc_3 are saturation curves for two machines with low densities in the iron; the gap in the second one is twice the length of that in the first. Oc_2 and Oc_4 are curves for similar machines with saturated magnetic circuits. The improvement in the regulation due to saturation is not so great in the machine with the long gap as in the other.

263. Short-circuit of Alternators.—If the impedance of the load circuit of an alternator, with normal excitation, is gradually reduced to zero, the short-circuit current is limited only by the synchronous impedance of the armature; it is

$$I_{sc} = \frac{E_0}{Z_s} = \frac{E_0}{\sqrt{r^2 + (x + x')^2}} \tag{294}$$

and ranges from one and one-half to three or four times full-load current, the larger values occurring in machines of low synchronous reactance and good regulation.

In the case of sudden short-circuits the momentary current will be much larger than the permanent value $\frac{E_0}{Z_s}$, because the component x' of the synchronous reactance, which represents

the effect of armature reaction, does not act instantaneously to limit the current. It represents a change in the flux which interlinks with the field circuit of the machine and on account of the inductance of the field winding with its large number of turns, this change of flux cannot take place instantaneously but may take several seconds to become complete. The initial wave of current is limited only by the true impedance of the armature and is

$$I'_{sc} = \frac{E_0}{\sqrt{r^2 + x^2}}. \qquad (295)$$

The amplitude gradually decreases as the armature reaction reduces the flux in the field and the generated e.m.f. E_0. With very large armature currents the path of the armature reactance flux becomes saturated and x is reduced below its normal value.

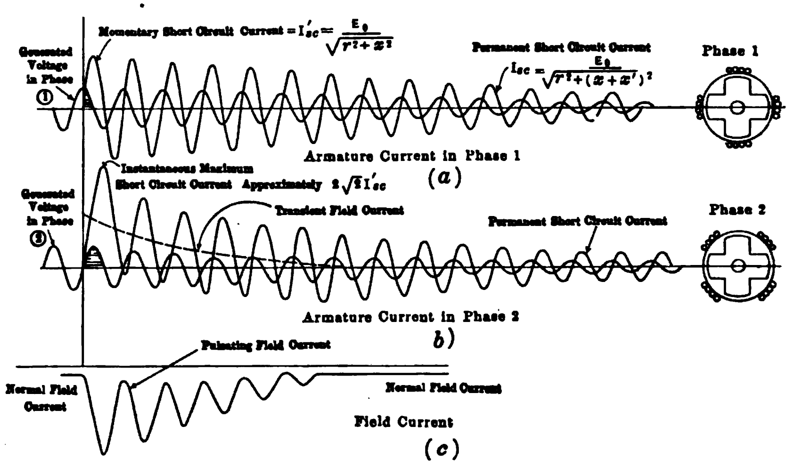


FIG. 316.—Short-circuit currents in a two-phase alternator.

$I'_{sc} = \dfrac{E_0}{\sqrt{r^2 + x^2}} = \dfrac{E_0}{x}$ will represent the short-circuit current only when the short-circuit occurs at the maximum point of the generated e.m.f. wave as illustrated in Fig. 316(a). Neglecting the armature resistance the generated e.m.f. is consumed by the back e.m.f. of armature inductance $= -L\dfrac{di}{dt}$. The cross-hatched area under the e.m.f. wave is $\displaystyle\int_0^{\sqrt{2}E_0} edt$ and is $= \displaystyle\int_{\sqrt{2}I'_{sc}}^{0} -L\frac{di}{dt}dt$ $= \sqrt{2}I'_{sc}L$ and it causes the current to increase from 0 to

$\sqrt{2}I'_{sc}$. The succeeding quarter wave of e.m.f. is negative and reduces the current to zero again; it then reverses and grows in the opposite direction to a value slightly less than before since the armature reaction will have caused a slight reduction in the flux and the generated e.m.f. The current is practically symmetrical about the zero line and its amplitude decreases until the reduction of flux by armature reaction is complete after which it remains constant. The transition from the initial to the permanent value will take from a few hundred to thousands of cycles depending on the inductance of the field winding. In Fig. 316 it is represented as covering only a few cycles.

Fig. 316(*b*) shows the short-circuit current in the second phase of the machine. The short-circuit occurs at the zero point of the generated e.m.f. wave and the positive area under the wave is twice as great as before and builds the current up to double the value in (*a*). The current then decreases and it alternates not about the line of zero current as before but about a line starting above the zero line by an amount $\sqrt{2}I'_{sc}$ on the current scale and sloping down to coincide with the zero line after a few cycles.

The armature current may be considered as being composed of a transient direct current, shown as a broken line, and, superimposed on it, the alternating current I'_{sc} with its decreasing amplitude.

The m.m.f. of the transient armature current, being fixed in position on the armature, revolves relative to the field poles and produces a single-frequency pulsation of the field flux which dies out with the passing of the transient current. This pulsation of the field causes the generated e.m.f. to become unsymmetrical about the zero line as shown and so after the first cycle the armature current crosses the zero line and after a few cycles becomes symmetrical.

The magnitude of the transient armature current depends on the point of the voltage wave at which the short-circuit takes place and therefore the short-circuit currents in the various phases of a polyphase alternator will not be identical.

In machines of high self-inductance and low armature reaction the ratio $\dfrac{I'_{sc}}{I_{sc}} = \dfrac{x + x'}{x}$ will be small but in large low-frequency turbo-alternators with low self-inductance and high armature reaction the ratio may be very large In extreme cases the instantaneous short-circuit current may reach twenty to thirty

times full-load current. Machines should be designed so that the instantaneous short-circuit current will not exceed five to eight times full-load current or if this is not possible external reactance must be added to limit the currents.

Circuit breakers are provided with time limits so that they will not open the circuit until the current has fallen to its steady value.

The pulsation of the field flux due to the m.m.f. of the transient armature current induces in the field winding an alternating current of normal frequency which opposes the pulsation of the flux; this pulsating field current is superimposed on the normal field current giving the resultant current shown in Fig. 316(*c*). In some cases the maximum field current may be ten to fifteen times its normal value.

If the field poles are solid or if they are provided with damper windings, the alternating current instead of appearing in the field winding may appear as eddy currents in the pole faces and dampers and the field current will scarcely be affected at all.

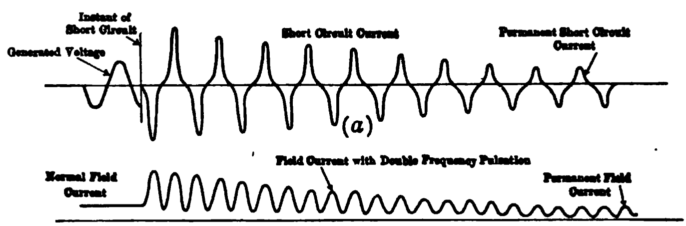


FIG. 317.—Single-phase short circuit currents.

Fig. 317(*a*) shows the short-circuit current of a single-phase alternator in which the circuit was closed at the maximum point of the voltage wave. There is no transient term and the current falls off from the large initial value to the permanent value as in the case of the polyphase machine. The current wave has, however, a decided peak showing the presence of a third harmonic due to the double-frequency pulsation of the field current as shown in Fig. 317(*b*). This phenomenon was discussed in Art. 255. If the circuit is closed at any other point on the voltage wave the current will have a transient term and the field current will have a single-frequency pulsation in addition to the double frequency pulsation; the single-frequency pulsation disappears after a few cycles while the double-frequency pulsation decreases

from a maximum at the first instant to its permanent value. Single-phase generators should always be provided with dampers which reduce the pulsation of the field current and the resulting third harmonic.

A short-circuit of one phase of a polyphase alternator is similar in its effects to a short-circuit of a single-phase alternator but in addition a third harmonic appears in the voltages of the phases not short-circuited.

264. Synchronous Motor.—A synchronous motor is similar to an alternator in construction and may be either single-phase or polyphase. The single-phase motor is not self-starting and must

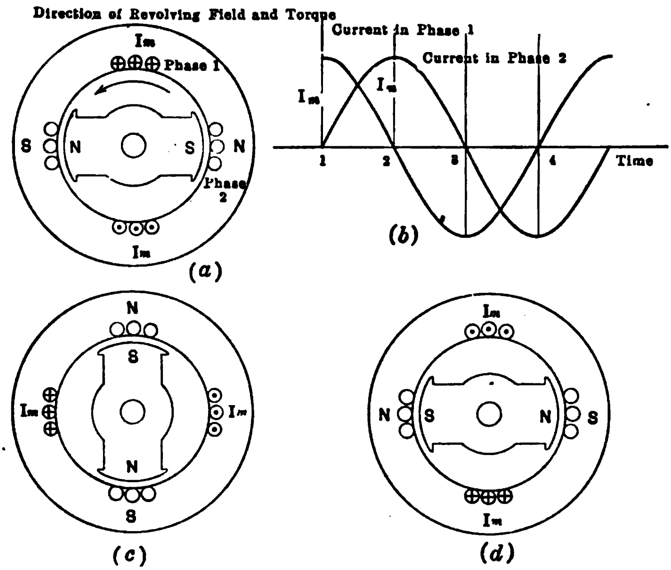


FIG. 318.—Two-pole, two-phase synchronous motor.

be brought up to synchronous speed before being connected to the supply. It is, therefore, not used except in special cases. The polyphase motor when connected to the supply will accelerate and run up to synchronous speed but only a low voltage should be impressed on it at start or very large lagging currents will be drawn from the supply lines. In many cases, however, polyphase synchronous motors are started by auxiliary motors to prevent voltage fluctuations on the line.

Fig. 318(a) represents a two-phase, two-pole motor. The

armature is stationary and is supplied with two-phase alternating currents, Fig. 318(*b*). The armature m.m.f. is constant in value as in the alternator and revolves at synchronous speed in the anti-clockwise direction and produces a revolving field of constant value. Figs. 318(*a*), (*c*) and (*d*) represent the armature m.m.f. at the instants (1), (2) and (3).

The speed of the field is directly proportional to the frequency of the impressed e.m.f. and inversely proportional to the number of pairs of poles; it is

$$n = \frac{f}{\frac{p}{2}} = \frac{2f}{p} \text{ rev. per sec.} \qquad (296)$$

This is the speed at which the motor operates and it is constant independent of the impressed e.m.f. of the field excitation and of the load.

265. Vector Diagrams for a Synchronous Motor.—If an alternating e.m.f. E is impressed on the terminals of a synchronous motor of resistance r and synchronous reactance x_s and a current I flows in the armature, the phase relation of the current and the impressed e.m.f. depends on the field excitation.

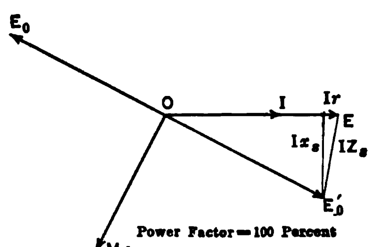


FIG. 319.

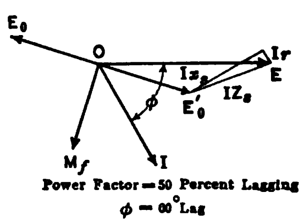


FIG. 320.

In Fig. 319:

E = impressed e.m.f., which remains constant.

I = armature current, in this case, in phase with E.

Ir = component of the impressed e.m.f. consumed by the resistance, in phase with the current.

Ix_s = component of the impressed e.m.f. consumed by the reactance x_s, in quadrature to the current.

IZ_s = component of the impressed e.m.f. consumed by the synchronous impedance, $IZ_s = I(r + jx_s)$.

E'_0 = component of the impressed e.m.f. consumed by the counter e.m.f. of the motor.

E_0 = counter e.m.f. of the motor, generated in the armature by cutting the flux produced by the field m.m.f.

M_f = field m.m.f. in quadrature ahead of the generated voltage E_0. This is the value of field excitation required to make the power factor of the motor unity under the assumed conditions of load.

Fig. 320 is the vector diagram when the current has the same value as before but lags behind the impressed e.m.f. by angle ϕ = 60 degrees.

Fig. 321 is the vector diagram when the current leads by angle ϕ = 60 degrees.

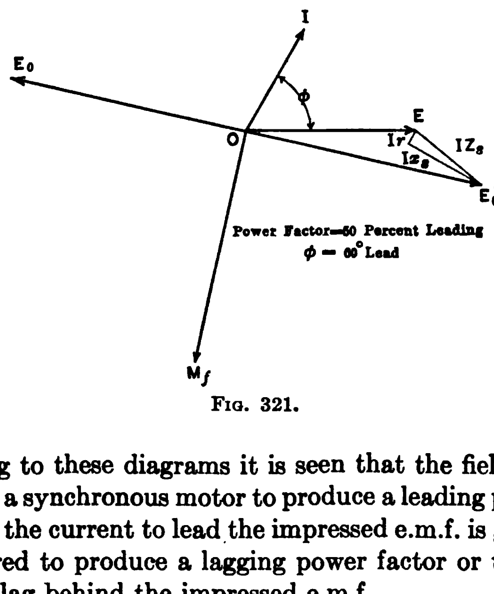


FIG. 321.

Referring to these diagrams it is seen that the field excitation required in a synchronous motor to produce a leading power factor or to cause the current to lead the impressed e.m.f. is greater than that required to produce a lagging power factor or to cause the current to lag behind the impressed e.m.f.

If, therefore, the field current of a synchronous motor is varied, there is no change in speed as in the direct-current motor, but the generated e.m.f. E_0 changes both its value and its phase relation with the impressed e.m.f. E and allows leading or lagging currents to flow to make up for the change in excitation; when the field current is decreased a component of current 90 degrees behind the impressed e.m.f. flows in the armature and magnetizes the field

and when the field current is increased a component of current 90 degrees ahead of the e.m.f. flows and demagnetizes the field.

266. Characteristic Curves.—The most important characteristic curves of the synchronous motor are (1) the compounding curves, or the relation between field current and armature current for given values of power factor, (2) the load characteristics showing the relation between armature current and output and power factor and output for a given value of field excitation, and (3) the phase characteristics or V curves showing the relation between armature current and field current for given values of motor output.

To predetermine these curves it is necessary to know the resistance r and synchronous reactance x_s of the armature and to have the no-load saturation curve showing the relation between the generated voltage E_0 and the field current I_f or field m.m.f. M_f.

The impressed e.m.f. E is constant.

267. Compounding Curves.—The voltage equation of the synchronous motor is

$$\dot{E} = E'_0 + \dot{I}\dot{Z}_s \quad (297)$$

or

$$E'_0 = \dot{E} - \dot{I}\dot{Z}_s \quad (298)$$

If the current I is taken as the line of reference, the impressed e.m.f. can be expressed in rectangular coördinates as

$$\dot{E} = E \cos \phi + jE \sin \phi,$$

where ϕ is the angle of lag of the current.

The impedance drop in the armature is

$$\dot{E}_1 = \dot{I}\dot{Z}_s = Ir + jIx_s$$

The e.m.f. consumed by the generated e.m.f. is, therefore,

$$E'_0 = (E \cos \phi - Ir) + j(E \sin \phi - Ix_s) \quad (299)$$

and its absolute value is

$$E'_0 = \sqrt{(E \cos \phi - Ir)^2 + (E \sin \phi - Ix_s)^2} \quad (300)$$

This relation can also be obtained by reference to the vector diagram in Fig. 322.

$$E'_0 = \sqrt{\overline{oa}^2 + \overline{ad}^2}$$

but

$$oa = ob - ab = E \cos \phi - I_r,$$

and

$$ad = bf - cf = E \sin \phi - Ix_s.$$

therefore,

$$E'_0 = \sqrt{(E \cos \phi - Ir)^2 + (E \sin \phi - Ix_s)^2}$$

Fig. 327 represents the compounding curves for unity power factor, 80 per cent. power factor leading and 80 per cent. lagging.

To predetermine these curves the impressed e.m.f. E is maintained constant, a definite value of power factor is chosen for each curve, the armature current I is varied and the values of E'_0 are calculated from equation (300). The values of field current I_f corresponding to the calculated values of E'_0 are obtained from the saturation curve, Fig. 325, and are plotted as ordinates.

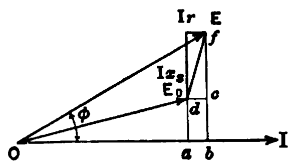


FIG. 322.

268. Load Characteristics.—The power input to the motor armature is the product of the current and the in-phase component of impressed e.m.f.; it is

$$P_1 = EI \cos \phi. \tag{301}$$

The electrical power transformed into mechanical power is the product of the current and the in-phase component of the e.m.f. E'_0, consumed by the generated e.m.f.; it is

$$P = I(E \cos \phi - Ir) = EI \cos \phi - I^2r, \tag{302}$$

and is less than the power input by the armature copper loss.

The power output is less than the mechanical power developed by the amount of the constant losses in the motor, namely, the iron, friction and windage losses; the output, therefore, is

$$\begin{aligned} P_2 &= P - \text{constant losses} \\ &= P_1 - I^2r - \text{constant losses} \\ &= EI \cos \phi - I^2r - \text{constant losses}. \end{aligned} \tag{303}$$

Fig. 328 represents the load characteristics for a given value of field current I_f. The value of I_f chosen here is such that the back e.m.f. of the motor is greater than the impressed e.m.f. and the motor is over excited at light load. If a lower value of I_f had been chosen the power factor would never have reached 100 per cent. and would have been lagging through the whole range of load. Since I_f is constant E'_0 is constant. I is varied and the corresponding values of $\cos \phi$ are obtained from equation 300. These values are substituted in equation 303 and the values of I and $\cos \phi$ are plotted on a base of power output.

At light loads the power factor is low and leading and the motor

is over-excited; as the load is increased the power factor increases until it reaches 100 per cent. at a value of load depending on the field excitation; beyond this point the power factor decreases again and becomes lagging. The current increases continually but is finally limited by the synchronous impedance of the armature.

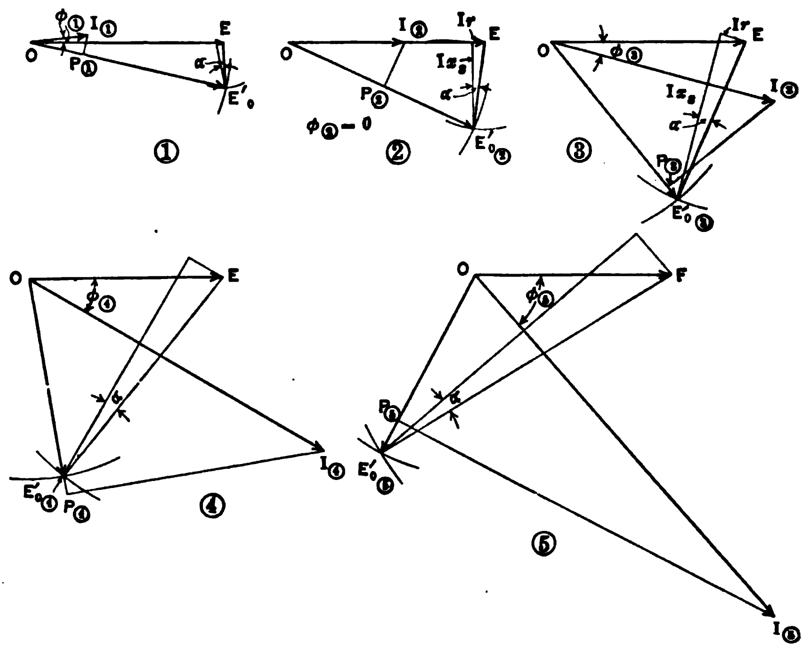


FIG. 323.

These results may be verified by referring to the diagrams in Fig. 323. In (2) the current I is in phase with the impressed e.m.f. E, that is, the power factor is 100 per cent. and $\phi = 0$. In (3) the power output is increased and the current is assumed to have increased to double the value in (2); the synchronous impedance drop is also doubled; E'_0 remains constant in all cases and its position can be found by drawing arcs of two circles of radii E'_0 and IZ_s about the two ends of the vector E. The synchronous impedance drop can be resolved into its two components Ir and Ix_s by setting off the constant angle $\alpha = \tan^{-1} \frac{r}{x_s}$. The current I is then drawn in the direction of Ir and it lags behind E by the

angle ϕ. The power factor is $\cos \phi$ and is lagging. The power output, neglecting the constant losses, is equal to the product of the current and the in-phase component of the e.m.f., E'_0, consumed by the generated voltage E_0 or it is equal to the product of the e.m.f., E'_0, and the component of the current in phase with it, and since E'_0 is constant the power output may be represented by the in-phase component of the current OP in each of the diagrams.

In (1) the power is decreased and the current is taken as half of that in (2); the position of E'_0 is found as before and the current leads E by the angle ϕ.

The power output cannot increase indefinitely but reaches its maximum when the decrease in power factor overcomes the increase in current or when the component of the current in phase with the generated e.m.f. begins to decrease; the motor then becomes unstable and falls out of synchronism and stops. In (4) the current is increased to three times the value in (2) its component OP in phase with E'_0 is greater than in (3) and therefore the power output is still increasing. In (5) the current is four times its value in (2) but the component OP in phase with E'_0 is less than in (4) and the output is therefore decreasing. The position of maximum output lies between (4) and (5).

The maximum power output of a synchronous motor is far beyond the limits set by temperature rise.

269. Phase Characteristics.—If the field excitation of a motor with constant output is varied, the armature current changes both its value and its phase relation with the impressed e.m.f. For each output there is a certain value of field excitation which makes the current a minimum and brings it in phase with the e.m.f.; as the excitation is decreased below this value the current increases and becomes lagging; as the excitation is increased the current increases and becomes leading.

In Fig. 329(a) are shown the phase characteristics for outputs, $P_2 = 0$ or at no load, $P_2 =$ full load and $P_2 =$ twice full load.

For each curve the output $P_2 = EI \cos \phi - I^2r -$ constant losses is kept constant; thus,

$$\cos \phi = \frac{P_2 + I^2r + \text{constant losses}}{EI} \tag{304}$$

$$E'_0 = \sqrt{(E \cos \phi - Ir)^2 + (E \sin \phi - Ix_s)^2}.$$

As I varies the corresponding value of $\cos \phi$ is found from equation (304) and by substituting I, $\cos \phi$ and $\sin \phi$ in equation (300),

the values of E'_0 are found. These are replaced by the corresponding values of field current I_f obtained from the no-load saturation curve.

The lowest point on each curve represents the smallest current input for the given output and thus represents the condition of unity power factor. The curve joining these lowest points is the compounding curve for unity power factor. If the phase characteristics are very steep a slight change in field excitation produces a large change in armature current, or a large component of wattless current is required to correct for a slight variation in field excitation. This is the case in a motor with small synchronous reactance or small armature reaction and the motor is unstable. If the synchronous reactance is large only a slight change in armature current is produced by a change in field excitation and the phase characteristics are flat and the motor is stable.

270. Torque.—Since the speed of the synchronous motor is constant, the torque developed at any output is directly proportional to the output.

If T is the torque in pounds at 1-ft. radius developed in the armature, the electrical power transformed into mechanical power is

$$P = \frac{2\pi \times \text{r.p.m.} \times T}{33{,}000} \times 746 \text{ watts} = EI \cos \phi - I^2r,$$

and thus the torque developed is

$$T = \frac{EI \cos \phi - I^2r}{2\pi \times \text{r.p.m.}} \cdot \frac{33{,}000}{746} \text{ lb.-ft.} \tag{305}$$

and the torque available for the load is

$$T_2 = \frac{EI \cos \phi - I^2r - \text{constant losses}}{2\pi \times \text{r.p.m.}} \times \frac{33{,}000}{746} \text{ lb.-ft.} \tag{306}$$

271. Blondel Diagram for a Synchronous Motor.—In Fig. 324 the triangle OAB is a voltage diagram for a synchronous motor. $OA = E$ is the constant impressed voltage, $AB = E_o$ is the back voltage of the motor and $OB = E' = IZ_s$ is the synchronous impedance drop due to current I. The angle $OAB = \alpha$ is called the coupling angle.

The current I lags behind E by angle ϕ and behind E' by angle $\theta = \tan^{-1} \frac{x_s}{r}$.

OC and AC are two straight lines making equal angles θ with the ends of the constant impressed e.m.f. vector OA.

The current may be represented by the vector OB if its scale is Z_s times the voltage scale and the phase relation of the current and the impressed e.m.f. is found as the inclination of OB to OC which is equal to the phase angle ϕ. The component of the current in phase with the impressed e.m.f. is Oa, the projection of OB on OC.

For constant power input to the motor this in-phase current Oa must remain constant and the locus of B is therefore a straight line through a perpendicular to OC.

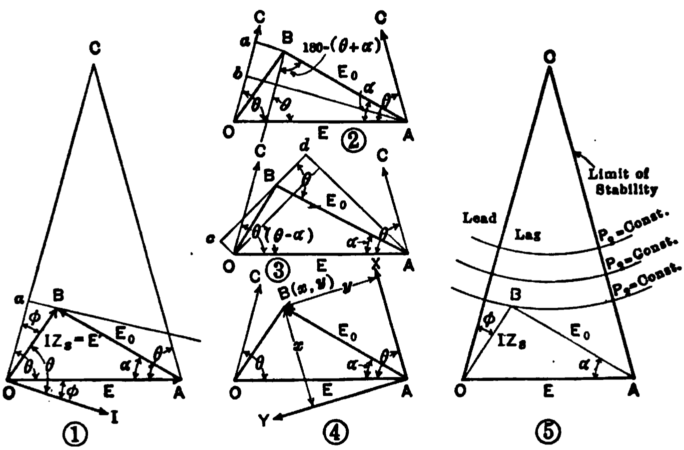


FIG. 324.—Blondel diagram.

The current Oa may be expressed in terms of E and E_0 by dropping a perpendicular from A on OC at b, Fig. 324 (2), remembering that voltage values are changed to current values by dividing by Z_s.

$$Oa = Ob + ba = \frac{E}{Z_s} \cos \theta + \frac{E_0}{Z_s} \cos \{180 - (\theta + \alpha)\}$$

$$= \frac{E}{Z_s} \cos \theta - \frac{E_0}{Z_s} \cos (\theta + \alpha)$$

and the power input is

$$P_1 = E \times Oa = \frac{E^2}{Z_s} \cos \theta - \frac{EE_0}{Z_s} \cos (\theta + \alpha) \tag{307}$$

A similar expression for the power output may be obtained by drawing the line cBd making angle θ with AB, Fig. 324(3) and dropping perpendiculars from O and A on it.

The component of current in phase with the generated voltage is

$$cB = cd - Bd = \frac{E}{Z_s} \cos (\theta - \alpha) - \frac{E_o}{Z_s} \cos \theta$$

and the power output is

$$P_2 = E_0 \times cB = \frac{EE_0}{Z_s} \cos (\theta - \alpha) - \frac{E_0^2}{Z_s} \cos \theta. \tag{308}$$

It is now required to find the equation of the locus of B which will make P_2 constant as the excitation and therefore the generated voltage E_0 varies and the current varies.

Take two rectangular axes, Fig. 324(4), the X-axis along AC and the Y-axis at right angles to it, then

$$x^2 + y^2 = E_0^2 \tag{309}$$

and

$$x = E_0 \cos (\theta - \alpha). \tag{310}$$

Substituting these in (308)

$$P_2 = \frac{E}{Z_s} x - (x^2 + y^2) \frac{\cos \theta}{Z_s}$$

and

$$x^2 + y^2 - \frac{E}{\cos \theta} x = - \frac{P_2 Z_s}{\cos \theta} = - \frac{P_2 r}{\cos^2 \theta}$$

and completing squares

$$\left(x - \frac{E}{2 \cos \theta}\right)^2 + y^2 = \frac{E^2}{4 \cos^2 \theta} - \frac{P_2 r}{\cos^2 \theta} = \frac{\frac{E^2}{4} - P_2 r}{\cos^2 \theta} \tag{311}$$

this is the equation of the locus of B and it is a circle of radius

$$R = \frac{\sqrt{\frac{E^2}{4} - P_2 r}}{\cos \theta} = \frac{Z_s}{r} \sqrt{\frac{E^2}{4} - P_2 r} \tag{312}$$

and having as its center the point $\left(\frac{E}{2 \cos \theta}, 0\right)$, which is the point C.

Thus the lines of constant power output P_2 are circles of radius $R = \frac{Z_s}{r} \sqrt{\frac{E^2}{4} - P_2 r}$ drawn about the point C, Fig. 324(5). The circle corresponding to $P_2 = 0$ passes through O and A.

The maximum possible power output would correspond to the

case where the radius = 0 and is represented by the point C. This maximum power P_2 is found by equating the equation for the radius to zero

$$\frac{Z_s}{r}\sqrt{\frac{E^2}{4} - P_2 r} = 0$$

and therefore

$$P_2 = \frac{E^2}{4r} \tag{313}$$

This is the maximum power output with maximum excitation and is far beyond the practical limit of output.

The maximum output with a given excitation I_f corresponding to a generated voltage E_0 may be found by differentiating P_2 with respect to α,

$$P_2 = \frac{EE_0}{Z_s}\cos(\theta - \alpha) - \frac{E_0^2}{Z_s}\cos\theta$$

$$\frac{dP_2}{d\alpha} = \frac{EE_0}{Z_s}\sin(\theta - \alpha) = 0, \text{ for maximum;} \tag{314}$$

the condition of maximum output is

$$\sin(\theta - \alpha) = 0, \text{ or } \alpha = \theta$$

and substituting this in the equation for P_2,

$$P_{2\text{max.}} = \frac{EE_0}{Z_s} - \frac{E_0^2}{Z_s}\cos\theta \tag{315}$$

This corresponds to the condition when B falls on AC. For positions of B to the right of AC the power decreases again. If a motor is loaded until the coupling angle α becomes greater than θ it will fall out of step and stop. The line AC, therefore, marks the limit of stable operation.

When the point B falls on the line OC the current is in phase with the impressed voltage and the power factor is unity. This is the position of minimum current for a given output. For points to the right of OC the current is lagging and for points to the left the current is leading.

The value $P_2 = \frac{E_2}{4r}$ may be found by differentiating $P_{2\text{max.}} = \frac{EE_0}{Z_s} - \frac{E_0^2}{Z_s}\cos\theta$ with respect to E_0,

$$\frac{d(P_{2\text{max.}})}{dE_0} = \frac{E}{Z_s} - \frac{2E_0}{Z_s}\cos\theta = 0 \text{ for maximum;}$$

therefore, $2E_0 \cos\theta = E$ and $E_0 = \dfrac{E}{2\cos\theta}$ and the maximum value of $P_{2max.}$ is

$$P_2 = \frac{E}{Z_s}\frac{E}{2\cos\theta} - \left(\frac{E}{2\cos\theta}\right)^2\frac{\cos\theta}{Z_s}$$
$$= \frac{E_2}{2r} - \frac{E^2}{4r} = \frac{E^2}{4r} \text{ as before.}$$

To find the relation between x_s and r which makes $P_{2max.}$ a maximum, substitute $Z_s = \sqrt{r^2 + x_s^2}$ and $\cos\theta = \dfrac{r}{Z_s} = \dfrac{r}{\sqrt{r^2 + x_s^2}}$, assume r to be constant and differentiate $P_{2\,max.}$ with respect to x_s.

$$P_{2max.} = \frac{EE_0}{\sqrt{r^2 + x_s^2}} - \frac{E_0^2 r}{r^2 + x_s^2}$$

$$\frac{d(P_{2max.})}{dx_s} = EE_0\left\{-\frac{1}{2}(r^2+x_s^2)^{-3/2}2x_s\right\} + E_0^2 r\,(r^2 + x_s^2)^{-2}2x_s$$

$$= -\frac{EE_0x_s}{(r^2 + x_s^2)^{3/2}} + \frac{2E_0^2rx_s}{(r^2 + x_s^2)^2} = 0 \text{ for maximum,}$$

and assuming $E_0 = E$, which would be approximately correct for normal operation,

$$-x_s\sqrt{r^2 + x_s^2} + 2rx_s = 0$$

and from this

$$x_s = \sqrt{3}r.$$

This is a much lower value of x_s than is usually found in such machines.

272. Synchronizing Power.—By synchronizing power is meant the change in power per degree change of the coupling angle α, it represents the stiffness of the coupling of the motor and was found in equation (314)

$$\frac{dP_2}{d\alpha} = \frac{EE_0}{Z_s}\sin(\theta - \alpha);$$

it is directly proportional to the excitation or E_0 and is inversely proportional to the synchronous impedance and therefore to the armature strength. It varies from a maximum value $\dfrac{EE_0}{Z_s}$ sin $\theta = \dfrac{EE_0x_s}{Z_s^2}$ when $\alpha = 0$ to zero when $\alpha = \theta$. If α becomes greater than θ the machine cannot pull into step again.

273. Construction of the Characterictic Curves of a Synchronous Motor from the Blondel Diagram.—A three-phase, 150-kw. (output) synchronous motor has a resistance per phase $r = 0.04$ ohms and a synchronous reactance $x_s = 0.6$ ohms. The constant losses may be taken as 5 per cent. of the rated output = 7,500 watts.

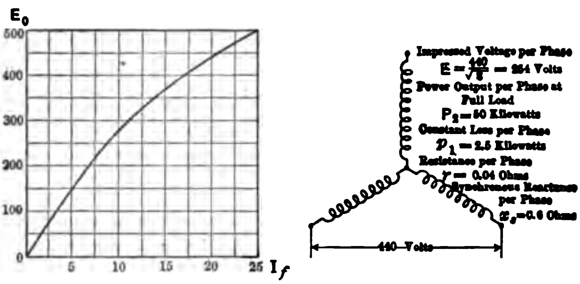


FIG. 325.

In constructing the Blondel diagram it is as well to work on one phase only with a full-load output of 50 kw. Assuming that the armature is Y-connected, the impressed voltage per phase is $E = \dfrac{440}{\sqrt{3}} = 254$ volts and this is maintained constant. The no-load saturation curve is shown in Fig. 325.

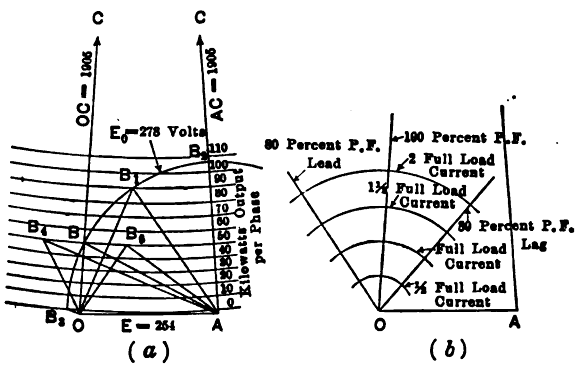


FIG. 326.

In Fig. 326 the horizontal $OA = E = 254$ volts and the two lines OC and AC are drawn making angles $\theta = \tan^{-1}\dfrac{x_s}{r} = \tan^{-1}$ 15 with the ends of OA; the point of intersection C is not shown.

The length of each of the two sides OC and AC is

$$\frac{E}{2 \cos \theta} = \frac{254}{2 \times \dfrac{0.04}{0.6}} = 1{,}905.$$

The circles representing constant power outputs per phase from 0 to 110 kw. are shown. Their radii may be found by substituting the given values of output in the equation.

$$R = \frac{Z_s}{r}\sqrt{\frac{E^2}{4} - P_2 r},$$

but here P_2 is the power developed at the shaft and is greater than the output by the amount of the constant losses per phase = 2,500 watts.

For example, the radius of the circle for full-load output is

$$R = \frac{Z_s}{r}\sqrt{\frac{E^2}{4} - P_2 r} = \frac{0.60}{0.04}\sqrt{\frac{(254)^2}{4} - (50{,}000 + 2{,}500)(0.04)} = 1{,}776.$$

and the radius for zero output is

$$R = \frac{0.60}{0.04}\sqrt{\frac{(254)^2}{4} - (2{,}500)(0.04)} = 1{,}899.$$

274. Load Characteristics.—To construct the load characteristics, take any value of motor back voltage E_0 and draw a circle about A with E_0 as radius. Here E_0 is taken as 278 volts corresponding to a field current I_f = 10 amp. and this gives unity power factor at full load. At the point B, Fig. 326, AB represents the constant motor voltage $E_0 = 278$, OB represents the synchronous impedance drop $E^1 = IZ_s$ at full load and it also represents the current. $OB = 129$ volts or $= \frac{129}{Z_s} = \frac{129}{0.6} = 215$ amp.; the power factor = 100 per cent. At the point B_1 the output = 90 kw. per phase, the current $= \frac{OB_1}{Z_s} = \frac{247}{0.6} = 413$ amp. and the power factor $\cos BOB_1 = \cos 19.5$ degrees $= 0.943 = 94.3$ per cent. lagging.

At B_3 the output = 0, the current $= \frac{OB_3}{Z_s} = \frac{24}{0.6} = 40$ amp. and the power factor $\cos \phi = \cos BOB_3 = \cos 75$ degrees = 0.259 = 25.9 per cent.

The maximum power output for the given excitation occurs at B_2 where the circle cuts the line AC. Here output = 106.5 kw.

per phase, the current = 605 amp. and the power factor = 79 per cent. To the right of AC operation is unstable.

These values of current and power factor are plotted on an output base in Fig. 328.

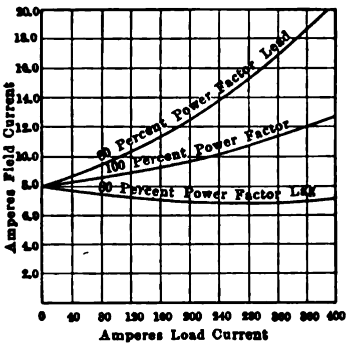


FIG. 327.—Compounding curves of a synchronous motor.

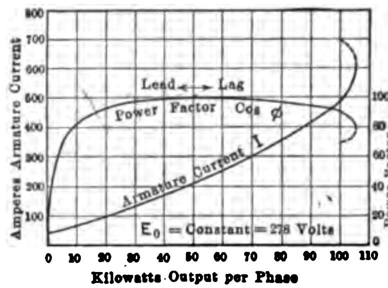
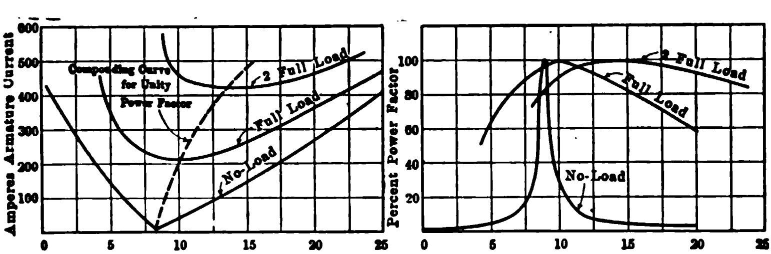


FIG. 328.—Load characteristics of a synchronous motor.

275. Phase Characteristics or "V" Curves and Compounding Curves.—To obtain the "V" curve for full-load output take points as B, B_4 and B_5 on the 50-kw. circle. At B the current is $\frac{OB}{Z_s} = \frac{129}{0.6} = 215$ amp., the power factor $= \cos \phi = 1.00$, the motor back-voltage $= AB = 278$ volts and the corresponding



(a) "V" curves.

(b) Power-factor curves.

FIG. 329.

field current is 10 amp., Fig. 325; at B_4 the current is 250 amp., the power factor is cos 30 degrees = 0.866 = 86.6 per cent. leading, the motor voltage is 350 volts and the field current is 13.8 amp.; at B_5 the current is 250 amp. the power factor is 86.6 per cent. lagging and the field current is 7.4 amp. The "V" curves

for zero output, full load and twice full load, are shown in Fig. 329(*a*). The minimum armature current for each output occurs where the power circle cuts the line *OC*, that is, where the power factor is unity. The corresponding values of the power factor are plotted on a field current base in Fig. 329(*b*).

Compounding Curves.—In Fig. 326(*b*) circles are drawn about center *O* with radii corresponding to currents from one-half to twice full-load current. Using the points where these cut the constant power-factor lines the compounding curves in Fig. 327 are drawn for unity power factor, 80 per cent. power-factor lead and 80 per cent. power-factor lag.

The theoretical maximum possible power output of this motor would be $P = \frac{E^2}{4r} = \frac{(254)^2}{4 \times 0.04} = 403{,}000$ watts $= 403$ kw. per phase which is more than eight times full load. Such an output would require a motor back voltage $= AC = 1{,}905$ volts.

276. Starting Synchronous Motors.—The single-phase synchronous motor is not self-starting and must be brought up to synchronous speed before being connected to the supply. This is due to the fact that single-phase armature reaction does not produce a revolving field. Single-phase motors are, therefore, not used except in special cases and in small sizes.

Polyphase synchronous motors are inherently self-starting and when connected to the supply will accelerate and run up to synchronous speed, but only a low voltage should be impressed at start or very large lagging currents will be drawn from the supply lines. In order to do away with the large starting currents a great many polyphase synchronous motors are started by auxiliary motors; they must be brought up to synchronous speed and synchronized just as alternators. If the exciter for the motor is mounted on the same shaft it may be used as a starting motor or, if the motor is one unit of a motor-generator set, the generator may be used as a starting motor, but in both cases a supply of direct current is necessary.

An induction motor mounted on the same shaft may be used to start the synchronous motor but it must have a smaller number of poles and therefore a higher synchronous speed than the motor to be started. The synchronous motor is raised above synchronous speed, the induction motor is then disconnected, and the synchronizing switch is closed as the motor passes through synchronous speed.

277. Self-starting Motors.—In order to improve the starting torque of synchronous motors and also to prevent hunting; short-circuited grids are placed in the pole faces or between the poles and sometimes complete squirrel-cage windings are carried by the rotors (Fig. 330). The field winding at start may be either open or short-circuited. In the following discussion it will be assumed that the field circuit is open. When a motor at rest is connected to a polyphase supply torque is developed in two ways: (1) the revolving field sweeps across the pole faces and grids and generates e.m.fs. and currents in them; these currents

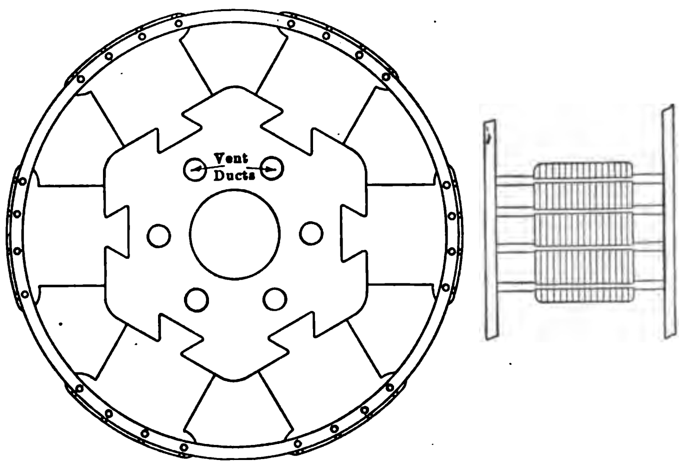


FIG. 330.—Rotor of a synchronous motor with a squirrel-cage winding.

react on the field and produce torque which causes the rotor to follow the field. The rotor can, however, never be brought up to synchronous speed by this torque because the e.m.fs. and currents are induced only below synchronous speed. The motor starts as a regular induction motor and when it is nearly up to speed the field locks with the revolving armature in the position of minimum reluctance.

2. The second method of producing torque for starting depends on hysteresis; as the armature north pole passes a field pole it induces a south pole in it and the attraction between the two tends to make the rotor follow the armature field. Due to hysteresis this induced field pole persists after the armature pole has passed; the succeeding armature south pole first repels the field pole then while passing changes it to a north pole and at-

tracts it in the direction of rotation. The resulting torque is small at low rotor speeds but becomes greater as synchronous speed is approached and the field pole remains longer under the influence of the armature pole. When the armature pole is moving very slowly across the field pole the two lock in the position of minimum reluctance.

Since the grids or squirrel-cage windings are carrying current only while starting up, or when there is relative motion between the armature reaction flux and the field poles, as when there is hunting, it is not objectionable if they have a comparatively high resistance; in fact, the higher the resistance the greater will be the starting torque, within limits. The bars of the squirrel cage can therefore be made of any suitable material and steel is often employed but the end rings are generally made of brass.

When the motor is running at synchronous speed the field circuit may be closed and the impressed e.m.f. raised to its full value. On closing the field circuit it may be found that the polarity is wrong and the rotor will then drop back in phase by 180 degrees and this change will be accompanied by a sudden rush of current. To avoid this it is better to excite the fields through a large resistance just before synchronism is reached, then increase the field to normal and raise the impressed voltage to full value. When starting in this way the motor draws a very large lagging current since the impressed voltage is consumed by the synchronous impedance of the armature, and the power factor is very low. The impressed voltage at start must be reduced to about one-third of its full value in order to reduce the starting current. It is the flux of armature reaction which produces the starting torque and, therefore, a motor with high armature reaction will give better starting torque than one of low armature reaction.

When the motor is running below synchronous speed large voltages sometimes reaching 5,000 volts or more are induced in the open field winding by the revolving armature flux. Such voltages are dangerous and may puncture the insulation of the field or endanger the lives of operators. Both the magnitude and frequency of these induced voltages become zero when the motor reaches synchronous speed.

The potential stresses may be reduced by breaking up the field into sections, by means of a suitable switch, during the starting period.

The approach of synchronism may be recognized by the sound or an alternating-current voltmeter connected across a section of the field winding will indicate synchronism by a zero reading.

If the field winding is short-circuited while coming up to synchronism these large induced voltages will not exist but the starting torque of the motor will in general be reduced. The short-circuited field winding tends to act in somewhat the same way as the grids but its inductance is so high that the flux produced through it by the armature m.m.f. is small and this reduction of the flux in the field poles more than counterbalances the torque due to induced currents. In the majority of cases, however, it is better to start the motor with the field winding short circuited since the safety of operation is thereby increased. When synchronous speed is reached the full excitation should be applied and then the impressed voltage raised to full value.

When individual exciters are used the motor field may be short-circuited by reducing the exciter field to its lowest value.

278. Synchronous Phase Modifier.—Since by varying the field excitation of a synchronous motor the power factor can be made either leading or lagging, such machines can be used to improve the power factor of transmission lines or distributing circuits by drawing wattless leading currents to compensate for the wattless lagging currents required by the load. The fields must be overexcited and the synchronous reactance should not be very large. This is one of the most important characteristics of the synchronous motor and is being applied to an ever-increasing extent.

A synchronous motor used in this way is called a synchronous phase modifier and is usually operated without load drawing the required wattless leading current and the small power current supplying its own losses. In some cases, however, it may be advantageous to supply some load from it.

In the case of long-distance transmission lines the synchronous phase modifier is used to obtain a constant terminal voltage under all conditions of load with a constant impressed voltage at the generating station. Assume a transmission line with a constant impressed voltage of 110,000 volts. Under normal conditions the terminal voltage at no load rises to 120,000 due to the compounding effect of the line capacity, while at full load 80 per cent. power factor the terminal voltage falls to 90,000 volts. To maintain a constant terminal voltage of 100,000 a synchronous phase modifier must be installed in the terminal station;

during periods of light load it is operated under excited drawing a large lagging current to pull down the voltage from 120,000 to 100,000; while under load it is over-excited and draws a leading current to raise the terminal voltage from 90,000 to 100,000.

The constant voltage regulation is made automatic by installing a voltage regulator operating on the field circuit of the phase modifier and having its alternating-current control magnet connected through a potential transformer across one phase of the circuit in which the voltage is to be maintained constant.

279. Parallel Operation of Alternators.—Before an alternator is connected in parallel with another machine which is supplying power, the incoming machine must be adjusted to give the same voltage, must have the same frequency and must be in phase.

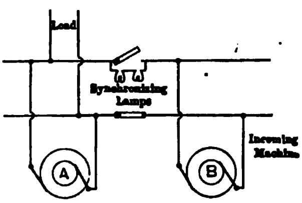


FIG. 331.—Alternators in parallel.

The condition of synchronism may be indicated by incandescent lamps or some form of synchroscope (Art. 452).

In Fig. 331 when the incoming machine B is in synchronism with A there is no voltage across the switch and the lamps are dark. The two machines are assumed to have been adjusted to give equal voltages.

If the frequency of B is lower or higher than that of A, there will be a slow pulsation of the light showing the difference between the two frequencies. When the pulsations are very slow and the periods of darkness long the switch may be closed and the two machines will operate in parallel. Lamps are not very satisfactory, since they do not show whether the incoming machine is running too slow or too fast.

280. Effect of Inequality of Terminal Voltage.—If two alternators are operating at the same frequency and are in phase but have not their fields adjusted to give the same terminal voltage, a wattless current will flow between the two machines leading and magnetizing in the machine of lower field excitation and lagging and demagnetizing in the machine of higher field excitation.

If E_B, the voltage of B, is lower than E_A, the voltage of A, then E' the difference between the two will act in the local circuit through the two armatures in series and will produce a current I' lagging nearly 90 degrees behind E' and E_A and leading E_B (Fig. 332).

The circulating current is

$$I' = \frac{E'}{Z_A + Z_B} \tag{316}$$

where Z_A and Z_B are the synchronous impedances of the two machines.

This current lowers the terminal voltage of A since it is lagging in A and raises the terminal voltage of B since it is leading in B and the two are made equal to the load voltage E.

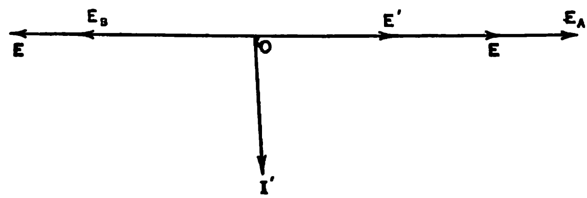


FIG. 332.

By adjusting the field rheostats the wattless circulating currents can be eliminated for any load, but if the two machines have different voltage characteristics, as the load varies wattless currents will circulate to correct for the differences of excitation.

With machines of reasonably high armature reaction the wattless cross currents are small even with large variations of excitation.

When two similar alternators in parallel are supplying an inductive load, they should operate at the same power factor. If one has a lower excitation than the other it will not supply its proper share of the watless kva. and will operate at a higher power factor than the second machine.

281. Effect of Inequality of Frequency.—Two alternators operating in parallel must have the same average frequency, but one may instantaneously drop behind or run ahead of the other.

Alternators driven by water turbines or steam turbines or electric motors will have a constant angular velocity but when the prime movers are steam engines or gas engines the angular velocity will pulsate about its average value during each revolution.

If two machines in parallel are excited to give the same terminal voltage and one falls behind the other, a power cross-current will circulate through the armatures and transfer energy from the leading to the lagging machine.

Fig. 333 shows the case of two machines of which A is ahead of its normal position by angle α and B is behind by angle α. A is therefore ahead of B by angle 2α and the resultant voltage causing a current to circulate through the two armatures is

$$E' = 2E \sin \alpha.$$

The circulating current is

$$I' = \frac{E'}{Z_A + Z_B} = \frac{2E \sin \alpha}{2Z} = \frac{E \sin \alpha}{Z} \tag{317}$$

where $Z = Z_A = Z_B$ is the synchronous impedance of each of the two machines which are assumed to be similar. Under ordinary circumstances the current I' lags nearly 90 degrees behind E'; it

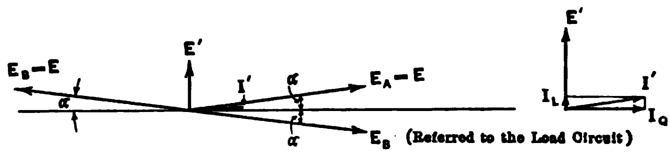


FIG. 333.

is approximately in phase with the terminal voltage E_A of A and in phase opposition to E_B; and it therefore transfers power from the leading to the lagging machine causing the leading machine to drop back in phase and the lagging machine to come up.

The component of I' in phase with E' represents the power consumed by the passage of I' through the resistance of the local circuit through the two armatures in series; the component in quadrature behind E' represents the transfer of power required to keep the machines in step.

Take the case of two similar 60-cycle alternators driven at 240 r.p.m. by single-cylinder steam engines of 1 per cent. speed variation. The driving effort and the speed reach a maximum twice during the revolution and fall to a minimum twice. For one-quarter of the revolution the alternator A runs at an average speed $\frac{1}{4}$ of 1 per cent. above normal and it therefore moves ahead of its normal position by $\frac{360}{1,600} = \frac{9}{40}$ of a mechanical degree. If at the same time the second machine B is running slow it will fall behind its normal position by an equal angle and the maximum phase difference between the two machines will be

$9/20$ of a mechanical degree. These alternators would have 30 poles and therefore the maximum phase displacement is

$$\frac{9}{20} \times \frac{p}{2} = \frac{9}{20} \times \frac{30}{2} = .6.75 \text{ electrical degrees} = 2\alpha.$$

The circulating current is by equation (317)

$$I' = \frac{E}{Z} \sin 3.37° = I_{sc} \sin 3.37°$$

where I_{sc} is the short-circuit current with full excitation and may be from two to six times full-load current; assuming $I_{sc} = 4I$ the circulating current is

$$I' = 4I \times 0.059 = 0.24I$$

and is approximately 25 per cent. of full-load current. With lower values of synchronous reactance and consequently higher values of I_{sc}, the cross-currents tend to become very large. Machines with very small synchronous reactance or armature reaction are not suitable for parallel operation. Too high synchronous reactance reduces the cross-currents and the synchronizing power too much.

These power cross-currents when of great magnitude tend to tear the machines out of synchronism and they also cause fluctuations of the voltage.

If the two machines above had been synchronized so that their angular velocities rose and fell together, there would not have been any power currents circulating between them but in that case the frequency of the voltage of the system would pulsate and power currents would circulate between the alternators and any synchronous motors operating on the system.

Alternators driven by steam engines or gas engines must be provided with flywheels large enough to reduce the pulsations of angular velocity during the revolution to a negligible amount.

If the speed characteristics of the prime movers are not the same and the speed of one machine tends to fall below the other as the load on the system is increased, then the machine driven by the prime mover of closer speed regulation takes more than its share of the load and so relieves the other machine and keeps its speed up.

Thus to insure a proper division of the load beween alternators operating in parallel it is necessary that their prime movers

have similar speed characteristics, that is, that their speed shall fall under load by the same amount and in the same manner. It is, therefore, preferable that the prime movers have drooping speed characteristics. The voltage characteristics have no effect on the division of the load but they do affect the amount of the reactive cross-currents between the machines.

Let R and X represent the resistance and reactance of the circuit through two alternators in parallel including the lines connecting them which may be very important in the case of alternators in different stations.

The circulating current is

$$I' = \frac{E'}{R+jX} = \frac{E'(R-jX)}{R^2+X^2} = \frac{E'R}{R^2+X^2} - j\frac{E'X}{R^2+X^2};$$

the quadrature component of the current which gives the power transfer is $I_Q = \frac{E'X}{R^2+X^2}$, Fig. 333, and thus it may be seen that reactance or armature reaction is necessary in alternators to be operated in parallel.

For a given value of R the synchronizing current I_Q is maximum when $X = R$; this result is obtained by differentiating I_Q with respect to X and equating to zero.

$$\frac{dI_Q}{dX} = \frac{E'\{(R^2+X^2) - X(2X)\}}{(R^2+X^2)^2} = 0, \text{ for maximum } I_Q,$$

and therefore $R^2 - X^2 = 0$, or $X = R$.

Such a low value of X would allow excessive currents to flow between the machines. In the majority of cases X is many times greater than R, but when two stations are to be operated in parallel R and X may approach equality, especially with underground distribution, and R may even be the greater. In such a case the operation would be improved by inserting extra reactance to make $X = R$ and so increase the synchronizing current.

282. Effect of Difference of Wave Form.—If two machines in parallel are adjusted to give the same effective value of voltage but have different wave shapes, then, since, due to the presence of the higher harmonics, the voltages are not equal at every instant, reactive cross-currents will flow to correct these inequalities in voltage. These currents will usually be very small since the voltages producing them are small and they are of high frequency and thus the path through the two machines offers a

high impedance to them; the impedance is, however, only the true impedance and not the synchronous impedance.

If two Y-connected alternators with neutrals grounded are connected in parallel and one has a prominent third harmonic in the voltage wave of each phase while the other gives a true sine wave, then, although the terminal voltages of the two machines may have the same effective values and the same wave form, still third harmonic circulating currents will flow between them returning through the neutral connections.

283. Conclusions.—Three kinds of circulating currents may exist in parallel operation of alternators: (1) reactive currents transferring magnetization between the machines due to a difference in excitation; (2) active currents transferring power between the machines due to phase displacements between their voltages; and (3) higher-frequency reactive currents due to differences of wave form.

The reactive cross-currents may be eliminated by regulating the excitation, that is, by electrical means, but the power cross-currents are due to differences in speed or phase and can be corrected only by regulating the speed of the prime movers. Parallel operation of alternators is therefore not so much an electrical as it is a mechanical problem.

When, therefore, an alternator is to be connected in parallel with machines supplying the load, the incoming machine must be brought up to synchronism and the switch connecting it to the load circuit closed, then the governor of its prime mover must be adjusted so that it supplies its proper share of the load and its field must be adjusted so that it supplies its proper share of the reactive current required by the load. Ordinarily the machines will be operated at approximately the same power factor. Further adjustment may be necessary as the load changes.

284. Hunting.—If two alternators are operating in parallel and one drops behind the other in phase due to a sudden decrease in the speed of its prime mover, the second machine supplies power to pull it into synchronism again. The impulse received causes it to swing past its mean position and it oscillates a few times before falling into step.

If the action producing the speed pulsation is repeated periodically and coincides with the natural period of the machine the oscillations instead of dying out will increase in amplitude until they are limited by the losses in the pole faces and the dampers or

until the machines fall out of step. When the oscillations tend to become cumulative the machines are said to be hunting.

Hunting may occur in a similar way in the case of a synchronous motor supplied by an alternator. If the load on the motor suddenly increases it falls back in phase to receive the extra power required and oscillates about its final phase position before running in synchronism again. This oscillation may become continuous as in the case of alternators in parallel.

The amplitude of the oscillations in hunting is very much reduced by the use of dampers in the form of grids or squirrel-cage windings placed in slots in the pole faces on the rotor. At synchronous speed the armature reaction flux is stationary relative to the fields and, therefore, does not produce any current in the grids but if the machine falls below or runs above synchronous speed, the flux sweeps across the grids and produces e.m.fs. in them and large currents flow which react on the field and tend to hold the machine exactly in synchronism.

Dampers are applied to alternators only in cases where hunting is liable to occur, due to the fact that the prime movers are reciprocating engines or gas engines but they should be used on all large synchronous motors since these are usually high-speed machines with small moment of inertia and a high natural frequency of oscillation which is more liable to coincide with some forced frequency on the system. The dampers are also required to make the motors self-starting.

285. Frequency of Hunting.—Referring to Fig. 333, the power supplied by A to pull B up in phase again is

$$P_s = EI' \cos \alpha,$$

but

$$I' = \frac{E'}{2Z_s} = \frac{2E \sin \alpha}{2Z_s} = \frac{E}{Z_s} \sin \alpha,$$

and thus the synchronizing power is

$$P_s = \frac{E^2}{Z_s} \sin \alpha \cos \alpha = \frac{E^2}{2Z_s} \sin 2\alpha = \frac{E^2}{2Z_s} 2\alpha,$$

since for small angles the sine is approximately equal to the angle, here α is expressed in electrical radians.

Changing α into mechanical radians,

$$P_s = \frac{E^2}{2Z_s} 2\alpha \frac{p}{2} = \frac{E^2 p}{2Z_s} \alpha \text{ watts.} \tag{318}$$

The synchronizing torque is

$$T_s = \frac{P_s}{2\pi \times \text{r.p.m.}} \times \frac{33{,}000}{746} = \frac{\dfrac{E^2 p\alpha}{2Z_s}}{2\pi \times \text{r.p.m.}} \times \frac{33{,}000}{746} = \frac{3.52\ E^2 p}{Z_s \times \text{r.p.m.}}\ \alpha.$$

and it is directly proportional to the angular displacement α.

Since the sum of the moments of the external forces acting on a rigid body is equal to the moment of inertia of the body about the axis of rotation multiplied by its angular acceleration, the equation for the resulting motion may be expressed as

$$-\ T_s = I\frac{d^2\alpha}{dt^2} = -\frac{3.52\ E^2 p}{Z_s \times \text{r.p.m.}}\ \alpha, \tag{319}$$

where I is the moment of inertia of the rotating field member including the flywheel. The negative sine is used because the torque tends to decrease the angular displacement α.

This equation may be written

$$\frac{d^2\alpha}{dt^2} = -\frac{3.52\ E^2 P}{Z_s I \times \text{r.p.m.}}\ \alpha = -\ k^2\ \alpha. \tag{320}$$

This is the equation of a simple harmonic motion of periodic time

$$T_h = \frac{2\pi}{k} = 2\pi\sqrt{\frac{Z_s I \times \text{r.p.m.}}{3.52 E^2 p}}\ \text{sec.} \tag{321}$$

and therefore the natural frequency of hunting is

$$f_h = \frac{1}{T_h} = \frac{1}{2\pi}\sqrt{\frac{3.52 E^2 p}{Z_s I \times \text{r.p.m.}}} = 0.3E\sqrt{\frac{p}{Z_s I \times \text{r.p.m.}}} \tag{322}$$

The frequency of hunting is directly proportional to the voltage E or to the air-gap flux Φ and is inversely proportional to the square root of the moment of inertia I and the synchronous impedance of the armature and connecting feeders.

The natural frequency may be increased by increasing E, that is, increasing the voltage of the system but this is not usually practicable and it may be decreased by inserting reactance in the lines between the machines.

The natural frequency of oscillation of an alternator should not approach within 20 per cent. of any forced frequency which is liable to occur on the system and the flywheel is usually designed so that it increases I to such a value that the natural frequency is at least 20 per cent. below the lowest forced frequency.

The same formula holds in the case of a synchronous motor

operating on a large constant-voltage system. E may be taken as the motor voltage which is dependent on the excitation. When the motor is under-excited E is low and the synchronizing power transfer per degree angular displacement is small and the natural period of oscillation is low. The electromagnetic coupling is soft and the motor may be thrown out of step by a sudden increase of load. When the motor is over-excited the synchronizing power is large and the natural frequency is high. The electromagnetic coupling is stiff and the motor is stable in operation. If, however, any forced frequency occurs approximating the natural frequency, serious hunting will occur.

Hunting may sometimes be reduced or eliminated by changing the field excitation and thus changing the natural period of oscillation of the machine.

Machines with high armature reaction are much less liable to hunt than machines of low armature reaction since the high armature reaction reduces the circulating currents produced by changes in phase and lowers the natural frequency of oscillation.

286. Design of Alternating-current Generators and Motors.— The design of alternating-current generators and synchronous motors is similar to that of direct-current generators and motors but without the difficulties and limitations due to commutation. Much larger outputs, higher voltages and greater peripheral speeds must be provided for in certain cases. The voltage regulation of alternators is not so good as that of direct-current generators, since a large component of the generated voltage is consumed by the armature reactance. The regulation also depends on the power factor of the load. To obtain reasonable values of regulation the air gap must be made long and therefore the ratio of field ampere-turns to armature ampere-turns must be large. For machines of very large output as turbo-alternators the peripheral speed becomes a limiting factor, the frame must be made long and forced ventilation is necessary to prevent undue rise of temperature.

Following are some of the formulæ and constants involved.

287. Electromotive Force Equation.

$$\text{E.m.f. per phase} = E = 4\delta\gamma f n\phi 10^{-8} \quad \text{(Art. 248)}$$

or assuming a distribution factor $\delta = 0.96$ and a form factor $\gamma = 1.11$, the e.m.f. is

$$E = 4.26 f n\phi 10^{-8} \text{ volts.} \qquad (323)$$

288. Output Equation.

Output in volt-amperes = $p'EI$ (where p' = number of phases)

$$= 4.26p'f\phi nI\ 10^{-8} = 4.26 \times \frac{\text{r.p.m.} \times p}{120} \times B_g\psi L_c\tau \times \frac{\pi D_a q}{2} \times 10^{-8}$$

$$= \frac{4.26}{120 \times 2} \times 10^{-8} \times \pi \times \text{r.p.m.} \times B_g\psi q \times p\tau \times D_aL_c$$

$$= \frac{4.26}{120 \times 2} \times 10^{-8} \times \pi^2 \times \text{r.p.m.} \times B_g\psi q \times D_a{}^2L_c,$$

or

$$D_a{}^2L_c = \frac{\text{volt-amperes} \times 5.7 \times 10^8}{\text{r.p.m.} \times B_g\psi q}. \tag{324}$$

$B_g = \dfrac{\phi}{\psi\tau L_c}$ = average gap density, varies from 40,000 to 60,000 lines per square inch, but is usually below 50,000.

ψ = pole enclosure = 0.60 to 0.65.

q = ampere conductors per inch, ranges from 400 to 1,000; a good average value is 500.

$\dfrac{L_c}{\tau}$ = ratio $\dfrac{\text{frame length}}{\text{pole pitch}}$ ranges from 0.6 to 2.0; a good average is 1.

p = number of poles = $\dfrac{120f}{\text{r.p.m.}}$ is fixed by the frequency and the r.p.m.

Approximate rated speed in r.p.m. of prime movers for generators of 250-, 500- and 1,000-kw. capacity.

Prime mover	Rated speed in r.p.m.		
	250-kw.	500-kw.	1,000-kw.
Moderate-speed steam engine	160	120	80
High-speed gas engine	250	200	120
High-speed steam engine	350	270	220
De Laval steam turbine	1,000		
Curtis steam turbine	2,500	1,800	1,200
Parsons steam turbine	3,000	2,500	1,900

The speed of hydraulic turbines is controlled by the head of water available.

Peripheral speed varies over a wide range; for low-speed machines a good average value is 3,500 ft. per minute but in very large turbo-alternators it may be as high as 25,000 ft. per minute.

289. Flux Densities.—The following table from the "Standard Handbook" gives the average flux densities used in the various parts of the magnetic circuit.

Part	Frequency	Flux density in lines per square inch	
		Ordinary iron	Silicon steel
Armature core....	25 cycles per second	60,000 to 70,000	70,000 to 80,000
	60 cycles per second	50,000 to 60,000	60,000 to 70,000
Armature teeth....	25 cycles per second	100,000 to 120,000	
	60 cycles per second	90,000 to 110,000	
Pole core.........	95,000 to 110,000 with wrought iron or steel poles.		
Yoke.............	90,000 to 100,000 for cast steel.		
	30,000 to 35,000 for cast iron.		

290. Current Densities.—Current densities in the armature conductors range from 1,500 to 2,500 amp. per square inch; the lower values should be used in machines which are difficult to cool, that is, machines with long cores or in high-voltage machines in which the thick slot insulation restricts the radiation of the heat due to the copper loss. Short machines of large diameter are easy to cool since the end connections are spread out and have a good radiating surface.

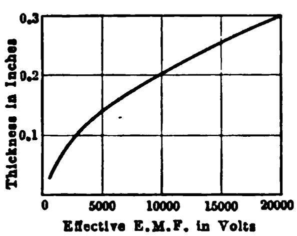


Fig. 334.—Thickness of slot insulation for alternators.

Current density in field windings varies from 1,200 to 2,500 amp. per square inch depending on the construction, insulation, available space and on the temperature rise allowed.

291. Insulation for High-voltage Alternators.—The very best materials must be used because the space is limited. Micanite or Bakelized paper tubes may be used or the insulation may be built up of a number of layers of empire cloth or similar material. The curve in Fig. 334 shows approximately the thickness of the slot insulation for various voltages. The built-up insulations may require more space than indicated by the curve.

The winding should not be distributed in more slots than is necessary since so much of the slot width is required for insulation.

292. Extra Insulation Required under Special Conditions.—If a three-phase, star-connected, 11,000-volt generator is operated with its neutral grounded through a resistance of 10 ohms, and a ground occurs on one line, the short-circuit current flowing through the ground resistance will raise the potential of the neutral point and will seriously increase the potential stresses from the other windings to the core. The voltage per phase is $\frac{11,000}{\sqrt{3}} = 6,350$ volts and taking the resistance in the short-circuit as 12 ohms the current is $\frac{6,350}{12} = 530$ amp. This will raise the neutral point to $530 \times 10 = 5,300$ volts above ground and may increase the stress from the ungrounded phases to the core to approximately $6,350 + 5,300 = 11,650$ volts.

293. Armature Windings.—Armature windings are discussed in Arts. 241 to 245. They should be distributed rather than concentrated to dissipate the heat more readily and to improve the wave form and reduce the reactance. Full-pitch windings give the highest voltage but fractional-pitch windings as low as 66 or even 50 per cent. of full pitch are used to eliminate objectionable harmonics or to reduce the length of the end connections.

294. Slots per Pole.—Large slow-speed machines with many poles have one to three slots per phase per pole, while high-speed machines have from four to nine slots per phase per pole.

Ratio $\frac{\text{tooth width}}{\text{slot width}}$ varies from 1 to 2 for open slots and the ratio $\frac{\text{slot depth}}{\text{slot width}}$ varies from 2 to 3.5 or 4.

295. Regulation.—The voltage regulation depends on: (1) armature resistance, (2) armature reactance, (3) armature reaction and (4) load power factor.

1. Armature resistance is determined more from considerations of current density and temperature rise than of regulation.

2. The armature reactance drop is produced by the various leakage fluxes and depends on the shape of the slots, the length of the gap, the pitch and distribution of the winding (see Arts. 252 and 253).

3. The effect of armature reaction is determined by the ratio $\frac{\text{field ampere-turns}}{\text{armature ampere-turns}}$, the length of the gap and the flux densities in the gap, the teeth and the poles. The ratio $\frac{\text{field ampere-turns}}{\text{armature ampere-turns}}$ usually lies between 2.0 and 3.0, the lower value being for high-speed machines.

4. The effect of load power factor or regulation is discussed in Art. 261.

For the best regulation use a high air-gap density, a long gap and a large ratio $\frac{\text{field ampere-turns}}{\text{armature ampere-turns}}$.

Air gaps are much longer than in direct-current machines ranging from 0.5 or lower to 2.5 in. in some turbo-alternators. Eighty-five or 90 per cent. of the field m.m.f. may be required for the gap.

Short gaps make the regulation poor and result in distortion of the gap flux and the wave of generated e.m.f.

Ordinary values of regulation are 5 to 8 per cent. at 100 per cent. power factor and 15 to 25 per cent. at 80 per cent. power factor.

With automatic voltage regulators, close regulation is not necessary and may be a distinct disadvantage since the armature current under conditions of short-circuit is likely to reach dangerous dimensions in large low-reactance alternators.

296. Excitation Regulation.—Excitation regulation or the per cent. increase of field excitation to maintain constant voltage from no load to full load ranges from 10 to 15 per cent. for non-inductive load to 25 to 35 per cent. for 80 per cent. power-factor load.

297. Excitation.—Alternators and synchronous motors are excited by a separate direct-current shunt generator called an exciter, usually at 125 or 250 volts.

The exciter should preferably be driven by a separate prime mover to reduce the variation of the alternator voltage with speed. When the exciter is mounted on the same shaft as the alternator a 1 per cent. decrease in speed produces more than 1 per cent. decrease in the exciter voltage and more than 2 per cent. decrease in the alternator voltage.

The rating of the exciter is usually from 1 to 2 per cent. of the alternator rating unless it is necessary to maintain the voltage to very low power factors, when it must be larger.

298. Losses.—The losses in alternators are similar to those in direct-current generators.

The field copper loss is usually greater than in direct-current machines since for good regulation the field must be stronger. This loss is about 1 per cent. for non-inductive loads but will be greater for inductive loads.

Armature copper loss is similar to that in direct-current machines but for the large outputs conductors of large section must be used and they must be laminated to prevent large extra losses due to eddy currents as discussed in Art. 188. Eddy-current and hysteresis losses in the teeth and other metal parts near the conductors which are due to the presence of the alternating current in the conductors add to the copper loss and increase the apparent or effective resistance of the armature. In some cases it may be 50 per cent. greater than the true ohmic resistance.

The no-load core loss may be from one and a half to four times the full-load copper loss depending on the rated speed and frequency. With silicon-steel this loss may be reduced by 30 or 40 per cent. The increase of core loss under load may be as great as 25 or 50 per cent. in some cases.

Windage and bearing-friction losses in moderate-speed machines range from 0.5 to 1 per cent. but in high-speed turbo-alternators with fans for cooling this loss may reach 1.5 per cent.

299. Ventilation.—The rate at which heat is dissipated from a surface is proportional to the difference in temperature between the surface and the cooling air. A continuous circulation of cool air is required to prevent dangerous temperature increases in generators and motors.

There is very little difficulty experienced in cooling slow-speed machines as the losses per pound of material are small and the radiating surfaces are large. The rotor with its salient poles acts as a fan and blows air over the armature core and end connections. If necessary, fans may be added to direct the flow of air and increase the cooling effect.

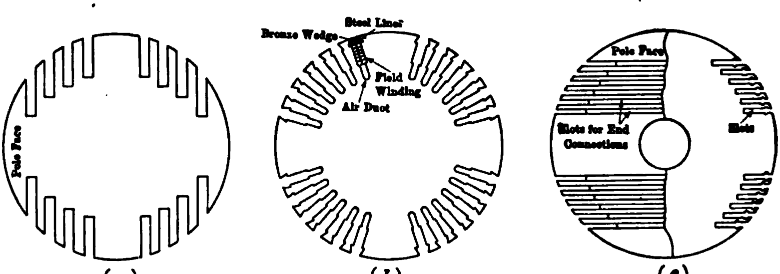
The insulation on the coils is a poor heat conductor and most of the heat generated by the copper losses passes along the conductors to the end connections where the insulation is thinner and is there radiated to the air. The end connections must be very well ventilated.

When the frame length is small and the diameter is large, the end connections are spread out and the machine is easy to cool.

When the frame length is great and the diameter is small there is very little space for the end connections and they are difficult to cool. Radial vent spaces are left in the core by placing special vent plates between blocks of punchings at suitable intervals. The vent spaces must be wide enough to allow the air to pass through them freely. The cooling air carries off the heat from the iron due to the core losses and also part of that due to the copper losses. The remainder of the heat due to the core losses is carried along the laminations to the air gap or to the frame. This is called radial ventilation (Figs. 242 and 336).

Another method of cooling makes use of axial ducts instead of or supplementing the radial ducts (Fig. 243). The axial ducts supply air which passes over the edges of the punchings and is very effective as a cooling agent, since the iron core conducts heat from 20 to 100 times better along the laminations than across them, depending on the method of stacking.

High-speed, steam turbine-driven alternators of large output are very difficult to cool. To limit the peripheral speed



(a) Two Pole, Cylindrical Rotor (b) Four Pole, Cylindrical Rotor (c) Two Pole, Cylindrical Rotor

FIG. 335.

the diameter must be made small and consequently the frame must be long. Forced ventilation is necessary and the machines must be completely enclosed to give control of the cooling air and to prevent objectionable noise. Openings are provided for the passage of cooling air. Fans are placed on the rotor and the air, after it has been cleaned and cooled, is drawn along the air gap and forced out through the stator radial ducts into passages which carry it to the outlet. Axial ducts may be provided in the core if it is considered necessary.

The rotor with its small diameter and restricted space for field copper is difficult to cool. Most of the heat developed in it is

carried to the air gap and dissipated there. In some cases axial ducts are cut at the bottom of the rotor slots and these are very effective (Fig. 335).

About 100 cu. ft. of cooling air per minute are required per kilowatt lost to limit the temperature rise at full load to permissible values.

For example, in a 25,000-kva. turbo-alternator of 97 per cent. efficiency the losses amount to 750 kw. and about 75,000 cu. ft. of air per minute are required. With a velocity of 5,000 ft. per minute the section of the air passages must be $\frac{75,000}{5,000} = 15$ sq. ft. Air may be admitted from both ends but even so the gap must be made very long to give the required area. The gap length in some cases must be determined in this way.

In the circumferential system of ventilation air is admitted at one side of the machine and passes circumferentially through the stator ducts to the other side where it is discharged. Air must at the same time be forced along the air gap to cool the rotor (Fig. 336).

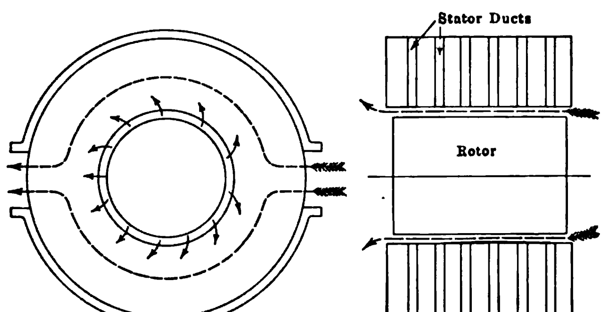


FIG. 336.—Circumferential ventilation.

300. Cylindrical Rotors.—The rotors of steam-turbine alternators present new features in design. To keep the peripheral speeds down to safe values, that is, below 25,000 ft. per minute, the diameter must be made small and the length great. The field winding is distributed and must be completely imbedded in slots. The rotor teeth must be designed with sufficient strength to stand the enormous centrifugal forces acting on them. At 25,000 ft. per minute a single pound of material is acted upon by a force of over 1 ton. Fig. 335 shows three sections of cylindrical

rotors. The four-pole rotor in (*b*) is ventilated by means of axial ducts below the slots. A section of the winding in one slot is shown. To force the winding solidly into place pressure is applied to the bronze wedge and then the steel liners are driven into place.

Since the space for the field winding is limited high current densities must be used and the rotors run hot. Only those materials which can stand high temperatures must be used for insulation.

CHAPTER X

TRANSFORMERS

301. The Constant-potential Transformer.—The constant-potential transformer consists of one magnetic circuit interlinked with two electric circuits, the primary circuit which receives energy and the secondary circuit which delivers energy. Its function is to transform electric power from low voltage and large current to high voltage and small current, or the reverse. In step-up transformers the primary is the low-voltage (L.V.) side and the secondary is the high-voltage (H.V.) side. In step-down transformers the primary is the high-voltage side.

In the following discussion letters with the subscript 1 will be used to represent primary quantities and with the subscript 2 to represent secondary quantities.

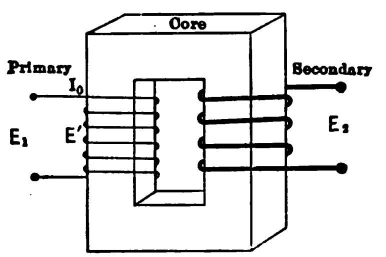


FIG. 337.—Constant potential transformer.

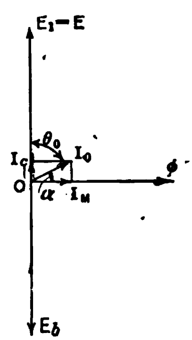


FIG. 338.

Fig. 337 represents a transformer. The core is made up of thin sheets of iron or steel of high permeability with small hysteresis and eddy-current loss.

The primary winding consists of n_1 turns in series and has a resistance of r_1 ohms, a self-inductive or leakage reactance of x_1 ohms and thus an impedance of $Z_1 = \sqrt{r_1^2 + x_1^2}$ ohms.

The secondary winding consists of n_2 turns in series. Its resistance is r_2 ohms, its reactance is x_2 ohms and its impedance is $Z_2 = \sqrt{r_2^2 + x_2^2}$ ohms

When an alternating e.m.f. E_1 is impressed on the primary winding with the secondary open, a current I_0 flows in the primary and produces an alternating flux through the core of maximum value Φ. The current I_0 is called the exciting current of the transformer and consists of two components (Fig. 338) I_M in phase with the flux Φ, called the magnetizing current, and I_c in quadrature ahead of the flux and in phase with the impressed e.m.f., called the core-loss current. The product of E_1 and I_c is the power wasted in the core loss, that is, in supplying the hysteresis and eddy current losses of the transformer. The exciting current, therefore, lags by an angle θ_0, which is less than 90 degrees, behind the impressed e.m.f. Cos θ_0 is the no-load power factor of the transformer.

The excitng current I_0 is from 3 to 10 per cent. of full-load current and the no-load power factor is of the order of 30 per cent.

The alternating flux produced by the magnetizing current links with the secondary winding and induces in it an e.m.f.,

$$e_2 = -n_2 \frac{d\phi}{dt} 10^{-8} \text{ volts.}$$

If the frequency is f cycles per second and the flux follows a sine wave of maximum value Φ, the instantaneous e.m.f. induced in the secondary is

$$\begin{aligned} e_2 &= -n_2 \frac{d}{dt} (\Phi \sin 2\pi ft) 10^{-8} \text{ volts} \\ &= -2\pi f n_2 \Phi \cos 2\pi ft 10^{-8} \text{ volts} \\ &= 2\pi f n_2 \Phi \sin (2\pi ft - 90) 10^{-8} \end{aligned} \tag{325}$$

this is a sine wave of e.m.f. in quadrature behind the flux, of maximum value

$$E_{2\text{max.}} = 2\pi f n_2 \Phi\, 10^{-8}$$

and effective value

$$E_2 = \frac{2}{\sqrt{2}} \pi f n_2 \Phi\, 10^{-8} = 4.44 f n_2 \Phi\, 10^{-8} \text{ volts.} \tag{326}$$

The flux also links with the primary winding and induces in it an e.m.f. of instantaneous value

$$\begin{aligned} e_b &= -n_1 \frac{d\phi}{dt} 10^{-8} \\ &= 2\pi f n_1 \Phi\, 10^{-8} \sin (2\pi ft - 90), \end{aligned}$$

a sine wave of maximum value

$$E_{b\text{max.}} = 2\pi f n_1 \Phi\, 10^{-8}$$

and effective value

$$E_b = 4.44 f n_1 \Phi\, 10^{-8} \text{ volts.} \tag{327}$$

This e.m.f. induced in the primary is almost equal in value and opposite in phase to the impressed e.m.f., the vector sum of the two being the small component of impressed e.m.f. required to drive the exciting current through the impedance of the primary winding. Thus

$$\dot{E}_1 + \dot{E}_b = \dot{I}_0 \dot{Z}_1.$$

This component has been neglected in Fig. 338. The induced e.m.fs. E_b and E_2 are directly in phase since they are produced by the same flux, and their intensities are in the ratio of the turns on the two windings; therefore,

$$\frac{E_b}{E_2} = \frac{4.44 f n_1 \Phi\, 10^{-8}}{4.44 f n_2 \Phi\, 10^{-8}} = \frac{n_1}{n_2} = \text{ratio of turns.} \tag{328}$$

If the secondary is connected to a receiver circuit of impedance $\dot{Z} = R + jX$, a current I_2 flows in it. The primary current is at the same time increased by a component I', the primary load current, which exerts a m.m.f. equal and opposite to that of the secondary current.

Thus

$$n_1 I' = n_2 I_2.$$

and

$$\frac{I_2}{I'} = \frac{n_1}{n_2} = \text{ratio of transformation} \tag{329}$$

The resultant m.m.f. acting on the magnetic circuit of the transformer is still that of the primary exciting current and the flux threading the two windings remains almost constant.

The primary current under load is I_1 and has two components I_0 the exciting current, which is proportional to the flux, and I' the load current which is proportional to the secondary current.

The exciting current $\dot{I}_0$ can be expressed as the product of the primary induced e.m.f. E_b and the primary exciting admittance $\dot{Y}_0 = g_0 - jb_0$; thus

$$\dot{I}_0 = \dot{E}_b\,(g_0 - jb_0) = \dot{E}'\,(g_0 - jb_o), \tag{330}$$

where E' is the component of impressed e.m.f. required to overcome the induced e.m.f. E_b.

The primary load current is $I' = \frac{n_2}{n_1} I_2$, and is opposite in phase to I_2.

As the load on the transformer is increased, the primary induced e.m.f. decreases (except when the power factor of the load is leading) because a larger component of impressed e.m.f. is consumed in driving the current through the primary impedance, thus,

$$\dot{E}' = -\dot{E}_b = \dot{E}_1 - \dot{I}_1\dot{Z}_1. \tag{331}$$

A smaller flux, is, therefore, required and a smaller exciting current. The decrease in flux from no-load to full-load non-inductive is 1 or 2 per cent. and for an inductive load of 50 per cent. power factor is only 5 or 6 per cent. With anti-inductive load the flux increases.

The secondary induced e.m.f., which is proportional to the primary, decreases with it.

The secondary terminal e.m.f. E is less than the secondary induced e.m.f. by the e.m.f. consumed by the secondary impedance thus

$$\dot{E} = \dot{E}_2 - \dot{I}_2\dot{Z}_2. \tag{332}$$

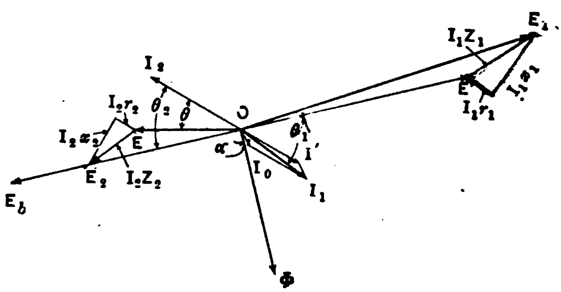


FIG. 339.—Vector diagram of a transformer with inductive load.

302. Vector Diagrams for the Transformer.—Fig. 339 is the vector diagram of e.m.fs. and currents in a transformer with a load impedance $\dot{Z} = R + jX$ and a load power factor $\cos\theta = \frac{R}{\sqrt{R^2 + X^2}}$.

E = secondary terminal e.m.f.

I_2 = secondary current lagging behind E by angle θ.

I_2r_2 = e.m.f. consumed by secondary resistance, in phase with I_2.

I_2x_2 = e.m.f. consumed by secondary reactance, in quadrature ahead of I_2.

I_2Z_2 = e.m.f. consumed by secondary impedance.

E_2 = e.m.f. induced in the secondary winding by the alternating flux Φ. $\dot{E}_2 = \dot{E} + \dot{I}_2(r_2 + jx_2)$.

E_b = e.m.f. induced in the primary winding by the alternating flux Φ. $E_b = \frac{n_1}{n_2}E_2$.

Φ = flux linking both primary and secondary windings, in quadrature ahead of E_b and E_2.

I_0 = primary exciting current leading the flux Φ by an angle $\alpha = 90 - \theta_0$, where $\cos\theta_0$ is the primary power factor at no load.

I' = primary load current in phase opposition to I_2, $I' = \frac{n_2}{n_1}I_2$.

I_1 = total primary current. $\dot{I}_1 = \dot{I}_0 + \dot{I}'$.

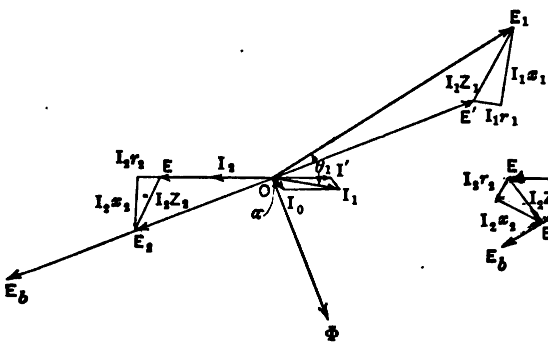


FIG. 340.—Transformer with non-inductive load.

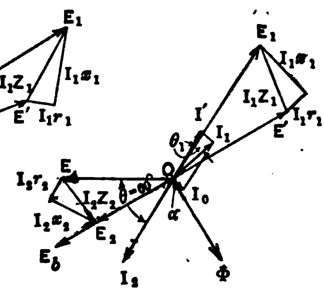


FIG. 341.—Transformer with a load power factor of 50 per cent. leading.

E' = component of primary impressed e.m.f. required to overcome the primary induced e.m.f E_b.

I_1r_1 = e.m.f. consumed by primary resistance, in phase with I_1.

I_1x_1 = e.m.f. consumed by primary reactance, in quadrature ahead of I_1.

I_1Z_1 = e.m.f. consumed by primary impedance.

E_1 = primary impressed e.m.f. $\dot{E}_1 = \dot{E}' + \dot{I}_1(r_1 + jx_1)$.

θ_1 = angle of lag of the primary current behind the primary impressed e.m.f.; $\cos\theta_1$ = primary power factor.

θ_2 = angle of lag of the secondary current behind the secondary induced e.m.f.

Fig. 340 shows the vector diagram of the transformer with a non-inductive load and Fig. 341 with a capacity load of 50 per cent. power factor leading.

The secondary current and secondary terminal e.m.f. are the same in the three cases and it may be seen that the required primary impressed e.m.f. is greatest in the case of the inductive load and least in the case of the capacity load.

303. Exciting Current.—When a sine wave of e.m.f. is impressed on the primary winding of a transformer, a sine wave of flux must be produced linking with the primary winding. The exciting current which produces the flux cannot be a sine wave on account of the lag of flux due to hysteresis. This is shown in Fig. 342. Curve (1) *abcf* is a hysteresis loop for the transformer

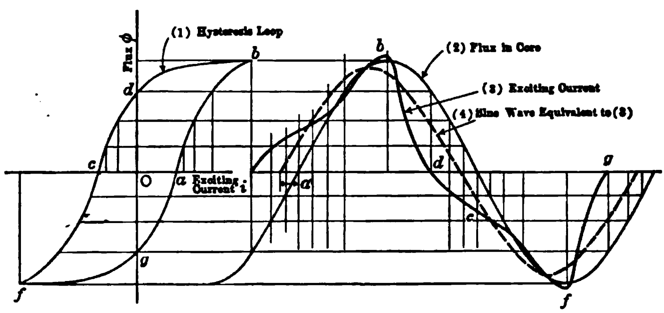


FIG. 342.—Exciting current.

iron, plotted with values of flux as ordinates on a base of exciting current. Curve (2) is the sine wave of flux in the core and curve (3) is the wave of exciting current. The method of obtaining curve (3) can be seen from the figure. The maximum values of flux and current must occur together; when the flux is zero the current has a value *oa* or *oc* and when the current is zero the flux has its residual value *od* or *og*.

For purposes of analysis the current wave (3) is replaced by the equivalent sine wave (4). The current wave (4) leads the flux wave (2) by an angle α, which is called the angle of hysteretic advance. If the eddy-current loss is small enough to be neglected, $\alpha = 90 - \theta_0$, where $\cos \theta_0$ is the no-load power factor.

304. Leakage Reactances.—Figs. 343 and 344 show the leakage paths around the windings of a "shell-type" and "core-type" transformer. Since the low-voltage windings are placed next to the iron, the leakage path surrounding the low-voltage winding is

of slightly lower reluctance than that surrounding the high-voltage winding and the reactance is correspondingly larger.

In the shell-type transformer the two windings are divided into a number of sections and high-voltage and low-voltage coils placed alternately to reduce the reactances.

The reactance voltage of a transformer with full-load current is about 10 per cent. of the total voltage. If, therefore, full vol-

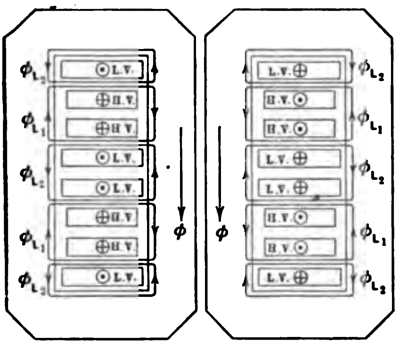


Fig. 343.—Leakage fluxes in a shell-type transformer.

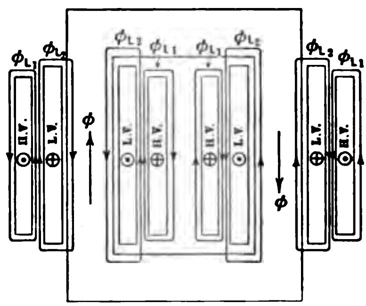


Fig. 344.—Leakage fluxes in a core-type transformer.

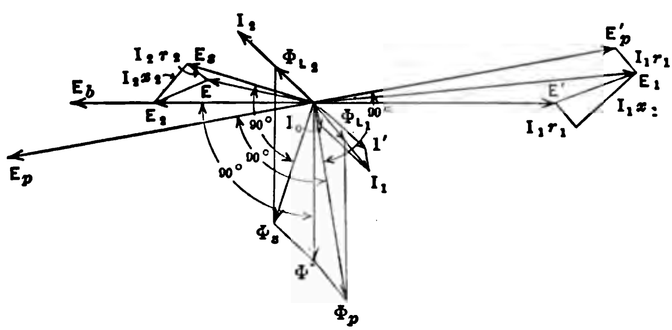


Fig. 345.

tage were impressed on a transformer primary with the secondary short-circuited about ten times full-load current would flow. When, however, full voltage is impressed on the primary with the secondary open, the exciting current which is less than 10 per cent. of full-load current flows. Thus the open-circuit reactance of a transformer is of the order of one hundred times the short-circuit reactance.

The vector diagram in Fig. 345 shows the various fluxes present in a transformer.

Φ = flux linking both primary and secondary windings.

Φ_{L_2} = leakage flux surrounding the secondary winding; it induces in the secondary an e.m.f. causing the secondary reactance drop I_2x_2.

Φ_s = total flux linking the secondary winding $\dot{\Phi}_s = \dot{\Phi} + \dot{\Phi}_{L_2}$; it induces in the secondary an e.m.f. E_s in quadrature behind Φ_s. $\dot{E}_s = \dot{E} + \dot{I}_2r_2$.

Φ_{L_1} = leakage flux surrounding the primary winding; it induces in the primary an e.m.f. causing the primary reactance drop I_1x_1.

Φ_P = total flux linking the primary winding $\Phi_P = \Phi + \Phi_{L_1}$, it induces in the primary an e.m.f. E_P in quadrature behind Φ_P.

E'_P = component of the impressed e.m.f. required to overcome E_P. $\dot{E}'_P = -\dot{E}_P = \dot{E}_1 - \dot{I}_1r_1$.

305. The Transformer as a Circuit.—The circuit in Fig. 346 represents a transformer supplying a load of resistance R and reactance X, or of power factor $\cos\theta = \dfrac{R}{\sqrt{R^2 + X^2}}$.

The relations between the various quantities may be expressed by the following equations:

$$\dot{E} = \dot{I}_2(R + jX) \tag{333}$$

$$\dot{E}_2 = \dot{I}_2\dot{Z}_2 + \dot{E} = \dot{I}_2(r_2 + jx_2) + \dot{I}_2(R + jX) \tag{334}$$

$$\dot{E}' = \frac{n_1}{n_2}\dot{E}_2 = \frac{n_1}{n_2}\dot{I}_2\{(r_2 + R) + j(x_2 + X)\} \tag{335}$$

$$\dot{I}' = \frac{n_2}{n_1}\dot{I}_2 \tag{336}$$

$$\dot{I}_0 = \dot{E}'(g_0 - jb_0) \tag{337}$$

$$\dot{I}_1 = \dot{I}_0 + \dot{I}' \tag{338}$$

$$\dot{E}_1 = \dot{I}_1\dot{Z}_1 + \dot{E}' \tag{339}$$

and these equations may be combined as follows:

$$\begin{aligned}
\dot{E}_1 &= \dot{I}_1\dot{Z}_1 + \dot{E}' \\
&= (\dot{I}_0 + \dot{I}')(r_1 + jx_1) + \frac{n_1}{n_2}\dot{I}_2\{(r_2 + R) + j(x_2 + X)\} \\
&= \dot{I}_0(r_1 + jx_1) + \dot{I}'(r_1 + jx_1) \\
&\qquad + \left(\frac{n_1}{n_2}\right)^2 \dot{I}'\{(r_2 + R) + j(x_2 + X)\} \\
&= \dot{I}_0(r_1 + jx_1) + \dot{I}'\left[\left\{r_1 + \left(\frac{n_1}{n_2}\right)^2(r_2 + R)\right\}\right. \\
&\qquad \left. + j\left\{x_1 + \left(\frac{n_1}{n_2}\right)^2(x_2 + X)\right\}\right].
\end{aligned} \tag{340}$$

This equation may be represented by the circuit in Fig. 347; secondary quantities are represented by equivalent primary quantities:

E.m.f. E_2 in the secondary is equivalent to e.m.f. $E' = \frac{n_1}{n_2}E_2$ in the primary.

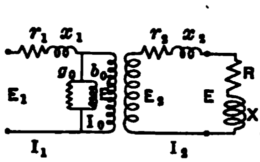


FIG. 346.—Circuit diagram of a transformer.

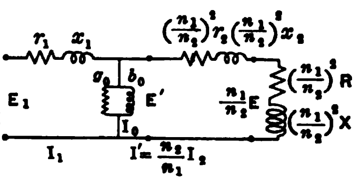


FIG. 347.—Equivalent circuit diagram of a transformer.

E.m.f. E in the secondary is equivalent to e.m.f. $\frac{n_1}{n_2}E$ in the primary.

Current I_2 in the secondary is equivalent to current $I' = \frac{n_2}{n_1}I_2$ in the primary.

Resistance r_2 in the secondary is equivalent to resistance $\left(\frac{n_1}{n_2}\right)^2 r_2$ in the primary.

Reactance x_2 in the secondary is equivalent to reactance $\left(\frac{n_1}{n_2}\right)^2 x_2$ in the primary.

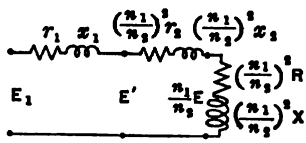


FIG. 348.—Simplified circuit diagram of a transformer.

If the exciting current is neglected, equation 340 becomes

$$E_1 = I'\left[\left\{r_1 + \left(\frac{n_1}{n_2}\right)^2(r_2 + R)\right\} + j\left\{x_1 + \left(\frac{n_1}{n_2}\right)^2(x_2 + X)\right\}\right] \quad (341)$$

and the simplified circuit diagram in Fig. 348 may be used. The resulting error is very small.

306. Examples. 1. A stepdown transformer with a ratio of turns of 10 : 1 delivers 100 kw. at 2,000 volts to a receiver of power factor 80 per cent. lagging. Determine the primary impressed voltage, the current and the power factor.

The primary impedance is

$$Z_1 = r_1 + jx_1 = 50 + 80j \text{ ohms.}$$

The secondary impedance is

$$Z_2 = r_2 + jx_2 = 0.6 + 0.8j \text{ ohms.}$$

The primary exciting admittance is

$$Y_0 = g_0 - jb_0 = (2 - 6j)\ 10^{-6}.$$

The current output or secondary current of the transformer is $\frac{100{,}000}{2{,}000 \times 0.80}$ $= 62.5$ and taking this as axis the various quantities may be expressed in rectangular coördinates as follows:

Secondary current

$$I_2 = 62.5 + 0j$$

Secondary terminal e.m.f.

$$E = 2{,}000 (\cos\theta + j\sin\theta) = 1{,}600 + 1{,}200j, \text{ where } \cos\theta = 0.8$$

is the load power factor.

Secondary impedance e.m.f.

$$I_2Z_2 = 62.5\ (0.6 + 0.8j) = 37.5 + 50j.$$

Secondary induced e.m.f.

$$E_2 = E + I_2Z_2 = 1{,}637.5 + 1{,}250j.$$

Primary induced e.m.f.

$$E_b = E' = \frac{n_1}{n_2} E_2 = 10E_2 = 16{,}375 + 12{,}500j.$$

Primary exciting current

$$I_0 = E'Y_0 = (16{,}375 + 12{,}500j)\ (2 - 6j)\ 10^{-6} = 0.11 - 0.07j.$$

Primary load current

$$I' = \frac{n_2}{n_1} I_2 = \frac{1}{10} I_2 = 6.25 + 0j.$$

Total primary current

$$I_1 = I' + I_0 = 6.36 - 0.07j.$$

Primary impedance e.m.f.

$$I_1Z_1 = (6.36 - 0.07j)\ (50 + 80j) = 324 + 505j.$$

Primary impressed e.m.f.

$$E_1 = E' + I_1Z_1 = 16{,}699 + 13{,}005j.$$

Taking absolute values,

primary impressed e.m.f.

$$E_1 = \sqrt{(16{,}699^2 + (13{,}005)^2} = 21{,}160 \text{ volts,}$$

primary current

$$I_1 = \sqrt{(6.36)^2 + (0.07)^2} = 6.36 \text{ amp.,}$$

exciting current

$$I_0 = \sqrt{(0.11)^2 + (0.07)^2} = 0.13 \text{ amp.,}$$

primary induced e.m.f.

$$E' = \sqrt{(16{,}375)^2 + (12{,}500)^2} = 20{,}600 \text{ volts.}$$

The primary impressed e.m.f. is inclined to the axis of coördinates at an angle θ', where

$$\tan \theta' = \frac{13{,}005}{16{,}699} = 0.7785 \text{ and therefore } \theta' = 37°50'.$$

The primary current is inclined to the axis at an angle θ'', where

$$\tan \theta'' = -\frac{0.07}{6.36} = -0.011 \text{ and therefore } \theta'' = -0° \ 40'.$$

The angle of phase difference between the primary current and the primary impressed e.m.f. is $\theta_1 = \theta' - \theta'' = 38° \ 30'$, and the primary power factor is $\cos \theta_1 = \cos 38° \ 30' = 0.782$, or 78.2 per cent.

The regulation of the transformer under these conditions of loading is $\frac{2{,}116 - 2{,}000}{2{,}000} \times 100$ per cent. $= 5.8$ per cent.

Primary copper loss is

$$I_1^2 r_1 = (6.36)^2 \times 50 = 2{,}020 \text{ watts.}$$

Secondary copper loss is

$$I_2^2 r_2 = (62.5)^2 \times 0.6 = 2{,}340 \text{ watts.}$$

Iron loss is

$$E'^2 g_0 = (20{,}600)^2 \times 2 \times 10^{-6} = 850 \text{ watts.}$$

The efficiency is therefore

$$\eta = \frac{\text{output}}{\text{output} + \text{losses}} 100 \text{ per cent.}$$

$$= \frac{100{,}000}{100{,}000 + 5{,}210} \times 100 \text{ per cent.} = 95 \text{ per cent.}$$

2. If the transformer in example 1, with 2,000 volts impressed, is used as a step-up transformer to charge a cable system of negligible resistance and supplies 5 amp., determine the secondary terminal e.m.f.

Primary impedance is now $\quad Z_1 = 0.6 + 0.8j.$
Secondary impedance is $\quad Z_2 = 50 + 80j.$
Primary exciting admittance is $Y_0' = 10^2 Y_0 = (2 - 6j)10^{-4}.$

Let the secondary terminal e.m.f. be E and take it as the real axis, the other e.m.fs. and currents may then be expressed in rectangular coördinates.
Secondary terminal e.m.f.

$$E = E + 0j.$$

Secondary current

$$I_2 = 0 + 5j.$$

Secondary impedance e.m.f.

$$I_2 Z_2 = 5j\,(50 + 80j) = -400 + 250j.$$

Secondary induced e.m.f.

$$E_2 = E + I_2 Z_2 = E - 400 + 250j.$$

Primary induced e.m.f.

$$E' = 1/10\, E_2 = 0.1E - 40 + 25j.$$

Primary load current

$$I' = 10\, I_2 = 0 + 50j.$$

Primary exciting current

$$\dot{I}_0' = \dot{E}'\dot{Y}_0' = (0.1E - 40 + 25j)(2 - 6j)\,10^{-4}$$
$$= \{(0.2\,E + 70) - (0.6E - 290)\,j\}10^{-4}$$

Total primary current

$$\dot{I}_1 = \dot{I}' + \dot{I}_0 = [\{(0.2E + 70) - (0.6E - 290)j\}10^{-4} + 50j].$$

Primary impedance e.m.f.

$$\dot{I}_1\dot{Z}_1 = [\{(0.2E + 70) - (0.6E - 290)j\}10^{-4} + 50j](0.6 + 0.8j)$$
$$= [\{(0.6E - 190)10^{-4} - 40\} + \{(-0.2E + 230)10^{-4} + 30\}].$$

Primary impressed e.m.f.

$$\dot{E}_1 = \dot{E}' + \dot{I}_1\dot{Z}_1 = (0.1E - 80) - 55j.$$

The absolute value of primary impressed e.m.f. is

$$E_1 = \sqrt{(0.1\,E - 80)^2 + (55)^2} = 2{,}000$$

and solving this gives the secondary terminal e.m.f.

$$E = 20{,}800.$$

307. Measurement of the Constants of a Transformer.—The constants are: (1) the ratio of turns $\frac{n_1}{n_2}$, (2) the primary exciting admittance $Y_0 = \sqrt{g_0^2 + b_0^2}$, (3) the primary impedance $Z_1 = \sqrt{r_1^2 + x_1^2}$ and (4) the secondary impedance $Z_2 = \sqrt{r_2^2 + x_2^2}$. The first two can be obtained from an open-circuit test and the last two from a short-circuit test.

I_0 / E_1

g_0

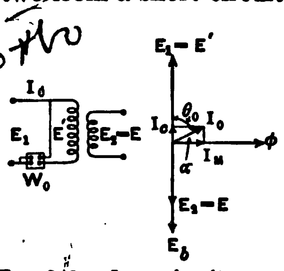


FIG. 349.—Open-circuit test.

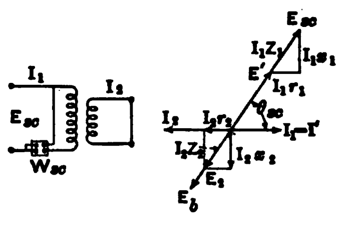


FIG. 350.—Short-circuit test.

For the open-circuit test, Fig. 349, impress full voltage E_1 on the primary, at rated frequency, and read primary impressed voltage E_1, secondary terminal voltage $E = E_2$, primary exciting current I_0 and the power input W_0 which gives the core loss plus a very small copper loss $I_0^2r_1$ which may be neglected.

The ratio of turns is found very approximately as $\frac{n_1}{n_2} = \frac{E'}{E_2} = \frac{E_1}{E}$; the primary exciting admittance is $Y_0 = \sqrt{g_0^2 + b_0^2} = \frac{I_0}{E_1}$; the core-loss current is $I_c = \frac{W_0}{E_1} = E_1g_0$, and thus, $g_0 = \frac{W_0}{E_1^2}$;

the magnetizing current is $I_M = \sqrt{I_0^2 - I_c^2} = E_1 b_0$, or b_0 may be found as $b_0 = \sqrt{Y_0^2 - g_0^2}$; the no load power factor is $\cos \theta_0 = \frac{W_0}{E_1 I_0}$ and the angle of hysteretic advance is $\alpha = 90 - \theta_0$, if the eddy-current loss is neglected.

In the short-circuit test, Fig. 350, the secondary is short-circuited and a voltage E_{sc} is impressed on the primary of such a value that full-load current I_1 flows; the secondary will then carry full-load current I_2; readings are taken of E_{sc}, I_1 and the power input W_{sc}; the power factor is found as $\cos \theta_{sc} = \frac{W_{sc}}{E_{sc} I_1}$. The exciting current is so small that it may be neglected.

Since the terminal voltage is zero, the impressed voltage E_{sc} is consumed by the impedance of the transformer and is the full-load impedance drop. When expressed as a per cent. of the rated primary voltage, it is the per cent. impedance drop. The per cent. resistance drop can be found by multiplying by $\cos \theta_{sc}$ and the per cent. reactance drop by multiplying by $\sin \theta_{sc}$.

$$
\begin{aligned}
E_{sc} &= I_1 Z_1 + \frac{n_1}{n_2} I_2 Z_2 \\
&= I_1 \left\{ Z_1 + \left(\frac{n_1}{n_2}\right)^2 Z_2 \right\} \\
&= I_1 \sqrt{\left\{ r_1 + \left(\frac{n_1}{n_2}\right)^2 r_2 \right\}^2 + \left\{ x_1 + \left(\frac{n_1}{n_2}\right)^2 x_2 \right\}^2} \\
&= I_1 \sqrt{r^2 + x^2} = I_1 Z,
\end{aligned}
$$

where r, x and Z are the equivalent resistance, reactance and impedance of the transformer as primary quantities.

The power input W_{sc} = primary and secondary copper losses plus a very small core loss, which may be neglected.

$$W_{sc} = I_1^2 r_1 + I_2^2 r_2 = I_1^2 \left\{ r_1 + \left(\frac{n_1}{n_2}\right)^2 r_2 \right\} = I_1^2 r$$

and thus

$$r = r_1 + \left(\frac{n_1}{n_2}\right)^2 r_2 = \frac{W_{sc}}{I_1^2};$$

to determine the values of r_1 and r_2, it may be assumed that the two windings have been designed for the same current density and that therefore the two copper losses are approximately the same and

$$r_1 = \frac{W_{sc}}{2I_1^2} = \frac{r}{2} = \left(\frac{n_1}{n_2}\right)^2 r_2.$$

In a similar way $x = x_1 + \left(\frac{n_1}{n_2}\right)^2 x_2$ may be separated into its two components $x_1 = \left(\frac{n_1}{n_2}\right)^2 x_2 = \frac{x}{2}$. This is correct if the leakage fluxes about the two windings are equal, which may be assumed to be the case, since the m.m.fs. are equal and the leakage paths are very similar.

308. Regulation.—The regulation of a transformer is the rise in secondary terminal voltage, when full load is thrown off, expressed as a per cent. of full-load voltage. The regulation of a transformer is very much better than that of an alternator because the resistance and reactance drops are very much smaller.

The regulation at 100 per cent. power factor ranges from 1.2 to 2 per cent. for large 25-cycle transformers and from 0.75 to 1.6 per cent. for large 60-cycle power transformers. Larger values are found in some small transformers. The regulation of large power transformers may be as high as 6 or 7 per cent. at 80 per cent. power factor.

If the terminal voltage at full load is $E = 100$ per cent. and the impedance drop is expressed in per cent., the vector sum of the two will give the terminal voltage at no load in per cent. Taking

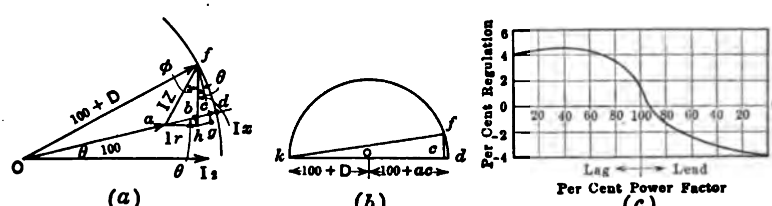


Fig. 351.—Regulation of a transformer.

the current as axis and a load power factor cos θ, the secondary terminal voltage is $\dot{E} = 100 \cos \theta + j100 \sin \theta$; the impedance drop is $\dot{I}\dot{Z} = Ir + jIx$, all in per cent., and the terminal voltage at no load is $\dot{E}_0 = \dot{E} + \dot{I}\dot{Z} = (100 \cos \theta + Ir) + j(100 \sin \theta + Ix)$; its absolute value is

$$E_0 = \sqrt{(100 \cos \theta + Ir)^2 + (100 \sin \theta + Ix)^2} \qquad (342)$$

and the per cent. regulation is $E_0 - 100$.

A formula giving the regulation directly may be found by referring to the vector diagram in Fig. 351(a). The per cent.

regulation for a load power factor $\cos\theta$ is represented by the length ad =

$$D = ab + bc + cd = Ir\cos\theta + Ix\sin\theta + cd.$$

From Fig. 351 (b), $cd \times ck = \overline{cf}^2$ and $cd = \frac{\overline{cf}^2}{ck} = \frac{(fg - cg)^2}{200 + D + ac} = \frac{(Ix\cos\theta - Ir\sin\theta)^2}{200 +}$ and the per cent. regulation is

$$D = Ir\cos\theta + Ix\sin\theta + \frac{(Ix\cos\theta - Ir\sin\theta)^2}{200} \qquad (343)$$

where Ir is the per cent. resistance drop and Ix the per cent. reactance drop in the transformer.

The last term is negligible except near unity power factor and if it is neglected, the per cent. regulation is

$$D = Ir\cos\theta + Ix\sin\theta = IZ\left(\frac{Ir}{IZ}\cos\theta + \frac{Ix}{IZ}\sin\theta\right)$$
$$= IZ(\sin\phi\cos\theta + \cos\phi\sin\theta) = IZ\sin(\theta + \phi); \qquad (344)$$

the angle $\phi = \tan^{-1}\frac{r}{x}$ is shown in Fig. 351.

The regulation is maximum when $\sin(\theta + \phi) = 1$, or $\theta = 90 - \phi$, that is, at a load power factor $\cos\theta = \cos(90 - \phi) = \sin\phi = \frac{r}{Z}$ and is equal to the per cent. impedance drop IZ.

Take for example a single-phase, 60-cycle, 33,000-volt transformer of 100 kva. output with a resistance drop of 1.5 per cent. and a reactance drop of 4 per cent., and calculate the regulation at 100 per cent. power factor and 80 per cent. power factor lag and lead, using equation (343):

At 100 per cent. power factor, the regulation

$$= 1.5 + \frac{4^2}{200} = 1.5 + 0.08 = 1.58 \text{ per cent.}$$

At 80 per cent. power factor lag the regulation

$$= 1.5 \times 0.8 + 4 \times 0.6 + \frac{(4 \times 0.8 - 1.5 \times 0.6)^2}{200}$$
$$= 1.2 + 2.4 + 0.026 = 3.626 \text{ per cent.}$$

At 80 per cent. power factor lead, the regulation

$$= 1.5 \times 0.8 - 4 \times 0.6 + \frac{(4 \times 0.8 + 1.5 \times 0.6)^2}{200}$$
$$= 1.2 - 2.4 + 0.08 = -1.12 \text{ per cent.}$$

The maximum regulation is $D_{max.} = IZ = \sqrt{\overline{Ir}^2 + \overline{Ix}^2} = \sqrt{1.5^2 + 4^2} = 4.27$ per cent. and corresponds to a load power-factor, $\cos\theta = \frac{r}{Z} = \frac{1.5}{4.27} = 0.35 = 35$ per cent.

The complete regulation curve for this transformer is shown in Fig. 351(*c*). For leading power factors the regulation becomes negative, that is, the voltage rises with load.

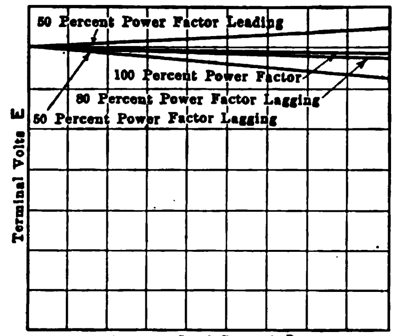


FIG. 352.—Voltage characteristics of a transformer.

309. Voltage Characteristics.—The voltage characteristics of a transformer show the relation between the secondary terminal voltage and the secondary load current for given values of power factor, with a constant impressed voltage E_1. Typical voltage characteristics are shown in Fig. 352 plotted in per cent. Since the no-load secondary voltage is constant $= E_0 = \frac{n_2}{n_1}E_1$, it has been taken as 100 per cent. and the values of E calculated from the equation,

$$E_0 = 100 = \sqrt{(E\cos\theta + Ir)^2 + (E\sin\theta + Ix)^2}.$$

310. Losses in Transformers.—The losses in a transformer are the copper losses and the iron losses.

The total copper loss is

$$W_c = I_1^2r_1 + I_2^2r_2 \tag{345}$$

and is almost evenly divided between the primary and the secondary since the two windings are designed for approximately the same current density.

The iron loss consists of the hysteresis and eddy-current losses and does not vary with load. It can be measured by applying full voltage to the primary with the secondary open and reading the watts input,

$$W_0 = E_1I_0\cos\theta_0 = E_1I_c = E_1^2g_0. \tag{346}$$

W_0 includes a small copper loss $I_0^2r_1$ which may be neglected.

311. Hysteresis Loss.—The hysteresis loss is the energy consumed in magnetizing and demagnetizing the iron. It is directly

proportional to the frequency f, varies as the 1.6th power of the maximum induction density $\mathcal{B}_m$, and also depends on the quality of the iron.

The hysteresis loss per cycle per cubic centimeter of iron is

$$w_h = \eta \mathcal{B}_m^{1.6} \text{ ergs,} \tag{347}$$

where η is the hysteresis constant for the iron and has a value of about 0.003 for good transformer punchings. In silicon-steel plates η may be as low as 0.001.

The hysteresis loss per second in a volume of V c.c. at a frequency f is

$$\begin{aligned} W_h &= \eta f V \mathcal{B}_m^{1.6} \text{ ergs per second} \qquad (348) \\ &= \eta f V \mathcal{B}_m^{1.6}\, 10^{-7} \text{ watts.} \end{aligned}$$

At constant frequency the hysteresis loss varies approximately as the 1.6th power of the impressed voltage.

312. Eddy-current Loss in Transformer Iron.—The eddy-current loss is the energy consumed by the currents induced in the iron of the transformer by the alternating flux cutting it.

In Fig. 353

t = thickness of sheets in centimeters,
$\mathcal{B}_m$ = maximum induction density,
f = frequency,
γ = electric conductivity of iron.

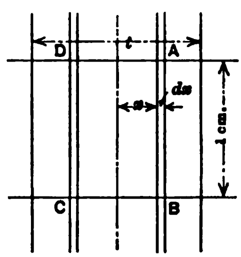


FIG. 353.—Eddy current loss in sheet iron.

AB is a section of length 1 cm., depth 1 cm. and width dx cm., parallel to the edge of the plate and distant x cm. from the center. DC is a similar section on the other side.

The maximum flux inclosed by $ABCD$ is $2\mathcal{B}_m x$ lines. Of this $\mathcal{B}_m x$ lines cut across the section AB and generate in it an e.m.f. of effective value,

$$dE = \sqrt{2}\pi f \mathcal{B}_m x \text{ c.g.s. units.}$$

The conductance of the section AB is $\gamma\, dx$, and thus the current induced in it is

$$dI = dE\gamma\, dx = \sqrt{2}\pi f \mathcal{B}_m \gamma x\, dx \text{ c.g.s. units.}$$

The power consumed by the induced current in section is

$$dw_e = dE\, dI = 2\pi^2 f^2 \mathcal{B}_m^2 \gamma x^2\, dx \text{ ergs per second}$$

and thus the power consumed per square centimeter of the plate is

$$\omega_\epsilon = 2\pi^2 f^2 \mathcal{B}_m{}^2 \gamma \int_{-\frac{t}{2}}^{\frac{t}{2}} x^2 dx$$

$$= 2\pi^2 f^2 \mathcal{B}_m{}^2 \gamma \left[\frac{x^3}{3}\right]_{-\frac{t}{2}}^{\frac{t}{2}}$$

$$= \pi^2 f^2 \mathcal{B}_m{}^2 \gamma \frac{t^3}{6} \text{ ergs per second.} \qquad (349)$$

The power consumed per cubic centimeter of iron is

$$\omega_1 = \frac{\omega_\epsilon}{t} = \frac{\pi^2 f^2 \mathcal{B}_m{}^2 \gamma t^2}{6} \text{ ergs per second.} \qquad (350)$$

Thus the eddy-current loss for a volume of V c.c. is

$$W_\epsilon = \frac{\pi^2 f^2 \mathcal{B}_m{}^2 \gamma t^2 V}{6} \text{ ergs per second}$$

$$= \frac{\pi^2 f^2 \mathcal{B}_m{}^2 \gamma t^2 V}{6} 10^{-7} \text{ watts.} \qquad (351)$$

The eddy-current loss, therefore, varies as the square of the frequency, the square of the maximum induction density and the square of the thickness of the plates. It also varies directly with the electric conductivity of the iron.

To reduce the eddy-current loss the core is usually built up of sheets of about 0.014 in. thick insulated with varnish. The electric conductivity of ordinary sheet iron is about 10^5. Silicon steel has a conductivity in some cases as low as 2×10^4 and it may be used in sheets 0.020 in. thick.

With a sine wave of impressed voltage the eddy-current loss varies as the square of the voltage, but if the wave is peaked the loss may be greater, (provided the virtual value of the applied emf is given

Since iron has a positive temperature coefficient for resistance the eddy-current loss will decrease slightly as the transformer heats up.

313. Extra Losses.—When the windings of a transformer are made up of a number of sections connected in multiple, if the impedances of the sections are not the same, circulating currents will flow and increase the copper losses. They will exist at no load as well as under load.

If wide copper strip is used for the winding, eddy currents may flow in the copper due to the fact that one side is in a stronger

field than the other. This may increase the copper loss as much as 20 per cent. in extreme cases. It is not advisable to use strip of width greater than 0.50 in. in 60-cycle transformers or greater than 0.75 in. in 25-cycle transformers.

314. Efficiency.—The efficiency of a transformer is

$$\eta = \frac{\text{output}}{\text{output} + \text{copper loss} + \text{iron loss}} 100 \text{ per cent.}$$

$$= \frac{EI_2 \cos\theta}{EI_2 \cos\theta + I_1^2 r_1 + I_2^2 r_2 + k} 100 \text{ per cent.}$$

$$= \frac{EI_2 \cos\theta}{EI_2 \cos\theta + I_2^2 r + k} 100 \text{ per cent.,} \tag{352}$$

where $r = \left(\frac{n_2}{n_1}\right)^2 r_1 + r_2$ is the equivalent secondary resistance and k = constant iron loss.

The efficiency is very high and remains high over a wide range of load; it reaches its maximum value, for a given load power factor, when the variable copper loss = the constant iron loss. This may be shown by differentiating η with respect to I_2 and equating the result to zero.

$$\frac{d\eta}{dI_2} = \frac{\{(EI_2 \cos\theta + I_2^2 r + k)E\cos\theta - EI_2\cos\theta\,(E\cos\theta + 2I_2 r)\}}{(EI_2\cos\theta + I_2^2 r + k)^2} 100 = 0$$

for maximum efficiency.

Therefore, $E \cos\theta\,(EI_2 \cos\theta + I_2^2 r + k - EI_2 \cos\theta - 2I_2^2 r) = 0$ and $I_2^2 r = k$, that is, the copper loss = the iron loss. Below one-fourth load the efficiency falls off rapidly due to the constant iron losses and above full load it falls due to the large copper losses.

The efficiency also depends on the power factor of the load, decreasing with it but not in direct proportion.

In the smaller sizes the efficiency at unity power factor varies from 93 per cent. at one-fourth load to 97 per cent. at full load and in the larger sizes from 97 per cent. at one-fourth load to 99 per cent. at full load.

Take for example a transformer rated at 100 kva. with a full-load copper loss of 2,000 watts and an iron loss of 1,600 watts.

The efficiency at full-load unity power factor is

$$\eta = \frac{100{,}000}{100{,}000 + 2{,}000 + 1{,}600} 100 \text{ per cent.} = 96.5 \text{ per cent.;}$$

the efficiency at full-load 80 per cent. power factor is

$$\eta = \frac{100{,}000 \times 0.80}{100{,}000 \times 0.80 + 2{,}000 + 1{,}600}\, 100 \text{ per cent.} = 95.7 \text{ per cent.}$$

the efficiency at one-fourth load unity power factor is

$$\eta = \frac{\dfrac{100{,}000}{4}}{\dfrac{100{,}000}{4} + \dfrac{2{,}000}{16} + 1{,}600}\, 100 \text{ per cent.} = 93.6 \text{ per cent.}$$

In the case of distributing transformers which are connected to the supply lines at all times but are delivering power for only a few hours during the day, the all-day efficiency is of more importance than the actual efficiency. It is the ratio of the energy output during the day to the energy input.

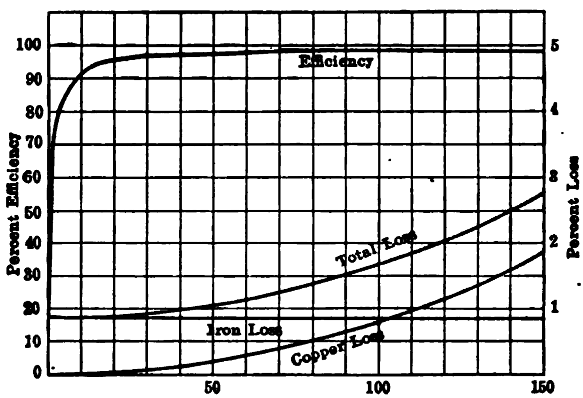


FIG. 354.—Efficiency and loss curves of a 550-kw., 10,500-volt, 60-cycle, air-blast transformer.

If the transformer mentioned above is operated at full-load unity power factor for 6 hr. during the day and is unloaded the remainder of the time, the all-day efficiency is

$$\eta \text{ all-day} = \frac{\text{energy output}}{\text{energy input}}\, 100 \text{ per cent.}$$

$$= \frac{100{,}000 \times 6}{100{,}000 \times 6 + 2{,}000 \times 6 + 1{,}600 \times 24}\, 100 \text{ per cent.} = 92 \text{ per cent.}$$

When a good all-day efficiency is required, the iron losses should be kept small by using the best grades of iron or by reducing the flux density.

Fig. 354 shows efficiency and loss curves for a large transformer.

315. Types of Transformers.—Transformers may be divided into two general types depending on the arrangement of the core and the windings: the core type, (Fig. 355); and the shell type (Fig. 356).

In the core type a single ring of iron is surrounded by two groups of windings, while in the shell type a single ring of copper formed of the two windings is surrounded by two or more rings of iron. The core type has a long magnetic path and a short

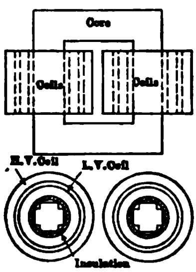


FIG. 355.—Core-type transformer.

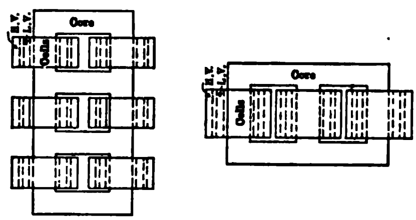


FIG. 357.—Three-phase core-type transformers.

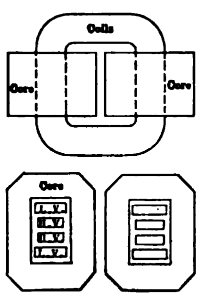


FIG. 356.—Shell-type transformer.

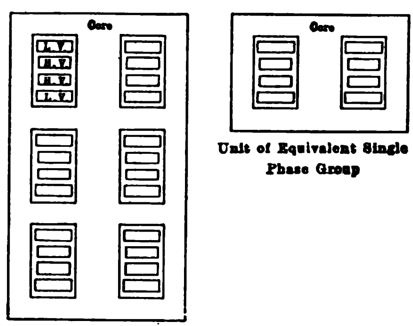


FIG. 358.—Three-phase shell-type transformer.

mean turn of winding and the shell type has a short magnetic circuit and a long electric circuit. The core type is more suitable for small high-voltage transformers as it is very easy to insulate the high-voltage from the low-voltage coils by placing a cylinder of insulating material between them. Large power transformers are usually made of the shell type but core-type transformers can be designed with equally good characteristics.

Three-phase transformers are built of both the core and the shell types (Figs. 357 and 358). In Fig. 358 a three-phase trans-

former is compared with one of the equivalent group of single-phase transformers. Since the fluxes in the three phases are combined at 120 degrees in the common paths a considerable saving in core material is possible.

In the majority of polyphase systems groups of single-phase transformers are used instead of three-phase units but the three-phase units have some advantages which tend to increase their application.

A three-phase transformer is less expensive than the equivalent single-phase group requiring a smaller weight of iron, a single tank, and fewer high-voltage brushings, since the connections between phases are made inside the tank. The floor space required is very much less and the bus-bar and switching layouts are simplified, as is also the cooling water system. A spare three-phase unit must be installed instead of a spare single-phase unit but the extra capacity may be very valuable at times of overload. For very large outputs three-phase units may be too heavy to handle easily in case repairs are necessary and single-phase units may be more satisfactory.

If one phase of a three-phase delta-connected transformer is damaged, operation may be carried on with reduced output using the other two windings in open-delta but both the primary and secondary windings of the damaged phase must be disconnected and short-circuited.

316. Methods of Cooling.—Very small transformers do not require any special method of cooling but are so designed that the exposed surface is large enough to radiate the heat generated by the power losses in the windings and core without a temperature rise exceeding the limits consistent with the life of the insulation. Transformers must be designed to operate for 24 hr. at full load with a temperature rise not exceeding 55°C. above the ambient temperature of 40°C.

Since the output and losses in a transformer increase in proportion to its volume or as the cube of its linear dimensions while the radiating surface increases only as the square of the linear dimensions, as the output is increased special methods of cooling must be adopted.

Transformers up to 500 kw. are usually immersed in tanks containing oil of good insulating qualities. This oil serves the double purpose of increasing the insulation of the transformer and conducting away the heat developed by the losses. Such trans-

formers are called oil-insulated self-cooled transformers. The cases are made with deep corrugations to give a larger radiating surface exposed to the air.

The oil, as it comes into contact with the coils and core, becomes heated, rises to the top of the tank and flows outward to the sides where it gives up its heat to the air and falls again to the bottom of the tank. A continuous circulation of oil is thus maintained and care must be taken that paths of ample section are left for the passage of the oil through the transformer (Fig. 359).

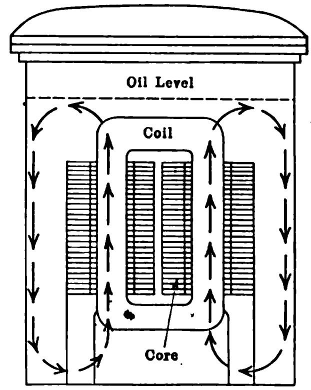
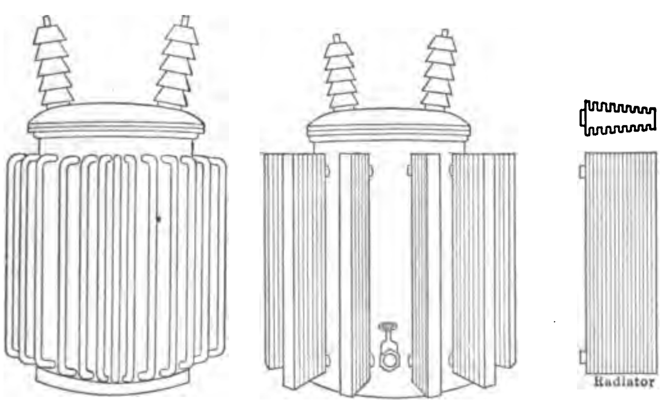


FIG. 359.—Oil circulation in a shell-type transformer.

Where larger self-cooled units are required, as in the case of large outdoor substances, special methods of design are necessary to provide sufficient radiating surface. Two successful types are shown in Fig. 360, the tubular tank and the external radiator tank.

The tubular tank has a large number of tubes welded to it, which draw the hot oil from the top of the tank and feed it back



Tubular tank. External radiator tank.

FIG. 360.—Oil-insulated self-cooled transformers.

near the bottom after its temperature has been reduced. The tubes are very efficient radiators and the effective surface of the tank may be increased many times by their use. Self-cooled units up to 2,000 kva. may be built in this way.

A second method of increasing the radiating surface of the tank is shown in Fig. 360. Radiators made of corrugated iron with very large surfaces are used to replace the tubes in the last design. They are made so that they can be easily detached when the transformer is to be shipped. Units of this type with an output of 5,000 kva. are in successful operation.

These special tanks are very bulky and are not suitable for installation where space is limited.

In the case of large banks of transformers it may be economical to provide a forced oil circulation. The hot oil is pumped from the transformers to a chamber where it is cooled by an air blast or in some other way.

A second method of getting rid of the heat developed by the losses is to blow air through the transformers to cool them. Transformers cooled in this way are termed "air-blast" and are used on electric locomotives where their light weight is an advantage and in places where oil cannot be used on account of the danger of fire. They cannot be operated safely above about 30,000 volts as the insulation rapidly deteriorates due to ozone set free in the air at high voltages. Oil is then a necessary protection.

For air-blast transformers about 150 cu. ft. of air are required per minute per kilowatt lost to keep the temperature within the permissible limits.

The majority of large transformers are cooled by placing in the upper part of the tank cooling coils which carry a continuous flow of cold water, which conveys the heat away from the oil. With the incoming water at 25°C., about ¼ gal. per minute pe- kilowatt lost is required. If too much water is supplied the transformer may be cooled below the temperature of the air and moisture may collect on it and cause a breakdown of the insulation. The cooling coils should have a surface in contact with the oil of about 1.0 sq. in. per watt lost.

Impurities in the water are liable to collect in the pipes and interfere with the flow.

Combined self-cooling and water-cooled transformers are sometimes built. The radiating surface of the tank is large enough to take care of the losses up to half load but above this point water must be circulated through the coils to keep down the temperature.

317. Transformer Connections.—If the primary and secondary windings are each divided into a number of coils, which

can be connected either in series or parallel, a number of different ratios of transformation can be obtained.

Take for example a standard lighting transformer with two coils for 110 volts on the low-voltage side and two coils for 1,100 volts

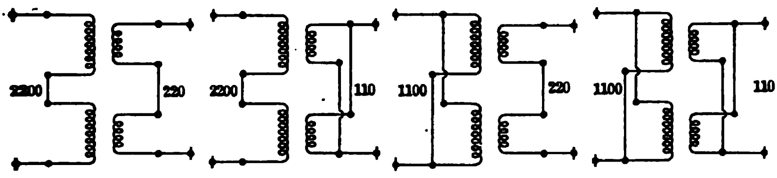


FIG. 361.—Transformer connections.

on the high-voltage side. The four possible connections are shown in Fig. 361.

If small percentage changes of ratio are required for line regulation a number of taps are brought out from the primary or secondary winding so that the number of turns in use may be changed by the required amount (Fig. 362).

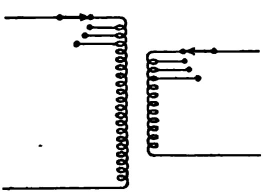


FIG. 362.—Variable ratio transformer.

The secondary voltage may be increased by cutting out some of the primary turns or it may be decreased by cutting out some of the secondary turns.

318. Single-phase Transformers on Polyphase Circuits.—Single-phase transformers are used in groups on polyphase circuits. The principal transformations are two-phase to two-phase, three-phase to three-phase, two-phase to three-phase or three-phase to two-phase, and three-phase to six-phase.

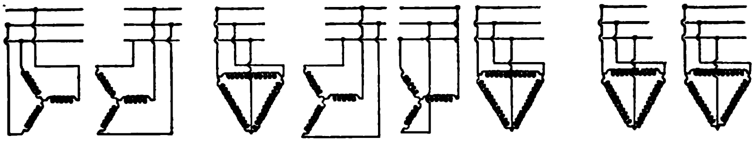


FIG. 363.—Single-phase transformers on three-phase circuits.

The transformation from two-phase to two-phase needs no discussion. The two phases are usually entirely separate but they may be interconnected if required.

Transformation from three-phase to three-phase can be

carried out in a large number of ways, the most important of which are, star-star, delta-star, star-delta, delta-delta, open-delta and tee (Figs. 363, 366 and 367).

Transformation from two-phase to three-phase is obtained by the use of the Scott connection, Fig. 369, and is reversible.

Six-phase power is required for the operation of many rotary converters. It can be obtained from a three-phase circuit by the double-delta connection, the diametrical or double-star connection and the ring connection, Fig. 370. Other methods of connection are possible such as the double tee. Six-phase power can be obtained from two-phase circuits by using two Scott connected banks of transformers.

319. Star-star Connection.—The star-star connection may be used to transform three-phase power to three-phase power at another voltage but it has some disadvantages which restrict its use to special cases.

In Art. 303 it was shown that, when a sine wave of voltage is impressed on a transformer and a sine wave of flux is produced in the core, the exciting current is not a sine wave but has the shape shown in Fig. 342. When such a wave is analyzed as in Art. 121 it is found to consist of a fundamental sine wave with a very prominent third harmonic and higher harmonics of smaller amplitude.

The fundamental sine wave of exciting current or the equivalent sine wave in Fig. 342 may be separated into two sine waves as in Fig. 364 the core-loss current, curve 3, in phase with the impressed e.m.f., curve 1, and the magnetizing current, curve 4, in quadrature behind the impressed e.m.f. and in phase with the flux, curve 2. Due to saturation a third harmonic magnetizing current, curve 5, must be added to the fundamental to produce the sine wave of flux. The shape of the resultant magnetizing current wave, curve 6, is therefore symmetrical with a peak at the center. The wave of exciting current, curve 7, is found as the sum of 3 and 6, and has the shape shown in Figs. 342 and 364. The amplitude of the third harmonic increases with the degree of saturation of the core.

In the three branches of the star connection, the three fundamental currents are displaced at 120 degrees while the third harmonic currents are displaced $120 \times 3 = 360$ degrees and are therefore in phase. For the third harmonics the ungrounded three-phase circuit is like a single-phase circuit without any

return. When the neutral point is not grounded the third harmonic and its multiples in the exciting current cannot flow and are therefore suppressed.

The result is that the flux wave is no longer a sine wave but may be considered to be made up of a sine wave, curve 3, Fig. 365, and a third harmonic, curve 4. The third harmonic flux is opposite in phase to the missing third harmonic of magnetizing current and the resultant flux wave, curve 2, is flat on the top and has very steep sides.

The sine wave of flux induces a sine wave of voltage, curve 8, in the two windings, in quadrature behind itself and the third

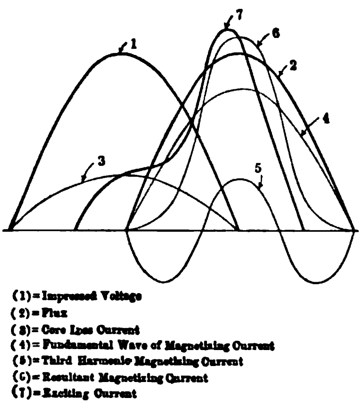


FIG. 364.

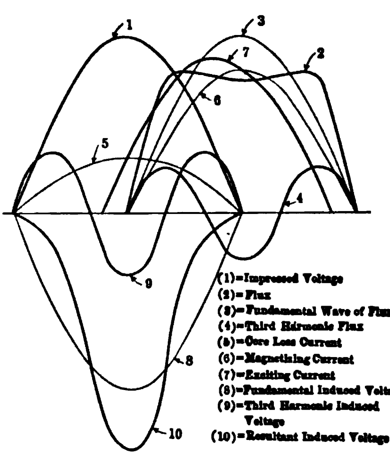


FIG. 365.

harmonic flux induces a third harmonic voltage, curve 9, in quadrature behind itself. The two components of the induced voltage combine to form the peaked wave, curve 10.

The maximum value of the third harmonic voltage induced in the secondary is $2\pi \times 3fn_2\Phi_3\, 10^{-8} = 6\pi fn_2\Phi_3\, 10^{-8}$ volts, where Φ_3 is the maximum value of the third harmonic flux. Since the secondaries are star-connected, the third harmonic cannot appear in the voltage between lines but it will increase the maximum value of the voltage in each transformer and may cause a breakdown of the insulation. The effective value of the secondary voltage may be increased only 5 or 10 per cent., while the maximum value is increased 40 or 50 per cent. This is a very dan-

gerous condition in high-voltage transformers where the factor of safety is small.

If the primary neutral is connected to the generator neutral the third harmonic of exciting current can flow, returning along the neutral and there will be no distortion of the voltage wave.

If the neutral on the secondary side only at the generating end of a transmission line is grounded and the neutral on the primary side at the receiving end, the third harmonics in the induced secondary voltage waves will cause third harmonic currents to flow in the three lines and since they are in phase with one another they may produce inductive disturbances in neighboring telephone lines.

The star-star connection cannot be used to supply a three-phase four-wire system since the three outers pulsate against the neutral connection at triple-frequency.

In star-star connected three-phase core-type transformers, on account of the mutual inductance between phases the third harmonic in the induced voltage wave is suppressed.

Another objection to the use of this connection is the fact that, if one transformer is short-circuited, the voltage across the other two increases by 73 per cent. unless the primary and generator neutrals are grounded. In any case if one transformer of the group is damaged the whole group must be shut down.

320. Delta-star and Star-delta Connections.—The delta-star connection is used at the generating end of high-voltage transmission lines and the star-delta connection at the receiving end.

The advantage of the star connection for the high-voltage side is that very high line voltages can be obtained with moderate transformer voltages, the ratio of the line voltage to the transformer voltage being $\sqrt{3}:1$, the cost of insulating the transformer is therefore less than if it were designed for the full-line voltage and the conductors being designed for larger currents are of greater section and better able to stand mechanical strains.

In stepping up with the delta-star connection the required third harmonic exciting current can flow in the three transformers and the induced voltages will be sine waves, if the impressed voltages are, and the secondary neutral can be grounded without causing any inductive disturbances.

When stepping down with the star-delta connection the required third harmonic currents cannot flow in the primary but the third harmonic e.m.fs. induced in the three transformers are in-

phase and cause a current to circulate through the delta and this acts as a magnetizing current and supplies the missing component of flux. The exciting current is therefore divided between the primary and the secondary.

321. Delta-delta Connection.—This connection is used very extensively even in the case of very high-voltage transmission lines, where it is thought advisable to make provision for an increase of the transmission voltage to meet the requirements of future extensions of the system. In such a case the secondary may be changed to star and extra insulation added to the line.

There is no trouble from wave distortion as in the star-star connection and if one transformer burns out the system can still be operated without any change in connections with the two remaining transformers in open delta but the load must be decreased to about 60 per cent. for the same temperature rise.

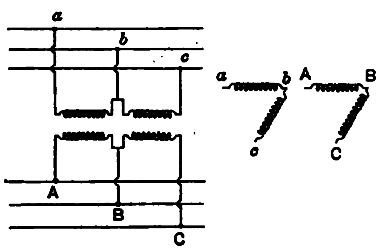


FIG. 366.—Open-delta connection.

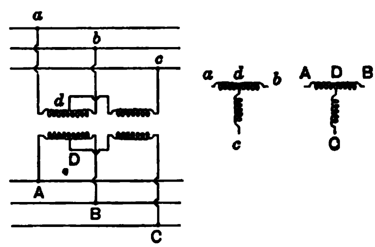


FIG. 367.—Tee connection.

322. Open-delta Connection.—If one transformer of a delta-delta bank is put out of commission the system can still be operated with the other two transformers without any change in connections. This arrangement of two transformers on a three-phase system, Fig. 366, is called the "open-delta." connection. It may be used as an emergency connection in case of accident or it may be employed in a new installation for temporary operation and a third transformer added to complete the delta-delta bank when conditions warrant the increase in capacity.

In this system each transformer carries the full-line current and for a given output the transformer currents in the open-delta bank will be 73 per cent. greater than in the closed-delta bank. To supply a load of 300 kva. from a three-phase system, three 100-kva. transformers are required connected delta-delta or two 173-kva. transformers connected in open-delta and therefore the open-delta bank must have a capacity of $2 \times 173 = 346$ kva.,

which is 15 per cent. greater than the closed-delta bank. Thus when one transformer of a delta-delta bank burns out the capacity of the bank is reduced to $\frac{100}{1.73} = 58$ per cent.

Fig. 368 shows a vector diagram for an open-delta system with a balanced inductive load. E_1, E_2 and E_3 are the e.m.fs. generated in the three transformers of a closed delta; they are also the terminal e.m.fs. at no load in the open delta. I'_1, I'_2 and I'_3 are the load currents in the closed delta.

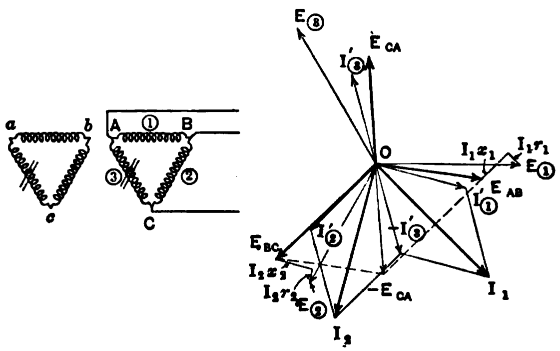


FIG. 368.—Vector diagram for an open delta.

Transformer No. 3 is disconnected and the two remaining transformers carry currents $I_1 = I'_1 - I'_3$ and $I_2 = I'_2 - I'_3$. The terminal e.m.f. of No. 1 is $E_{AB} = E_1 - I_1Z$, where Z is the impedance of each of the transformers: the terminal e.m.f. of No. 2 is $E_{BC} = E_2 - I_2Z$ and the e.m.f. between the lines C and A is $E_{CA} = -(E_{AB} + E_{BC})$ since the sum of the three must equal zero.

The three terminal e.m.fs. are slightly unbalanced under load but not enough to have any serious effect on operation. If an extra impedance $= \frac{2}{3}Z$ is added to the primary at b the three secondary line e.m.fs. will be equal. Here Z is the impedance of one transformer.

The impedance drops in Fig. 368 are very much exaggerated and the terminal e.m.fs., therefore, appear to be very badly unbalanced.

323. "T" Connection.—The "T" connection is a second method of obtaining a three-phase to three-phase transformation using only two transformers. One transformer is designed for only 87 per cent. of the three-phase e.m.fs. while the second one

is designed for 100 per cent. and has a tap at its center point to which one terminal of the 87 per cent. winding is connected (Fig. 367). Between the lines A, B, C three equal e.m.fs. are obtained at no load and they become only slightly unbalanced under load. If the short-circuit impedance of the 87 per cent. transformer is equal to one-half the impedance of the 100 per cent. transformer the e.m.fs. will be balanced. External impedance may be added to obtain this condition.

The two halves of the winding of the 100 per cent. transformer carry currents which are out of phase and they should be well interleaved.

324. Transformation from Two-phase to Three-phase.—The Scott connection, Fig. 369, is an arrangement of two transformers by means of which three-phase power can be obtained from a two-phase circuit. This connection is reversible.

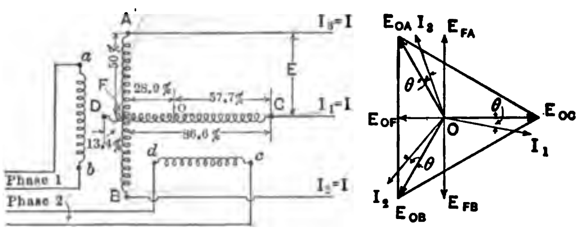


FIG. 369.—Two-phase to three-phase (Scott connection).

Two similar transformers are used with their primaries connected to the two-phase system. The secondary AB of one has a tap at its center point and the secondary DC of the other has a tap 86.6 per cent. from the end C. The two taps are connected at F and three-phase power can be taken off from the terminals A, B and C. The section DF is not used.

If the e.m.f. generated in the winding DC is

$$e_{DC} = E_m \sin \theta,$$

and that in AB is

$$e_{AB} = E_m \sin (\theta - 90),$$

then the e.m.fs. generated in the various sections are

$$e_{FC} = \frac{\sqrt{3}}{2} E_m \sin \theta,$$

$$e_{FA} = \tfrac{1}{2} E_m \sin (\theta + 90)$$

$$e_{FB} = \tfrac{1}{2} E_m \sin (\theta - 90);$$

the neutral point of the three-phase system is at 0, which is $\frac{100}{\sqrt{3}}$ = 57.7 per cent. from the end C, and thus

$$e_{OC} = \frac{E_m}{\sqrt{3}} \sin \theta$$

$$e_{FO} = \left(\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{3}}\right) E_m \sin \theta = \frac{1}{2\sqrt{3}} E_m \sin \theta.$$

The three-phase star voltages are e_{OA}, e_{OB} and e_{OC};

$$e_{OA} = e_{OF} + e_{FA} = -\frac{E_m}{2\sqrt{3}} \sin \theta + \frac{E_m}{2} \sin (\theta + 90)$$

$$= \frac{E_m}{\sqrt{3}}\left(\frac{\sqrt{3}}{2} \cos \theta - \tfrac{1}{2} \sin \theta\right) = \frac{E_m}{\sqrt{3}} \sin (\theta + 120)$$

$$e_{OB} = e_{OF} + e_{FB} = -\frac{E_m}{2\sqrt{3}} \sin \theta + \frac{E_m}{2} \sin (\theta - 90)$$

$$= \frac{E_m}{\sqrt{3}}\left(-\frac{\sqrt{3}}{2} \cos \theta - \tfrac{1}{2} \sin \theta\right) = \frac{E_m}{\sqrt{3}} \sin (\theta + 240).$$

Thus the three star voltages e_{OA}, e_{OB} and e_{OC} are equal in value and displaced in phase by 120 degrees and form a three-phase system.

These star voltages may be combined to form the three delta or line voltages, $e_{AB} = E_m \sin (\theta - 90)$, $e_{BC} = E_m \sin (\theta + 30)$ and $e_{CA} = E_m \sin (\theta + 150)$.

If the loads on the three phases are balanced the terminal voltages will be very slightly unbalanced, since the impedance from 0 to the two terminals A and B is greater than that from 0 to C. To correct this an external impedance Z_C may be added at C and its value must be such that

$$Z_C + Z_{OC} = Z_{OF} + Z_{FA}$$

and taking Z as the short-circuit impedance of each transformer

$$Z_C + \frac{Z}{\sqrt{3}} = \left(\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{3}}\right)Z + \frac{Z}{2} = \frac{Z}{2\sqrt{3}} + \frac{Z}{2}$$

and

$$Z_C = \frac{Z}{2\sqrt{3}} + \frac{Z}{2} - \frac{Z}{\sqrt{3}} = \frac{\sqrt{3} - 1}{2\sqrt{3}} Z = 21.2 \text{ per cent. of } Z.$$

Fig. 369 shows a vector diagram for a balanced load of power factor $\cos \theta$.

The power delivered by the secondary DC is

$$P_1 = E_{FC} I_1 \cos \theta = \frac{\sqrt{3}}{2} EI \cos \theta$$

where E and I are the line voltage and line current respectively. The total power is $\sqrt{3}EI \cos \theta$ and thus each transformer supplies one-half of the load.

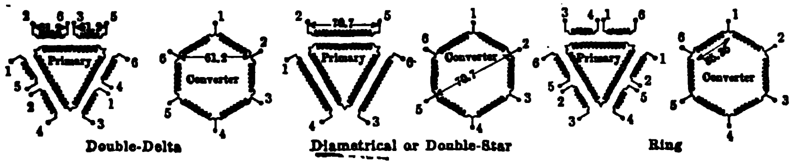


FIG. 370.—Three-phase to six-phase.

325. Single-phase Power from Three-phase Circuits.—Single-phase power can be drawn from any phase of a three-phase circuit but when the amounts of power required are large this leads to an unbalance of the terminal voltages and to unequal heating of the three phases. Fig. 371 shows one method of connecting three transformers on a three-phase four-wire system to supply a single-phase load by means of which equal currents are drawn from the three phases. The power factors of the loads on the three transformers are very different but the temperature rise will be the same. The single-phase voltage is only double the voltage of one transformer and therefore the transformer capacity must be 150 per cent. of the load.

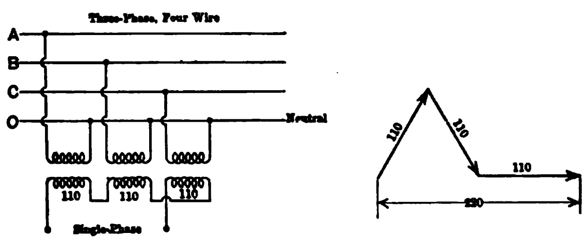


FIG. 371.—Three-phase to single-phase.

326. Multiple Operation of Transformers.—In order that two transformers connected in multiple shall divide the load in proportion to their capacities it is necessary (1) that they have the same ratio of turns and (2) that the per cent. resistance drops and the per cent. reactance drops are the same in both.

If the ratios are not the same they may be equalized by using an auto-transformer and differences in the impedances may be corrected by adding external impedance to one transformer.

In three-phase systems only those groups of transformers can

be operated in multiple which give the same change in phase between the primaries and secondaries, a star-star bank can be operated in multiple with a similar star-star bank or with a delta-delta bank, but neither of these banks can be operated in multiple with a delta-star or star-delta bank. One delta-star bank may be operated in multiple with another delta-star bank, or it may be operated in multiple with a star-delta bank with its secondary voltages reversed, if the ratios are such as to make the voltages equal.

327. Booster Transformers.—Booster transformers are connected with their primaries across the line and their secondaries in series with the line to raise or lower the voltage (Fig. 372).

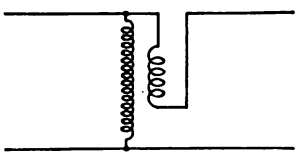


FIG. 372.—Booster transformer.

If the primary becomes disconnected while the load is on, the booster transformer becomes a series transformer with open secondary. The load current acts as an exciting current and very high voltages may be generated in the high-voltage winding, resulting in danger to persons and damage to the insulation. To disconnect a booster transformer under load the secondary must be short-circuited at the same instant that the primary is opened.

328. Auto-transformers.—An auto-transformer has only one winding. The primary includes all the turns, while the secondary includes only a part of them. The secondary voltage is usually made variable by bringing out a number of taps.

Auto-transformers may be used in almost every case in which ordinary transformers are used but are not satisfactory where it is desirable to have the secondary insulated from the primary.

They are used very extensively to obtain a variable voltage for starting induction motors, synchronous motors, single-phase series motors, etc., and as balance coils on three-wire distributing circuits. They are necessary when transformers of unequal ratios are to be operated in multiple. Three-phase auto-transformers are often used to interlink two systems of unequal voltage.

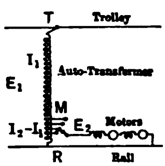
Fig. 373 shows an auto-transformer for starting and operating single-phase series motors from a high-voltage trolley in electric-railway service. The section of the winding between T and M carries the primary current, which is small, while the section from M to R carries the difference between the secondary and

primary currents which is very large if the ratio of voltages is large and, therefore, it must be made of large current capacity.

The copper loss in an auto-transformer is smaller than in an equivalent two-coil transformer and the efficiency is therefore higher but this advantage decreases as the ratio of turns increases.

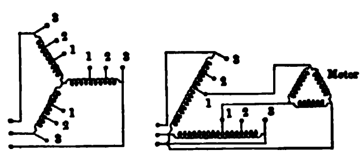
Auto-transformers may be used either to step up or step down the voltage.

When starting induction motors or synchronous motors, a low voltage must be impressed at start to prevent objectionable current surges. From 40 to 70 per cent. of full voltage is usually impressed at the first step depending on the size of motor. Starters for medium-sized motors will usually have only two taps while for large-sized motors three or more taps may be required.



Auto-Transformer for Single-Phase Electric Railway Service

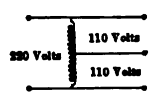
FIG. 373.



Star-Connected Auto-Starter

Open-Delta Auto-Starter

FIG. 374.



Auto-Transformer as Balance Coil for a Three-Wire System

FIG. 375.

For three-phase motors three auto-transformers connected in star may be used or two connected in open delta or tee (Fig. 374).

Fig. 375 shows an auto-transformer used as a balance coil on a 220-volt, three-wire system.

329. Instrument Transformers.—Instrument transformers are of two kinds, shunt or potential transformers and series or current transformers.

Potential transformers are used to reduce the line voltage to a value suitable for operating, indicating, integrating and recording instruments as voltmeters, wattmeters, power-factor meters, etc., and relays for voltage regulation or protection. They also serve to insulate these instruments from the high-voltage circuit.

The ratio of primary impressed voltage to secondary terminal voltage should be as nearly as possible constant and the phase angle between these two voltages should be small. Fig. 376 shows a vector diagram for a potential transformer with a 1 to 1 ratio and non-inductive load. This is the usual case since the voltage coils of instruments are practically non-inductive. The same symbols are used as in Art. 302.

The ratio $\frac{E'}{E_2}$ is constant and is equal to the ratio of turns but the ratio $\frac{E_1}{E}$ is always greater than $\frac{E'}{E_2}$ on account of the impedance drops in the primary and secondary. Thus to obtain the primary impressed voltage E_1 from the secondary terminal voltage E, it is necessary to multiply E by the nominal ratio and by a

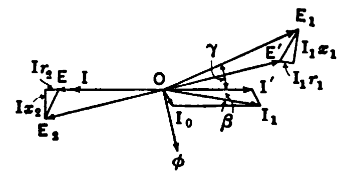


FIG. 376.—Instrument transformer.

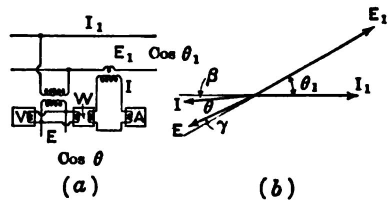


FIG. 377.

ratio correction factor k_1. This factor varies with the impressed primary voltage or secondary terminal voltage and it also varies with the secondary load, that is, with the number and type of instruments connected to the secondary. The ratio correction factor is usually plotted on a base of secondary terminal voltage for a specified secondary load resistance. Fig. 378 shows values of k_1 for a Weston portable transformer it increases with the secondary terminal voltage E.

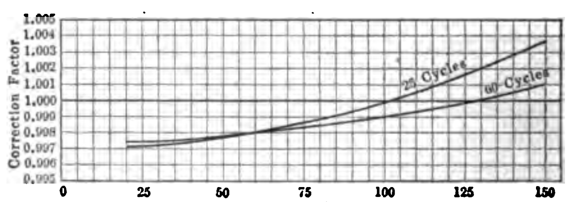


FIG. 378.—Ratio-correction curves for a Weston portable potential transformer.

The angle of phase shift or the phase angle of the transformer under given conditions is the angle by which E_1 and E are out of line and is shown as γ in the diagram. The phase angle depends on the impedance drops in the transformer and increases with the secondary load current and with the secondary terminal voltage. The phase angles of standard potential transformers are very small, unless they are overloaded, and they can be neglected except in the most accurate measurements or at low power fac-

tors. Phase-angle corrections do not apply to voltmeter readings but only to the indications of wattmeters and power-factor meters which are affected by change of phase.

Potential transformers are designed with very low impedances by subdividing the coils well and placing them as close together as possible. The very best insulating materials must be used. They are designed with very low flux densities to prevent distortion of wave form due to the exciting current.

Calibrations must be made with a definite instrument load in the secondary.

330. Current Transformers.—Current transformers are connected with their primaries in series with the circuit and are used to reduce the line current to a value suitable for current relays and for indicating and integrating instruments. They also insulate the instruments from the high-voltage circuits.

The ratio of primary to secondary current should be as nearly as possible constant and the phase angle between them should be very small.

Fig. 376 may also be used as a vector diagram for a current transformer. The meter load is again assumed to be non-inductive and its resistance is given as the ratio of secondary terminal voltage to secondary current $= \frac{E}{I_2}$. The exciting current I_0 added to the primary load current prevents the ratio of currents from being constant and equal to the ratio of turns and it also causes the currents to be out of line by the phase angle β.

The exciting current can be kept low by using iron of very high permeability and low loss and designing the secondary with a low impedance. The core-loss current is nearly in phase with the primary load current and has a large effect on the ratio but little effect on the phase angle while the magnetizing current has a large effect on the phase angle and a small effect on the ratio. Increasing the secondary load resistance by adding extra instruments increases the voltage generated in the secondary for a given value of secondary current and so increases the flux required and the exciting current. This increases both the phase angle and the ratio correction. Reactance added to the secondary circuit causes the exciting current to swing more nearly into line with the load current and therefore increases the ratio correction but decreases the phase angle.

Fig. 379 shows ratio and phase angle curves for a Weston

standard current transformer. To find the primary current, corresponding to a given secondary current I_2, multiply I_2 by the nominal ratio and by the ratio correction factor k_2.

The ratio correction factors and phase angles are smaller on 60 cycles than on 25 cycles since the flux and exciting currents for a given load current are less. The phase angles of current transformers are greater than those of potential transformers and a correction should always be made for them.

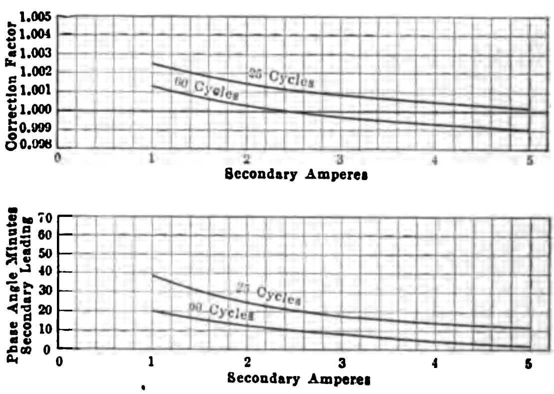


FIG. 379.—Ratio-correction and phase-angle curves for a Weston portable current transformer.

The primary winding of a current transformer usually consists of a single turn in series with the line while the secondary may have any number of turns depending on the reduction in current required. The current in the primary is the load current of the circuit and does not depend in any way on the transformer. It sets up a magnetic flux in the core which links with both the primary and secondary windings. If the secondary were open-circuited all the primary m.m.f. would be magnetizing and would produce a very large flux in the core and in a transformer having a large ratio thousands of volts would be induced in the secondary, resulting in danger to operators and damage to the instrument. The iron core is designed to carry only the flux required with closed secondary and it would be very highly saturated under open-circuit conditions and would become hot due to excessive iron losses. The transformer would require to be recalibrated before its readings could be relied on. Current transformers should always be provided with a short-circuiting link which must be closed before the secondary is opened.

Potential transformers are built for all voltages up to 100,-000 volts but for the high voltages they become very expensive. The secondary is usually designed for 100 volts.

Current transformers are built with a rating of about 25 volt-amp. and a secondary current of 5 amp. The primary current ranges from 5 amp. to 3,000 or more. On circuits up to 20,000 volts they are usually air-cooled but above this they are oil-immersed. For primary currents above 600 amp. no primary winding is supplied but the transformer is slipped over the bus-bar which then forms its single turn.

331. Example.—In the circuit in Fig. 377(*a*) it is required to find the primary voltage E_1, current I_1, power factor $\cos\theta_1$ and power W_1, knowing the secondary voltage E, current I and power W. The phase angle of the potential transformer is γ, Fig. 376, and the ratio correction factor is k_1; the phase angle of the current transformer is β and the ratio correction factor is k_2.

The primary voltage is $E_1 = E \times \text{nominal ratio} \times k_1$.

The primary current is $I_1 = I \times \text{nominal ratio} \times k_2$.

The secondary power factor is $\cos\theta = \dfrac{W}{EI}$, or $\theta = \cos^{-1}\dfrac{W}{EI}$.

Referring to Fig. 377(*b*) the primary angle of lag is $\theta_1 = \theta + \beta + \gamma$ and the primary power factor is $\cos(\theta + \beta + \gamma)$.

The secondary voltage always lags behind the primary voltage and the secondary current always leads the primary current.

Errors due to the phase angles β and γ are much more serious on low power factors than on high power factors. When using two wattmeters to read three-phase power, phase-angle corrections should always be made.

332. The Constant-current Transformer.—The constant-current transformer is shown diagrammatically in Fig. 380. The primary coil P is fixed in position and receives power at constant voltage. The secondary coil S is movable and regulates for constant current in the receiver circuit which it supplies irrespective of the load. The transformer is used to obtain a constant current for series arc-light circuits.

When the secondary coil is close to the primary there is very little leakage and most of the flux produced by the primary links with the secondary and the secondary voltage is, therefore, a maximum. Primary and secondary currents are in opposite directions and the two coils repel one another. The weight W is so adjusted that the pull due to it together with the force of repulsion of the coils just balances the weight of S and allows the coil to remain in such a position that the required current flows in it. Fig. 381 shows the flux in the core.

If the resistance or impedance of the load circuit decreases due to the cutting out of one or more arc lamps an increase of the current in both secondary and primary follows and the repulsion between the coils, which is proportional to the product of their currents, increases. The secondary, therefore, rises and increases the leakage reactances of both coils and so less of the primary magnetism links with the secondary; its voltage is, therefore, decreased and its current drops to the required value. The moving arm must be designed to give the required regulation with a fixed weight W.

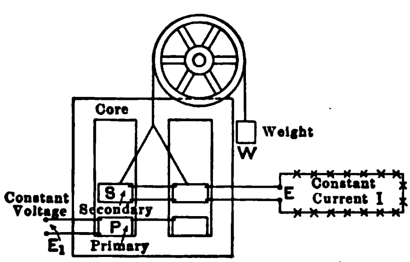


Fig. 380.—Constant-current transformer.

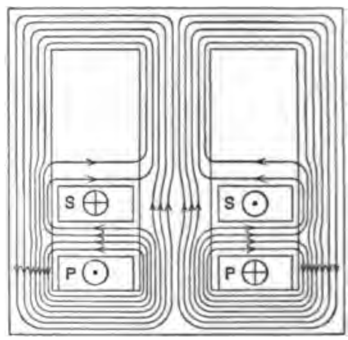


Fig. 381.—Flux in the core of a constant-current transformer.

Such an arrangement regulates for constant current between the limits of secondary voltage set by the two extreme positions of the moving coil.

Neglecting the primary exciting current the relation between the terminal voltages may be written:

$$\dot{E}_1 = \dot{I}'\left\{r_1 + jx_1 + \left(\frac{n_1}{n_2}\right)^2(r_2 + jx_2)\right\} + \frac{n_1}{n_2}\dot{E}$$

$$= \dot{I}_2\left\{\frac{n_2}{n_1}(r_1 + jx_1) + \frac{n_1}{n_2}(r_2 + jx_2)\right\} + \frac{n_1}{n_2}\dot{E}.$$

E_1, I_2, r_1 and r_2 are constant and x_1 and x_2 increase as E decreases.

333. Regulation and Power Factor.—The secondary current should be maintained within 0.1 amp. of the rated current from no load to full load.

The full-load power factor of an alternating arc-lamp system seldom exceeds 70 per cent. and is much lower at light loads. In series tungsten-lamp systems the power factor varies from about 23 per cent. at one-fourth load to 85 per cent. at full load.

334. Cooling.—Small constant-current transformers are cooled by natural air circulation; above 100 kva. they are oil-immersed and may be water-cooled.

335. Induction Regulator.—Induction regulators are special transformers used to vary the voltage of an alternating-current distributing circuit or the voltage impressed on a rotary converter.

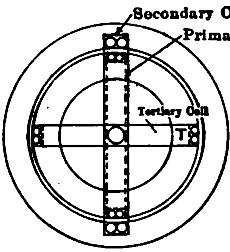


FIG. 382.

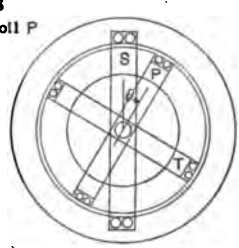


FIG. 383.

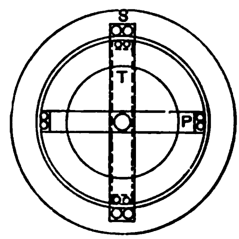


FIG. 384.

Single-phase induction regulator.

There are two types of induction regulators, single-phase and polyphase.

1. The single-phase regulator is illustrated in Figs. 382 to 384. The primary coil P is carried on a movable core built of laminated iron and is connected across the line. The secondary coil S is carried on a stationary core and is connected in series with the line to raise or lower the voltage (Fig. 385).

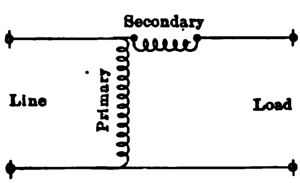


FIG. 385.

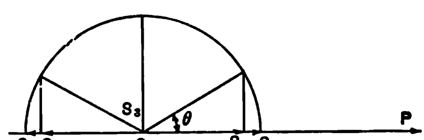


FIG. 386.

The primary exciting current produces an alternating flux, which induces a voltage in the secondary. This secondary voltage varies with the position of the primary winding, but it is always in phase with or in opposition to the impressed voltage or line voltage. In Fig. 382 all the primary flux (neglecting leakage) passes through the secondary coil and the secondary voltage is a maximum and is in opposition to the impressed voltage, and so gives the minimum load voltage.

In Fig. 386 OP is the impressed voltage or line voltage, OS_1 is

the maximum secondary induced voltage and S_1P is the load voltage.

The load current of the circuit flows in the secondary, and there must also be a load current in the primary of equal and opposite m.m.f. as in the ordinary transformer.

When the movable core is turned through an angle θ, Fig. 383, only part of the primary flux passes through the secondary and the secondary voltage is reduced approximately in the ratio $1 : \cos \theta$, but is still in opposition to the line voltage. The load voltage is represented by S_2P, Fig. 386.

With the primary coil at right angles to the secondary coil, Fig. 384, none of the primary flux passes through S and there is no secondary induced voltage.

When the core is turned through 180 degrees the secondary voltage is again maximum, but is in phase with the impressed voltage and so raises the load voltage to S_5P. Thus the total variation of the load voltage is from S_1P to S_5P and is equal to twice the maximum secondary voltage.

In Fig. 384 the m.m.f. of the load current in the secondary cannot be opposed by any primary m.m.f. since the coils are at right angles. To supply the m.m.f. required to balance the secondary load m.m.f., and so prevent a large reactance drop in the winding, the coil T called the "tertiary" coil is placed on the movable core at right angles to the primary coil. It is short-circuited and exerts a m.m.f. equal and opposite to the secondary m.m.f., and so reduces the secondary reactance to the value corresponding to the leakage flux.

The only current carried by the primary coil in this position is the exciting current.

In intermediate positions of the rotor the secondary m.m.f. is partly balanced by the induced current in the tertiary or compensating coil and partly by a load current in the primary coil.

2. The polyphase induction regulator has a polyphase winding on the moving core, which is connected to the polyphase supply. The secondary or stator is wound with the same number of phases, but the phase windings are kept entirely separate so that they can be connected in the different lines to raise or lower the voltage.

When polyphase currents flow in the primary windings a revolving magnetic field is produced of constant value as in the alternator or induction motor. This field cuts the secondary

windings and generates e.m.fs. in them of the same frequency as the primary impressed e.m.fs., but less in the ratio of turns.

As the primary is turned the magnitude of the revolving field is not changed, and therefore the magnitudes of the secondary e.m.fs. are not changed but their phase relations with the impressed e.m.fs. are changed and the load voltage is varied as shown in Fig. 387. By turning the primary through 180 degrees the phase of the secondary e.m.f. is changed from direct opposition to the line voltage to direct addition to it and thus the load volt tage is varied by an amount equal to twice the secondary voltage.

The rotor must be clamped in the required position or it will tend to rotate at a high speed in the direction opposite to that of the revolving field.

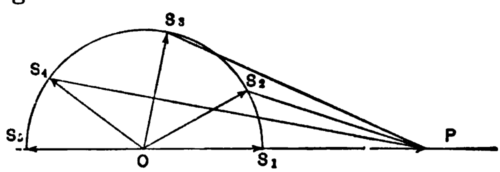


FIG. 387.

The advantage of the induction regulator over a transformer with variable voltage taps on the secondary or primary is that the variation of voltage is uniform over the entire range. Regulators are, however, very expensive and require a large exciting current and have large leakage reactances.

The regulator is operated either by hand or by means of a small motor placed on the top or it can be made automatic if required.

336. Design of a Transformer.—Given the type and method of cooling, number of phases, frequency, capacity in kilovolt-amperes, ratio of voltages and the class of service, a transformer may be designed using the equations and constants listed below.

The e.m.f. equation is

$$E = 4.44 f n \Phi 10^{-8} \text{ volts} \qquad \text{Art. 301.} \tag{353}$$

$$\text{Volts per turn} = e = \frac{\sqrt{\text{output in volt-amperes}}}{k}, \tag{354}$$

where k is a constant for which the following approximate values may be used:

For shell-type power transformers, $k = 25$,
For core-type power transformers, $k = 50$,
For core-type distributing transformers, $k = 80$.

Flux densities. The flux densities to be used depend on the quality of the iron, the frequency, and the service in which the transformer is to be placed.

With good alloyed iron densities from 65,000 to 90,000 are used. The lower values should be used for 60-cycle distributing transformers to keep down the iron losses. When using silicon steel with its low iron losses higher densities may be employed but care must be taken that the magnetizing current does not become too large since the permeability of silicon steel is comparatively low.

Ratio of losses. In distributing transformers the core loss is from 30 to 50 per cent. of the copper loss while in power transformers the two are approximately equal.

Knowing the voltage and the volts per turn the number of turns may be found and substituting in equation (353) the flux Φ is determined. Dividing Φ by the assumed flux density gives the section of the magnetic circuit. Fig. 388 shows the magnetic circuits of shell- and core-type transformers and their relative dimensions.

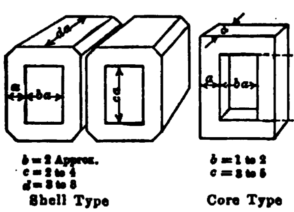


Fig. 388.

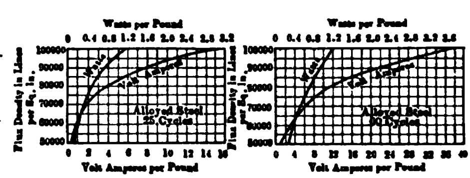


Fig. 389.—Magnetic characteristics of transformer sheets.

Before deciding on the size of the opening in the iron the sections and arrangement of the winding must be determined.

Design of the windings.

(*a*) Volts per coil should not exceed 5,000.

(*b*) Volts per layer should not exceed 350.

(*c*) Current density should lie between 625 and 1,250 amp. per square inch or 2,000 to 1,000 circ. mils per ampere. In large water-cooled transformers higher densities may be used up to 2,000 amp. per square inch.

(*d*) The strip used in the windings should not be wider than 0.5 in. for 60 cycles nor 0.75 in. for 25 cycles to prevent excessive eddy currents in the copper.

(*e*) Coils should not be made more than 1 in. in thickness and must have a surface in contact with the oil of at least 2 to 3 sq. in. per watt copper loss, the lower value being for water-cooled transformers.

Copper losses should be calculated at 75°C. and if they are found to be too large either the current density must be decreased or the radiating surface increased by further subdivision of the winding. Decrease of current density will improve the efficiency while an increase in the number of coil groups will decrease the reactance and therefore improve the regulation but will not affect the efficiency.

(*f*) The spacings between coils and between coils and iron depend on the voltage and approximate values may be obtained from Fig. 393. The fullerboard insulation between the high-voltage (H. V.) and low-voltage (L. V.) coils may be assumed to take up from one-third to one-half the available space. Spacing blocks are used to strengthen the coils and to keep the paths for oil circulation open. Low-voltage coils are always placed next the iron and the windings are arranged in groups of high- and low-voltage coils to reduce the reactance.

Having determined the space required for the windings and the opening in the iron the size of plates may be found and the total volume and weight of iron calculated.

Fig. 389 shows characteristic curves for good alloyed iron at 25 and 60 cycles plotted with flux density against watts iron loss per pound and volt-amperes per pound. Knowing the flux density the watts iron loss per pound may be obtained and this value multiplied by the total weight of iron gives the total iron loss or core loss in the transformer. If the iron loss is too large it may be decreased by using a lower flux density or by making the core of silicon steel with lower losses. If the total flux in the transformer is decreased the number of turns must be increased and this will result in increased copper losses.

The iron must have a radiating surface in contact with the oil of at least 1 sq. in. per watt lost in the core.

Self-cooled transformers are placed in corrugated tanks and are immersed in oil. From 5 to 8 sq. in. of tank surface are required for each watt lost depending on the depth of the corrugations.

In water-cooled transformers with cooling coils 1¼ in. in

diameter a coil surface of 1 sq. in. per watt lost is required and about ¼ gal. of water per minute per kilowatt lost.

337. Reactance.—The reactance of the windings of a transformer can be determined very accurately since the leakage paths are of simple form.

Fig. 390 shows a section through the coils of a shell-type transformer and an enlarged section of one group of coils. w = width of the iron opening, a_1 and a_2 are the widths of high-voltage and low-voltage coils respectively, and d_1 and d_2 are the thicknesses of these coils. The distance between the coils s. When a_1 and a_2 are only slightly smaller than w the leakage flux may be assumed to cross the opening from one side to the other but in high-voltage transformers where a_1 is much less than w, a more approximate expression for the length of the leakage path in air may be found by inspection.

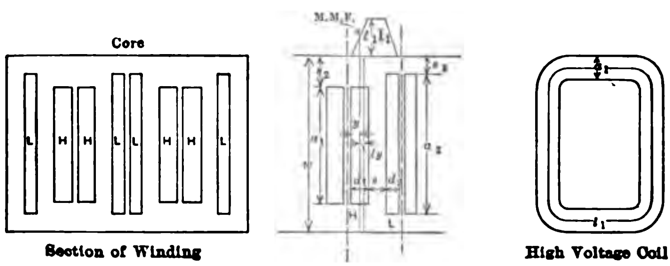


Fig. 390.

Assuming that the two windings are divided into the same number of coils = g, let t_1 be the number of turns in one primary coil carrying current I_1 and t_2 be the number of turns in one secondary coil carrying current I_2, then $t_1 I_1 = t_2 I_2$. The leakage flux about any coil may be separated into two parts: (a) that crossing the coil, and (b) that passing through between the high- and low-voltage coils. One-half of the flux between the coils is assumed to surround each of them.

At a distance y from the inside of coil A the m.m.f. acting is $t_1 I_1 \frac{y}{d_1}$ and this produces a flux, in the path of section $l_1 dy$ and length w, of a value

$$d\phi' = \frac{4\pi t_1 I_1 \frac{y}{d_1}}{\frac{w}{l_1 dy}}.$$

Where l_1 is the length of the mean turn of the coil; this flux surrounds only $\frac{t_1 y}{d_1}$ turns and is equivalent to a flux $d\phi$ surrounding t_1 turns, where

$$d\phi = d\phi' \frac{y}{d_1} = \frac{4\pi t_1 I_1 l_1 y^2 dy}{w d_1^2}$$

and the flux equivalent to the total flux crossing the coil is

$$\phi_1 = \int_{y=0}^{y=d_1} d\phi = \frac{4\pi t_1 I_1 l_1}{w d_1^2} \int_0^{d_1} y^2 dy = \frac{4\pi t_1 I_1 l_1}{w} \frac{d_1}{3}.$$

The flux in the path between the coils is

$$\phi_s = \frac{4\pi t_1 I_1 l_1 s}{w}$$

and the part of this surrounding coil A is

$$\phi_2 = \frac{\phi_s}{2} = \frac{4\pi t_1 I_1 l_1}{w} \frac{s}{2}.$$

The total leakage flux surrounding coil A is

$$\phi_A = \phi_1 + \phi_2 = \frac{4\pi t_1 I_1 l_1}{w} \left(\frac{d_1}{3} + \frac{s}{2}\right)$$

and the self-inductance of the coil is

$\mathfrak{L}_A = \frac{t_1 \phi_A}{I_1} = \frac{4\pi t_1^2 l_1}{w} \left(\frac{d_1}{3} + \frac{s}{2}\right)$ absolute units, if all dimensions are in centimeters.

Using inch units, the inductance of each coil is

$$\mathfrak{L}_A = \frac{4\pi t_1^2 l_1}{w} \left(\frac{d_1}{3} + \frac{s}{2}\right) 2.54 \text{ absolute units,}$$

or

$$L_A = \frac{4\pi t_1^2 l_1}{w} \left(\frac{d_1}{3} + \frac{s}{2}\right) 2.54 \times 10^{-9} \text{ henry.}$$

$$= 3.2 \times 10^{-8} \frac{t_1^2 l_1}{w} \left(\frac{d_1}{3} + \frac{s}{2}\right) \text{ henry.}$$

If the g coils are connected in series the inductance of the winding is

$$L_1 = 3.2 \times 10^{-8} \frac{t_1^2 l_1}{w} \left(\frac{d_1}{3} + \frac{s}{2}\right) g$$

and the reactance is

$$X_1 = 2\pi f L_1 = 2\pi f \times 3.2 \times 10^{-8} \frac{t_1^2 l_1}{w} \left(\frac{d_1}{3} + \frac{s}{2}\right) g$$

$$= 20.2 \times 10^{-8} f\, t_1^2 g \frac{l_1}{w} \left(\frac{d_1}{3} + \frac{s}{2}\right)$$

$$= 20.2 \times 10^{-8} f \frac{n_1^2}{g} \frac{l_1}{w} \left(\frac{d_1}{3} + \frac{s}{2}\right) \text{ ohms,} \qquad (355)$$

where $n_1 = t_1g$ = total primary turns. The reactance is inversely proportional to the number of coil groups.

The reactance of the secondary winding is

$$X_2 = 20.2 \times 10^{-8} f \frac{n_2^2}{g} \frac{l_2}{w} \left(\frac{d_2}{3} + \frac{s}{2}\right) \text{ ohms.} \tag{356}$$

The reactance of a core-type transformer may be worked out in a similar way.

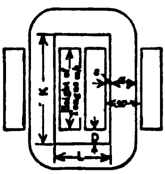


Fig. 391.

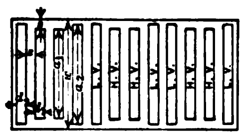


Fig. 392.

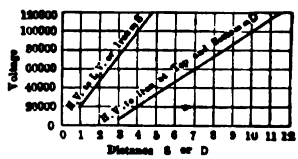


Fig. 393.

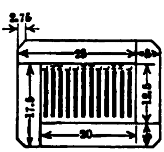


Fig. 394.

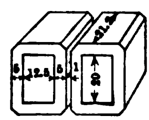


Fig. 395.

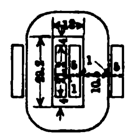


Fig. 396.

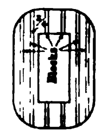


Fig. 397.

338. Design of a Shell-type, Water-cooled Transformer, Single-phase, 60 Cycles, 1,000 kva., 22,000 to 2,200 Volts for Power Transmission.—

$$\text{Volts per turn} = \frac{\sqrt{\text{output in volt-amperes}}}{k} = \frac{\sqrt{1,000,000}}{25} = 40.$$

Number of primary turns $= \frac{22,000}{40} = 550$, assume 540. Make the primary winding of six coils four of 92 turns and two outer coils of 86 turns, this leaves space for extra insulation on the end turns to take care of strains due to switching or lightning.

Circular mils per ampere = 1,000.

Current density = 1,270 amp. per square inch.

Primary current $= \frac{1,000,000}{22,000} = 45.4$ amp.

Section of conductor = 45.4 × 1,000 = 45,400 circ. mils = 0.0368 sq. in.

Size of strip = 0.5 × 0.08 bare or 0.515 × 0.095 d.c.c.

Actual current density = 1,170 amp. per square inch.

Actual circular mils per ampere = 1,085.

Insulation between layers = 2 ply 0.007 fullerboard.

Thickness of coil = d_1 = 0.515 in.

Width of coil = a_1 = 0.095 × 92 + 2 × 0.007 × 91 = 10.014 = 10.5 in. to allow for warping (Fig. 398).

The end coils have 5 ply 0.007 fullerboard between turns 1 to 11 and 3 ply between turns 11 to 31 making the coil width 10.5 as before.

Number of secondary turns $= \frac{540}{10} = 54$.

Number of secondary coils = 6.

Turns per coil = 18, connect coils in series multiple.

Secondary current = 454 amp.

Current per coil = 227 amp.

Amperes per square inch = 1,270 as in primary.

Section of conductor = 0.2 sq. in. = 0.4 in. × 0.5 in.

Use four strips 0.4 × 0.125 in parallel separated by 1 ply 0.007 fullerboard and taped with half-lapped tape.

Insulation between layers = 2 ply 0.007 fullerboard.

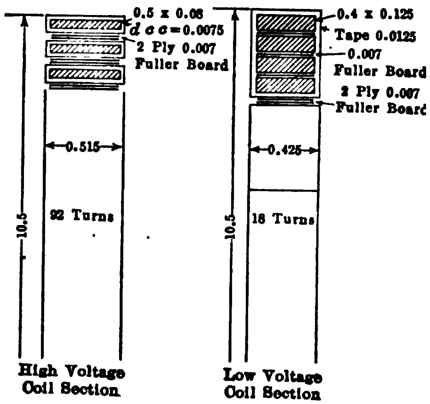


FIG. 398.

Thickness of low voltage coil $= d_2 = 0.425$ in.

Width of low voltage coil $= a_2 = 18(4 \times 0.125 + 3 \times 0.007 + 0.025) + 17 \times 2 \times 0.007 = 10.066$.

Take the width = 10.5 in. to allow for warping (Fig. 398).

Spacing

H.V. to L.V. = 3/8 in. block + 2 (1/8) in. fullerboard + 3/8 in. block = 1.0 in.

H.V. to H.V. = 3/8 in. block + 1/8 in. fullerboard + 3/8 in. block = 7/8 in.

L.V. to L.V. = 3/4 in. block.

L.V. to iron = 1/4 in. block + 1/8 in. fullerboard + 3/8 in. block = 3/4 in.

H.V. to iron = 1 in. (Fig. 393).

H.V. to iron at top and bottom of cores $= D = 4$ in. (Fig. 393).

Arrangement of coils

Opening in iron = 12.5 × 20 } Fig. 394.

Size of plates 5 × 25 and 5 × 17.5 }

Flux density = 80,000 lines per square inch, assumed.

$$\text{Flux required} = \Phi = \frac{E_1}{4.44 f n_1 10^{-8}} = \frac{22{,}000 \times 10^8}{4.44 \times 60 \times 540} = 15.3 \times 10^6 \text{ lines.}$$

$$\text{Section of iron} = \frac{15.3 \times 10^6}{80{,}000} = 191 \text{ sq. in.} = (5+5) \times h \times 0.9 \quad \text{(Fig. 391).}$$

Height of tongue $= h = \dfrac{191}{10 \times 0.9} = 21.2$ in.

Weight of iron $= 2\left\{30 \times 22.5 - 20 \times 12.5 - \dfrac{(2.75)^2}{2} \times 4\right\}$
$\times 21.2 \times 0.9 \times 0.28 = 5{,}538$ lb.

where, 0.28 is the weight of 1 cu. in. of iron in pounds.

Watts lost per pound = 0.70 (Fig. 389).

Total iron loss $= 0.70 \times 5{,}538 = 3{,}877$ watts.

Core-loss current $= I_C = \dfrac{3{,}877}{22{,}000} = 0.176$ amp.

Volt-amperes per pound = 11.5 (Fig. 389).

Volt-amperes at no load $= 11.5 \times 5{,}538 = 63{,}680$.

Exciting current $= \dfrac{63{,}680}{22{,}000} = I_0 = 2.89$ amp.

Per cent. exciting current $= \dfrac{2.89}{45.4} \times 100 = 6.36$ per cent. of full-load current.

Magnetizing current $= I_M = \sqrt{I_0^2 - I_C^2} = \sqrt{(2.89)^2 - (0.176)^2} = 2.88$ amp.

No-load power factor $\cos\theta_0 = \dfrac{3{,}877}{63{,}680}\,100$ per cent. = 6 per cent.

Primary exciting admittance $= Y_0 = \sqrt{g_0^2 + b_0^2} = \dfrac{I_0}{E_1} = \dfrac{2.89}{22{,}000} = 1.31 \times 10^{-4}$.

Conductance $= g_0 = \dfrac{I_c}{E_1} = \dfrac{0.176}{22{,}000} = 0.08 \times 10^{-4}$.

Susceptance $= b_0 = \dfrac{I_M}{E_1} = \dfrac{2.88}{22{,}000} = 1.3 \times 10^{-4}$.

Radiating surface of the iron = 8,900 sq. in. (Fig. 395).

Square inches per watt iron loss $= \dfrac{8{,}900}{3{,}877} = 2.3$.

Length of the mean turn of high-voltage winding $= 2K + 2L + \pi a_1$ = 117.4 in. = 9.8 ft. (Figs. 391 and 395).

Resistance of the H.V. winding at 75°C. $= r_1 = \rho \dfrac{l}{\text{circ. mils.}} = 13.2$
$\dfrac{9.8 \times 540}{1{,}085 \times 45.4} = 1.42$ ohms.

Primary copper loss $= I_1^2 r_1 = (45.4)^2 \times 1.42 = 2{,}940$ watts.

Length of the mean turn of L.V. winding = 9.8 ft.

Resistance of the L.V. winding $= r_2 = \dfrac{r_1}{10^2} = 0.0142$ ohms.

Secondary copper loss $= I_2^2 r_2 = I_1^2 r_1 = 2{,}940$ watts.

Total copper loss = 5,880 watts.

Surface of each coil = mean turn $\times$ width $\times 2 = 117.4 \times 10.5 \times 2$
= 2,470 sq. in.

Area covered by spacing blocks = 360 sq. in. (Fig. 397).

Radiating surface per coil = 2,470 − 360 = 2,110 sq. in.

Radiating surface of H.V. winding $= 2{,}110 \times 6 = 12{,}660$ sq. in.

Square inches per watt lost $= \dfrac{12{,}660}{2{,}940} = 4.3$.

Square inches per watt lost on L.V. winding = 4.3.

Total watts lost = 3,877 + 5,880 = 9,757.

Cooling coil surface required $= 9{,}757 \times 1.0 = 9{,}757$ sq. in.

Diameter of pipe = 1.25 in.

Length of pipe $= \dfrac{9{,}757}{1.25 \times 3.14 \times 12} = 207$ ft.

Diameter of case = 5 ft.

Length of one turn of cooling coil $= 3.14 \times 5 = 15.7$ ft.

Number of turns of cooling coil $= \dfrac{207}{15.7} = 13$ turns.

Efficiency at full-load unity power factor $= \dfrac{1{,}000{,}000}{1{,}000{,}000 + 9{,}757}$
$\times\ 100$ per cent. = 99 per cent.

Primary reactance $= x_1 = 20.2 \times 10^{-8} \times 60 \times \dfrac{(540)^2 \times 117.4}{12.5} \times \dfrac{1}{6}$
$\left(\dfrac{0.515}{3} + \dfrac{1}{2}\right) = 3.7$ ohms.

Primary reactance drop $= I_1x_1 = 45.4 \times 3.7 = 168$ volts
$= \dfrac{168}{22{,}000} \times 100 = 0.76$ per cent.

Secondary reactance $= x_2 = 20.2 \times 10^{-8} \times 60 \times (18)^2 \times \dfrac{3}{2} \times \dfrac{117.4}{12.5}$
$\left(\dfrac{0.425}{3} + \dfrac{1}{2}\right) = 0.035$ ohms.

Secondary reactance drop $= I_2x_2 = 454 \times 0.035 = 15.9$ volts
$\dfrac{15.9}{2{,}200} \times 100 = 0.72$ per cent.

Total reactance drop = 1.48 per cent.

Primary resistance drop $= I_1r_1 = 45.4 \times 1.42 = 64.5$ volts $= \dfrac{64.5}{22{,}000}$
$\times\ 100 = 0.294$ per cent.

Secondary resistance drop $= I_2r_2 = 454 \times 0.0142 = 6.45$ volts $= \dfrac{6.45}{22{,}000}$
$\times\ 100 = 0.294$ per cent.

Total resistance drop = 0.588 per cent.

Regulation at unity power factor $= 0.588 + \dfrac{(1.48)^2}{200} = 0.6$ per cent.

Regulation at 80 per cent. power factor $= 0.588 \times 0.8 + 1.48 \times 0.6$
$+ \dfrac{(1.48 \times 0.8 - 0.588 \times 0.6)^2}{200} = 1.35$ per cent.

CHAPTER XI

CONVERTERS

339. Types of Converters.—Converters are rotating machines which change electrical energy from one form to another. They are of several types:

1. A synchronous converter or rotary converter converts from an alternating to a direct current or *vice versa.*

2. A frequency converter converts the power of an alternating-current system from one frequency to another with or without a change in the number of phases or in the voltage.

3. A rotary phase converter converts from an alternating-current system of one or more phases to an alternating-current system of a different number of phases but of the same frequency. The most usual change is from single phase to polyphase (see Art. 414). Changes from one polyphase system to another are usually carried out by means of static transformers (Art. 318).

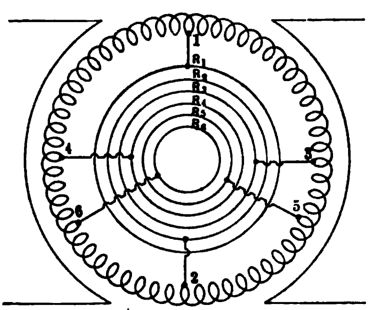


FIG. 399.—Ring-wound rotary converter.

Any of these conversions can be made with motor-generator sets.

340. Synchronous Converter.—The synchronous converter or rotary converter is a combination of a synchronous motor and a direct-current generator. It receives alternating current and converts it into direct current.

The fields are excited by a shunt winding connected between the direct-current brushes or from a separate exciter. In some cases a series winding is added for compounding.

The armature winding is an ordinary direct-current winding and may be either series or multiple. It is connected to a commutator and taps are taken out from it at equidistant points and connected to slip rings. The alternating current is delivered to the slip rings either single-phase, two-phase, three-phase or six-

phase and drives the armature as a synchronous motor. The same armature conductors generate and carry the direct current.

Fig. 399 shows a ring-wound bipolar armature tapped for single-, two-, or three-phase currents; single-phase 1 to 2 or 3 to 4; two-phase 1 to 2 and 3 to 4; three-phase 1 to 5, 5 to 6 and 6 to 1.

Fig. 400 shows a six-circuit multiple winding for a six-pole, three-phase rotary converter and Fig. 401 shows a two-circuit or series winding for an eight-pole, three-phase converter.

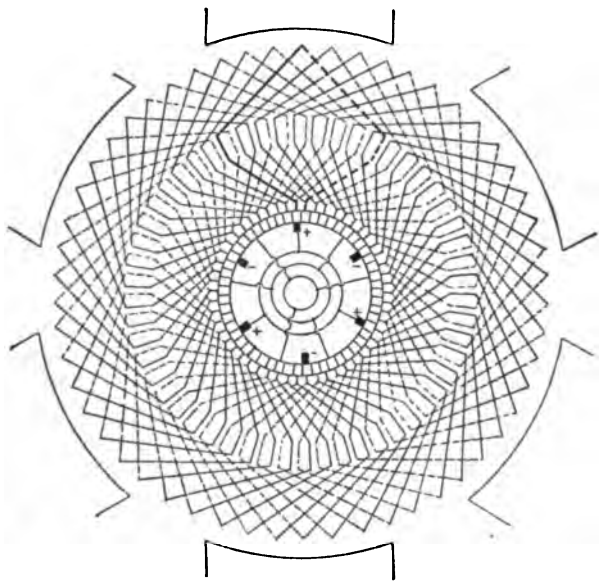


FIG. 400.—Multiple-drum winding for a three-phase rotary converter.

In a series-wound armature the total number of coils must be divisible by the number of phases and in a multiple-wound armature the number of coils per pair of poles must be divisible by the number of phases.

Six-phase converters are operated from three-phase circuits. Practically all converters above 200 or 300 kw. output are now built with six collector rings and six phases. Three methods of connecting the supply transformers are shown in Fig. 370.

341. Ratios of E.M.Fs. and Currents.—With the brushes fixed on the no-load neutral line, the direct and alternating e.m.fs. generated in the converter bear a definite relation to each other and one cannot be varied without varying the other. At unity

power factor the alternating and direct currents in the armature also bear a definite relation to each other if the current required to supply the converter losses is neglected.

Since the alternating and direct currents flow in the same armature conductors and in opposite directions, the e.m.f. consumed in the armature is small and the power loss is small. In the following analysis these quantities will be neglected and the alternating current will be assumed to be in phase with the impressed e.m.f.

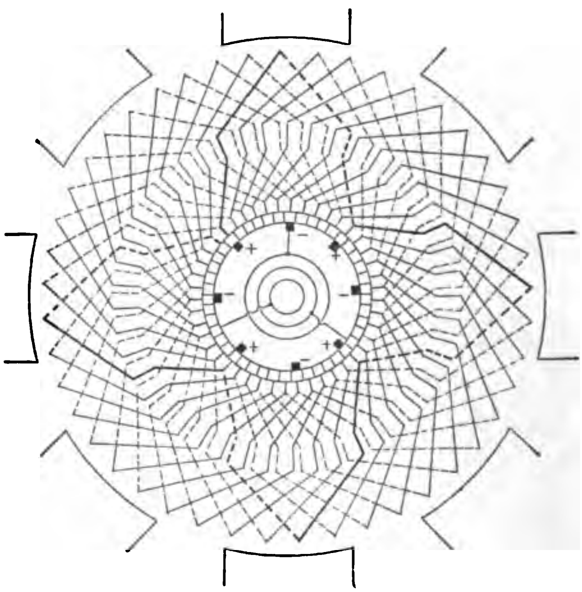


FIG. 401.—Two-circuit, retrogressive winding for a three-phase rotary converter.

This condition can be obtained in practice by properly adjusting the exciting current.

Take first the case of the single-phase converter, Fig. 402.

Let E = direct voltage of the converter.
I = direct-current output.
E_1 = effective value of the alternating supply voltage, whether single phase or polyphase.
I_1 = alternating current in the supply lines.
I' = alternating current in the armature winding.

The voltage between the leads l_1 and l_2 or between the slip rings R_1 and R_2 is alternating and reaches its maximum value

when l_1 and l_2 are under the brushes and it is then equal to the direct voltage of the machine. Therefore, in a single-phase converter the direct voltage is equal to the maximum value of the alternating voltage and thus

$$E = \sqrt{2}E_1 \tag{357}$$

or

$$E_1 = \frac{E}{\sqrt{2}}. \tag{358}$$

Neglecting losses and phase displacements the output of the converter is equal to the volt-amperes input or

$$EI = E_1 I_1$$

and the alternating current in the line is

$$I_1 = \frac{EI}{E_1} = \frac{EI}{\frac{E}{\sqrt{2}}} = \sqrt{2}I. \tag{359}$$

The alternating current in the winding is

$$I' = \frac{I_1}{p},$$

where p is the number of circuits in multiple through the winding. In the bipolar machine in Fig. 402, $p = 2$ and, therefore,

$$I' = \frac{I_1}{2} = \frac{\sqrt{2}I}{2} = \frac{I}{\sqrt{2}}. \tag{360}$$

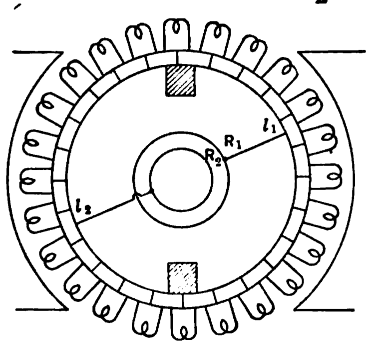


FIG. 402.—Single-phase converter.

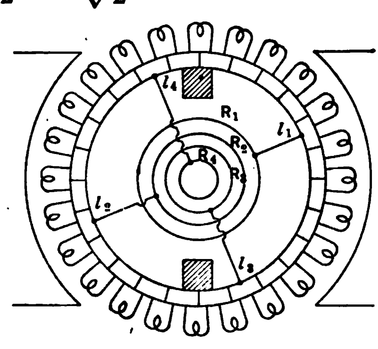


FIG. 403.—Two-phase or quarter-phase converter.

342. Two-phase or Quarter-phase Converter.—When four collector rings R_1, R_2, R_3 and R_4, Fig. 403, are connected to four equidistant points l_1, l_2, l_3 and l_4, the machine is a two-phase or quarter-phase converter. The two voltages R_1 to R_2 and R_3 to R_4 are equal and are in quadrature, forming a two-phase system;

the four voltages R_1 to R_3, R_3 to R_2, R_2 to R_4 and R_4 to R_1 are all equal and form a four-phase or quarter-phase system.

The voltage between lines or the voltage per phase of the two-phase supply is

$$E_1 = \frac{E}{\sqrt{2}}. \tag{361}$$

The voltage between adjacent rings or the quarter-phase voltage is

$$E'_1 = \frac{E_1}{\sqrt{2}} = \frac{E}{2}. \tag{362}$$

Assuming the volt-amperes input two-phase to be equal to the direct-current output, that is,

$$2E_1I_1 = EI,$$

the alternating current per line is

$$I_1 = \frac{EI}{2E_1} = \frac{EI}{2\frac{E}{\sqrt{2}}} = \frac{I}{\sqrt{2}}. \tag{363}$$

The alternating current in the winding is the resultant of two currents $\frac{I_1}{2} = \frac{I}{2\sqrt{2}}$ in quadrature and its value is therefore

$$I' = \sqrt{2}\frac{I}{2\sqrt{2}} = \frac{I}{2}. \tag{364}$$

343. Three-phase Converter.—With three collector rings R_1, R_2 and R_3, Fig. 404, connected to three equidistant points l_1, l_2 and l_3 the machine is a three-phase converter.

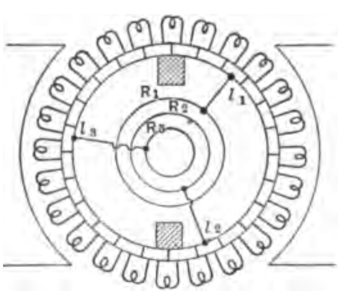


FIG. 404.—Three-phase converter.

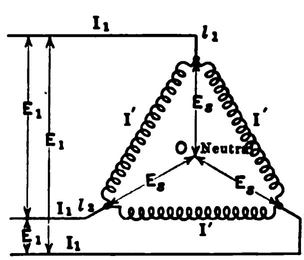


FIG. 405.—E.m.fs. and currents in a three-phase converter.

The e.m.f. between each of the rings and the neutral point or the "star" e.m.f. is equal to half of the single-phase voltage. It is shown as E_S in Fig. 405.

$$E_S = \frac{E}{2\sqrt{2}}. \tag{365}$$

The e.m.f. between rings or "delta" e.m.f. is

$$E_1 = \sqrt{3}E_S = \frac{\sqrt{3}E}{2\sqrt{2}} = 0.612E. \tag{366}$$

The power input is

$$\sqrt{3}E_1 I_1 = 3E_s I_1 = 3E_1 I'$$

and is equal to the output EI.

Thus the line current is

$$I_1 = \frac{EI}{\sqrt{3}E_1} = \frac{EI}{\sqrt{3}\cdot\frac{\sqrt{3}E}{2\sqrt{2}}} = \frac{2\sqrt{2}}{3}I = 0.943I, \tag{367}$$

and the alternating current in the winding is

$$I' = \frac{I_1}{\sqrt{3}} = \frac{2\sqrt{2}}{3\sqrt{3}}I = 0.545I. \tag{368}$$

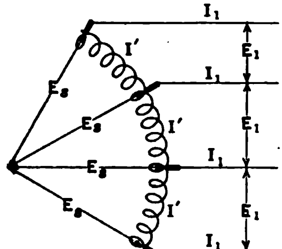


FIG. 406.—n-phase converter.

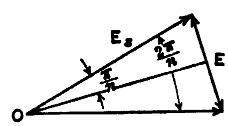


FIG. 407.

344. n-phase Converter.—For an n-phase converter, Fig. 406 the winding must be tapped at n equidistant points. The e.m.f. between each of the rings and the neutral point or the "star" e.m.f. is as before

$$E_S = \frac{E}{2\sqrt{2}}.$$

The e.m.f. between rings or the e.m.f. between lines is the vector difference between two e.m.fs. E_s displaced at $\frac{2\pi}{n}$ radians (Fig. 407). Thus

$$E_1 = 2E_s \sin\frac{\pi}{n} = \frac{E \sin\frac{\pi}{n}}{\sqrt{2}}. \tag{369}$$

The power input is

$$nE_s I_1 = nE_1 I'$$

and is equal to the output EI.

Therefore the alternating current in the line is

$$I_1 = \frac{EI}{nE_s} = \frac{EI}{n\dfrac{E}{2\sqrt{2}}} = \frac{2\sqrt{2}}{n} I, \tag{370}$$

and the alternating current in the winding is

$$I' = \frac{EI}{nE_1} = \frac{EI}{n\dfrac{E \sin\frac{\pi}{2}}{\sqrt{2}}} = \frac{\sqrt{2}I}{n \sin\frac{\pi}{n}}. \tag{371}$$

The values obtained above for the e.m.fs. and currents in single-phase, two-phase and three-phase converters can also be obtained by substituting the proper values of n in equations 369, 370 and 371; single-phase $n = 2$, two-phase or four-phase $n = 4$, and three-phase $n = 3$. The results are tabulated in Fig. 408.

Type	Single-phase $n = 2$	Three-phase $n = 3$	Two-phase or four-phase $n = 4$	Six-phase $n = 6$	n-phase
E.m.f. between collector rings or line e.m.f. E_1...	$\frac{E}{\sqrt{2}}$	$\frac{\sqrt{3}E}{2\sqrt{2}}$	$\frac{E}{2}$	$\frac{E}{2\sqrt{2}}$	$\frac{E \sin\frac{\pi}{n}}{\sqrt{2}}$
Current per line I_1.......	$\sqrt{2}I$	$\frac{2\sqrt{2}I}{3}$	$\frac{I}{\sqrt{2}}$	$\frac{\sqrt{2}I}{3}$	$\frac{2\sqrt{2}I}{n}$
Current in the winding I'	$\frac{I}{\sqrt{2}}$	$\frac{2\sqrt{2}I}{3\sqrt{3}}$	$\frac{I}{2}$	$\frac{\sqrt{2}I}{3}$	$\frac{\sqrt{2}I}{n \sin\frac{\pi}{n}}$

FIG. 408.

These ratios of currents only hold on the assumption that the power factor is unity and that the efficiency is 100 per cent. The power factor can be maintained approximately unity by adjusting the excitation, but the power losses cannot be eliminated and the values of alternating current in the table must be increased by the small component required to supply the losses in the machine. When the power factor is not unity the reactive or wattless components of current must be added to the power components.

The ratios of e.m.fs. are the ratios of the generated e.m.fs. and can only approximately represent the ratios of terminal

e.m.fs. since components of e.m.f. are consumed in the resistance and reactance of the armature. The ratios also depend on the assumption that the alternating e.m.f. wave is a sine wave. If the wave is peaked, the ratio of the effective value to the maximum value is less than $\frac{1}{\sqrt{2}}$ and the values of the alternating e.m.fs. must be reduced. If the wave is flat topped the ratio of effective to maximum value is greater than $\frac{1}{\sqrt{2}}$ and the values of the e.m.fs. must be increased.

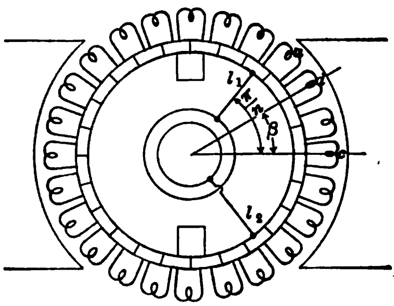


FIG. 409.

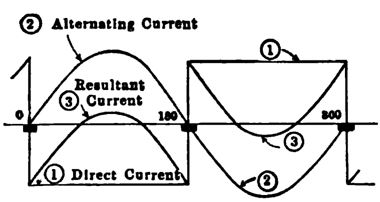


FIG. 410.—Current in coil c, Fig. 409, at unity power-factor.

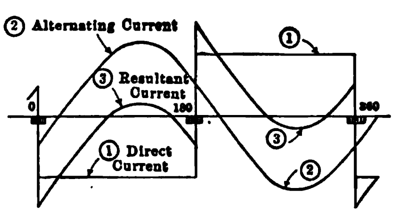


FIG. 411.—Current in coil a, Fig. 409, at unity power-factor.

345. Wave Forms of Currents in the Armature Coils.—The current in any armature coil is the difference between the alternating-current input and the direct-current output.

In Fig. 409, l_1 and l_2 are the two leads of one of the n-phases of a converter, a is the coil next to one lead and c is the coil in the center of the phase. The alternating e.m.f. and the power component of the alternating current in the phase l_1 to l_2 are maximum when this section of the winding is midway between the brushes and they are both zero when the center coil c is under the brush.

Fig. 410 shows the resultant of the alternating and direct

currents in coil c during one revolution. The alternating current is opposed to the direct current and is zero when the direct current reverses as the coil passes under the brush. The current in the center coil is, therefore, less than the current in any other coil in the phase when the power factor is unity.

Fig. 411 shows the current in coil a next to one of the leads. The alternating and direct currents are not directly opposing and the resultant current is greater than in the center coil c.

The coils next to the leads, therefore, carry larger currents than the coils in the center of the phases and they rise to a higher temperature.

The worst condition of local heating occurs in the coil next the lead of a single-phase converter, Fig. 412. The alternating current has its maximum value when the direct current reverses.

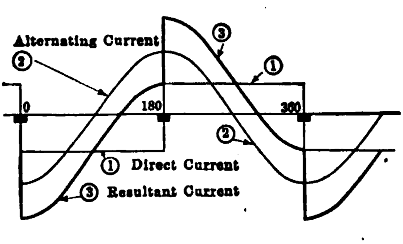


FIG. 412.—Current in the coil next to the lead in a single-phase converter.

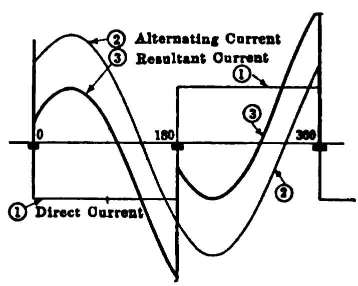


FIG. 413.—Current in coil c, Fig. 409, at 70 per cent. power factor.

When the power factor is not unity the minimum resultant current will not occur in the center coil of a phase but in a coil displaced from it by the angle of lag or lead of the current. Fig. 413 shows the current in coil c when the power factor is 70 per cent. and a component of lagging current equal to the power current flows in the armature.

346. Heating Due to Armature Copper Loss.—Take the case of a two-pole, n-phase armature, Fig. 409, and use the same notation as before. In the center coil c of the phase, the direct current is $\frac{I}{2}$ and the effective value of the alternating current is

$$I' = \frac{\sqrt{2}I}{n \sin \frac{\pi}{n}}.$$

The instantaneous value of the alternating current is

$$i = \sqrt{2}\, I' \sin\theta = \frac{2I}{n \sin\frac{\pi}{n}} \sin\theta, \tag{372}$$

and the instantaneous value of the resultant current is

$$i_0 = \frac{2I}{n \sin\frac{\pi}{n}} \sin\theta - \frac{I}{2}. \tag{373}$$

In an armature coil d displaced by angle β from the center of the phase the alternating current is

$$i = \sqrt{2} I' \sin(\theta - \beta) \tag{374}$$

and the instantaneous value of the resultant current is

$$\begin{aligned} i_\beta &= \frac{2I}{n \sin\frac{\pi}{n}} \sin(\theta - \beta) - \frac{I}{2} \\ &= \frac{I}{2}\left\{\frac{4 \sin(\theta - \beta)}{n \sin\frac{\pi}{n}} - 1\right\}. \end{aligned} \tag{375}$$

The effective value of the resultant current is

$$I_\beta = \sqrt{\frac{1}{\pi}\int_0^\pi i_\beta^2 d\theta} = \frac{I}{2}\sqrt{\frac{1}{\pi}\int_0^\pi \left\{\frac{4 \sin(\theta - \beta)}{n \sin\frac{\pi}{n}} - 1\right\}^2 d\theta} \tag{376}$$

$$= \frac{I}{2}\sqrt{\frac{1}{\pi}\int_0^\pi \left\{\frac{16 \sin^2(\theta - \beta)}{n^2 \sin^2\frac{\pi}{n}} - \frac{8 \sin(\theta - \beta)}{n \sin\frac{\pi}{n}} + 1\right\} d\theta}$$

$$= \frac{I}{2}\sqrt{\frac{1}{\pi}\int_0^\pi \left[\frac{8\{1 - \cos 2(\theta - \beta)\}}{n^2 \sin^2\frac{\pi}{n}} - \frac{8 \sin(\theta - \beta)}{n \sin\frac{\pi}{n}} + 1\right] d\theta}$$

$$= \frac{I}{2}\sqrt{\frac{1}{\pi}\left[\frac{8}{n^2 \sin^2\frac{\pi}{n}}\left\{\theta - \frac{\sin 2(\theta - \beta)}{2}\right\} + \frac{8 \cos(\theta - \beta)}{n \sin\frac{\pi}{n}} + \theta\right]_0^\pi}$$

$$= \frac{I}{2}\sqrt{\frac{1}{\pi}\left[\frac{8\pi}{n^2 \sin^2\frac{\pi}{n}} - \frac{16 \cos\beta}{n \sin\frac{\pi}{n}} + \pi\right]}$$

$$= \frac{I}{2}\sqrt{\frac{8}{n^2 \sin^2\frac{\pi}{n}} - \frac{16 \cos\beta}{n\, \pi \sin\frac{\pi}{n}} + 1}. \tag{377}$$

Since $\frac{I}{2}$ is the current in the coil when the machine is operating as a direct-current generator, the ratio of the power lost in the coil when operating as a converter to that lost when operating as a direct-current generator with the same output is

$$h_\beta = \left[\frac{I_\beta}{\frac{I}{2}}\right]^2 = \frac{8}{n^2 \sin^2 \frac{\pi}{n}} - \frac{16 \cos \beta}{n \pi \sin \frac{\pi}{n}} + 1 \tag{378}$$

and this is the ratio of the coil heating under the two conditions.

This ratio is a maximum for the coil next to the alternating leads l_1 or l_2, where $\beta = \frac{\pi}{n}$, and it is

$$h_{\text{max.}} = \frac{8}{n^2 \sin^2 \frac{\pi}{n}} - \frac{16 \cos \frac{\pi}{n}}{n\pi \sin \frac{\pi}{n}} + 1. \tag{379}$$

It is a minimum for the center coil of the phase, where $\beta = 0$, and is

$$h_0 = \frac{8}{n^2 \sin^2 \frac{\pi}{n}} - \frac{16}{n\pi \sin \frac{\pi}{n}} + 1. \tag{380}$$

The ratio of the total power lost in the armature of the converter to that lost when the machine is operating as a direct-current generator with the same output is found by integrating the ratio h_β over one-half phase from $\beta = \frac{\pi}{n}$ to $\beta = 0$ and taking the average. It is

$$H = \frac{n}{\pi}\int_0^{\frac{\pi}{n}} h_\beta d\beta = \frac{n}{\pi}\int_0^{\frac{\pi}{n}}\left(\frac{8}{n^2 \sin^2 \frac{\pi}{n}} - \frac{16 \cos \beta}{n\pi \sin \frac{\pi}{n}} + 1\right)d\beta.$$

$$= \frac{n}{\pi}\left[\frac{8\beta}{n^2 \sin^2 \frac{\pi}{n}} - \frac{16 \sin \beta}{n\pi \sin \frac{\pi}{n}} + \beta\right]_0^{\frac{\pi}{n}}$$

$$= \frac{8}{n^2 \sin^2 \frac{\pi}{n}} - \frac{16}{\pi^2} + 1, \tag{381}$$

and this is the relative armature heating under the two conditions.

To get the same loss in the armature of a converter and the

same heating as in the direct-current generator, the armature current and the output may be increased in the ratio $\frac{1}{\sqrt{H}}$.

The values of H and $\frac{1}{\sqrt{H}}$ for the various polyphase converters are tabulated in Fig. 414.

Type	Direct-current generator	Single-phase $n = 2$	Three-phase $n = 3$	Two-phase or four-phase $n = 4$	Six-phase $n = 6$
Relative armature heating H	1.00	1.37	0.55	0.37	0.26
Rating by armature heating $\frac{1}{\sqrt{H}}$	1.00	0.85	1.34	1.64	1.96

FIG. 414.

For a single-phase converter, $n = 2$, the value of $\frac{1}{\sqrt{H}}$ is 0.85, and, therefore, the output of a machine as a single-phase converter is only 85 per cent. of its output as a direct-current generator for the same temperature rise.

For a three-phase converter $\frac{1}{\sqrt{H}} = 1.34$ and therefore the output is 34 per cent. greater than as a direct-current generator. For a six-phase converter the output is increased 96 per cent.

These values only hold if the alternating current is in phase with the impressed e.m.f. When leading or lagging currents flow in the armature the heating is very largely increased and the rating must be decreased.

The reduction in rating due to reactive lagging or leading currents may be found as follows:

If the alternating current lags behind the voltage by an angle ϕ, then, since the ratio of the power components of the alternating and direct currents must be the same as before, the alternating current input is increased by the amount of the reactive lagging current and its effective value is

$$I' = \frac{\sqrt{2}I}{n \sin\frac{\pi}{n} \cos\phi}; \tag{382}$$

and the instantaneous value of the resultant current in the coil d is

$$i_\beta = \frac{2I}{n \sin\frac{\pi}{n} \cos\phi} \sin(\theta - \beta - \phi) - \frac{I}{2}; \qquad (383)$$

its effective value is

$$I_\beta = \frac{I}{2}\sqrt{\frac{8}{n^2 \sin^2\frac{\pi}{n} \cos^2\phi} - \frac{16\cos(\beta+\phi)}{n\pi \sin\frac{\pi}{n}} + 1}; \qquad (384)$$

the ratio of the power lost in the coil of a converter to that of a direct-current generator is

$$h_\beta = \left[\frac{I_\beta}{\frac{I}{2}}\right]^2 = \frac{8}{n^2 \sin^2\frac{\pi}{n} \cos^2\phi} - \frac{16\cos(\beta+\phi)}{n\pi \sin\frac{\pi}{n} \cos\phi} + 1 \qquad (385)$$

and the relative armature heating is

$$H = \frac{n}{2\pi}\int_{-\frac{\pi}{n}}^{\frac{\pi}{n}} h_\beta d_\beta = \frac{8}{n^2 \sin^2\frac{\pi}{n} \cos^2\phi} - \frac{16}{\pi^2} + 1. \qquad (386)$$

To find the power factor at which the rating of the converter is reduced to that of the direct-current generator, equate H to unity and solve for $\cos\phi$.

Thus,

$$\frac{8}{n^2 \sin^2\frac{\pi}{n} \cos^2\phi} - \frac{16}{\pi^2} + 1 = 1 \text{ and } \cos\phi = \frac{\pi}{\sqrt{2}n \sin\frac{\pi}{n}}. \qquad (387)$$

When $n = 3$, $\cos\phi = 0.85$ and when $n = 6$, $\cos\phi = 0.745$; therefore the rating of a machine as a three-phase converter will be the same as when operated as a direct-current generator when the power factor is 85 per cent. and the rating of a machine as a six-phase converter will be the same as when operated as a direct-current generator when the power factor is about 75 per cent.

Synchronous converters should be operated at approximately unity power factor at full load and overload.

347. Armature Reaction.—The armature reaction of a rotary converter is the resultant of the armature reactions of the machine as a direct-current generator and as a synchronous motor. The direct-current brushes are usually placed on the no-load neutral points and therefore the direct-current exerts a m.m.f. in

quadrature behind the field m.m.f. (Fig. 415). The power component of the armature current in a synchronous motor exerts a m.m.f. in quadrature ahead of the field m.m.f. and it is therefore opposed to the m.m.f. of the direct current.

If Z is the number of conductors on the armature of a bipolar generator and $\frac{I}{2}$ is the direct current in each conductor, the armature m.m.f. is the resultant of $\frac{Z}{2}$ m.m.fs. of magnitude $\frac{I}{2}$ uniformly distributed over the circumference of the armature and

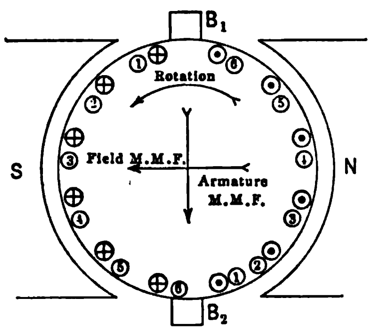


FIG. 415.—M.m.fs. in a direct-current generator.

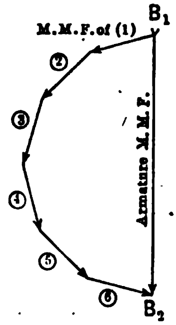


FIG. 416.

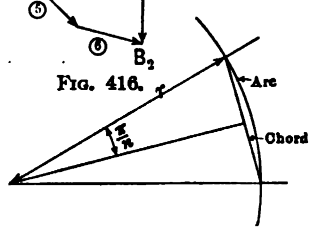


FIG. 417.

it is therefore less than that of a concentrated winding in the ratio $\frac{2}{\pi}$ (Fig. 416). The m.m.f. of the direct current in the armature of a converter is thus

$$M_d = \frac{Z}{2} \cdot \frac{I}{2} \cdot \frac{2}{\pi} = \frac{ZI}{2\pi} \tag{388}$$

in quadrature behind the field m.m.f.

If the machine is connected as an n-phase converter, the number of turns per phase is $\frac{Z}{2n}$ and the effective value of the alternating current in each is

$$I' = \frac{\sqrt{2}I}{n \sin \frac{\pi}{n}},$$

and the m.m.f. per phase in effective ampere-turns is

$$m' = \frac{ZI'}{2n} = \frac{ZI}{\sqrt{2}\, n^2 \sin\frac{\pi}{n}}. \tag{389}$$

These ampere-turns are distributed over $\frac{1}{n}$th of the circumference of the armature and their resultant is, Fig. 417,

$$m = \frac{ZI'}{2n} \cdot \frac{\text{chord}}{\text{arc}} = \frac{ZI'}{2n} \cdot \frac{2r \sin\frac{\pi}{n}}{\frac{2\pi r}{n}}, \text{ (Fig. 417),}$$

$$= \frac{ZI}{\sqrt{2} n^2 \sin\frac{\pi}{n}} \cdot \frac{2r \sin\frac{\pi}{n}}{\frac{2\pi r}{n}}$$

$$= \frac{ZI}{\sqrt{2}\pi n}. \tag{390}$$

The maximum value of the m.m.f. per phase is

$$m_m = \sqrt{2} m = \frac{ZI}{\pi n} \text{ ampere-turns.} \tag{391}$$

To find the resultant m.m.f. of the alternating current in the armature it is necessary to combine n m.m.fs. of maximum value $m_m = \frac{ZI}{\pi n}$ displaced in direction by angle $\frac{2\pi}{n}$ and displaced in phase by $\frac{1}{n}$th of a period or by angle $\frac{2\pi}{n}$.

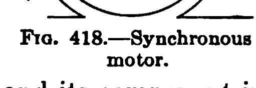


Fig. 418.—Synchronous motor.

In Fig. 418 phase 1 is shown in the position of maximum m.m.f. if the power factor is unity. The direction of the m.m.f. is OB and it is in quadrature ahead of the field m.m.f. If time and angular displacement are measured from OB, then at time t and angle θ the m.m.f. of phase 1 is $m_m \cos\theta$ in direction OB_1 and its component in direction OB is $m_m \cos^2\theta$.

At time t the m.m.f. of phase 2 is $m_m \cos\left(\theta + \frac{2\pi}{n}\right)$ making an angle $\left(\theta + \frac{2\pi}{n}\right)$ with OB and its component in direction OB is $m_m \cos^2\left(\theta + \frac{2\pi}{n}\right)$.

The resultant m.m.f. of the n phases in the direction OB at any time t is

$$M_a = m_m \left\{ \cos^2\theta + \cos^2\left(\theta + \frac{2\pi}{n}\right) + + + \cos^2\left(\theta + \frac{2(n-1)\pi}{n}\right) \right\}$$
$$= m_m \times n \times \text{average } (\cos)^2 = m_m \frac{n}{2}$$

since the average $\cos^2$ is $= \frac{1}{2}$.

The resultant m.m.f. of the n phases in the direction at right angles to OB or in line with the field m.m.f. is zero.

Therefore the resultant m.m.f. of the alternating current in the converter armature has a constant value

$$M_a \doteq m_m \frac{n}{2} = \frac{ZI}{\pi n} \cdot \frac{n}{2} = \frac{ZI}{2\pi} \qquad (392)$$

and is in quadrature ahead of the field m.m.f. It is thus equal to the m.m.f. of the direct current and is opposed to it and the resultant armature reaction of the direct current and of the corresponding power component of the alternating current is zero.

The armature reaction due to the power current required to supply the losses remains but it is very small and produces only a slight distortion of the flux in the air gap.

The effective armature reaction of a six-phase converter is from 7 to 20 per cent. of that of the corresponding direct-current generator.

When the power factor is not unity the wattless currents in the armature exert m.m.fs., as in the synchronous motor, which act in line with the field m.m.f. and are either magnetizing or demagnetizing but are not distorting.

Thus in the rotary converter there is very little field distortion or very little shifting of the neutral points under load. As a result the limit of overload set by commutation is much higher than in the direct-current generator. This is very important in the case of converters supplying railway loads where the load factor is usually below 50 per cent. The overload capacities for short periods must be high. In some cases when using interpoles momentary overloads of 200 per cent. are permitted.

348. Control of the Direct-current Voltage.—Since the ratio of alternating- to direct-current voltage is fixed, for a given flux distribution, the direct voltage cannot be controlled as simply as in the direct-current generator. There are three practicable methods of control:

(*a*) Variation of the alternating voltage.

(*b*) Variation of the direct-current voltage by means of a direct-current booster.

(*c*) Variation of the flux distribution as in the split-pole converter.

349. Methods of Varying the Alternating Voltage.—The impressed alternating voltage may be varied: (1) by variable ratio supply transformers; (2) by induction regulators in the supply lines; (3) by introducing reactances in the supply lines and drawing lagging or leading currents through them by varying the field excitation of the converter; and (4) by a synchronous booster.

The first method has the disadvantage of a step-by-step regulation and the contacts are liable to be burned as the circuit is opened when changing from one tap to another; the second method requires expensive apparatus but gives very good regulation and may be made automatic; the third is inexpensive and is made automatic by placing a series winding on the converter poles. The fourth method requires the use of an additional synchronous machine. It is very satisfactory and is being employed very extensively especially in large installations.

350. Compounding by Reactance.—When the field current of a synchronous converter is varied, reactive lagging or leading currents flow in the armature and magnetize or demagnetize the field and bring it back to its former strength and the direct-current voltage remains as before. To vary the direct-current voltage the impressed alternating voltage must be varied. This may be accomplished by introducing reactance coils in the supply lines and placing a series-field winding on the converter poles. The e.m.f. of inductance lags 90 degrees behind the current; thus, when the converter is under-excited a component of current lagging 90 degrees behind the impressed e.m.f. flows through the reactance coils, and the e.m.f. of inductance due to it lags 180 degrees behind the impressed e.m.f. and therefore opposes and lowers it. When the converter is over-excited and a component of current 90 degrees ahead of the e.m.f. flows through the reactance, the e.m.f. of inductance due to it is in phase with the impressed e.m.f. and raises it.

Therefore, when reactance coils are connected in the supply lines an increase of the field excitation raises the impressed e.m.f. and so raises the direct-current voltage, and a decrease of the excitation lowers the impressed e.m.f. and also the direct-current

voltage. The result is the same as in the direct-current generator but it is produced in a somewhat different way.

The series winding causes the direct-current voltage to rise automatically with load. The excitation is so adjusted that at light load the converter is under-excited and the power factor is low and lagging while at full load it operates at a power factor of about 98 per cent. leading. This results in only a slight increase in the heating.

In some cases the required reactance may be provided in the step-down transformers but in 25-cycle systems it is difficult to provide sufficient reactance in this way.

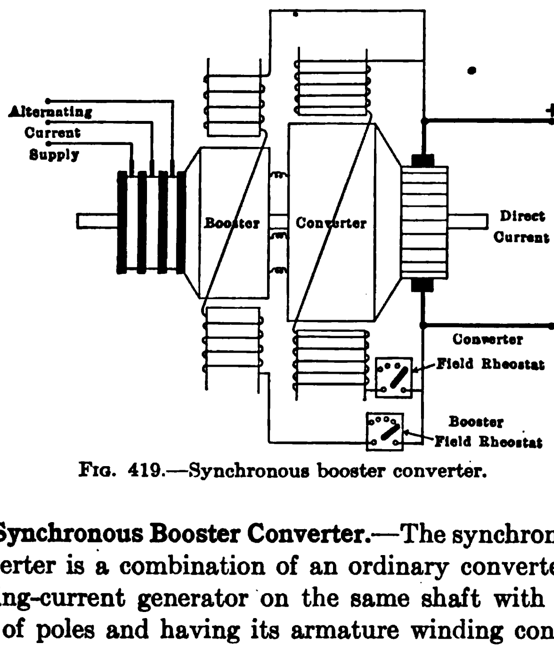


Fig. 419.—Synchronous booster converter.

351. Synchronous Booster Converter.—The synchronous booster converter is a combination of an ordinary converter and an alternating-current generator on the same shaft with the same number of poles and having its armature winding connected in series with the converter armature. The excitation of the booster is so arranged that it can be reversed, and therefore the booster voltage reversed, giving a uniform voltage variation of double the booster voltage. The booster may be shunt-excited and have its voltage controlled by an automatic regulator or it may be series-wound. Fig. 419 shows the diagram of connections for a shunt-excited booster.

352. Direct-current Booster Converter.—The direct-current voltage can be varied by inserting a booster in the direct-current

circuit. It may be direct-connected to the converter or driven by a separate motor. In either case additional floor space is required and the commutator of the booster must have the same current capacity as that of the converter.

353. Split-pole Converter.—In the split-pole converter the variation of direct-current voltage is secured, not by a variation of the impressed alternating voltage but by changing the shape of the magnetic field so that the total flux and the direct-current voltage are changed while the alternating voltage remains unchanged. This is possible because in three-phase windings third harmonics and their multiples do not appear in the terminal voltage.

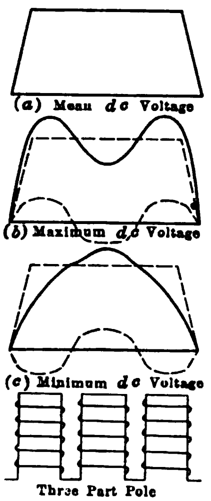


FIG. 420.—Split-pole converter.

If the pole is made in three parts the excitation of the various parts may be changed so as to introduce a third harmonic in the flux wave. With the three sections equally excited the flux wave is flat as shown at (*a*) in Fig. 420 and the direct-current voltage has its mean value. If the excitation of 1 and 3 is increased while that of 2 is decreased a third harmonic is produced in the flux wave and in the generated voltage but will not appear in the terminal voltage of the three-phase converter with its taps at 120 degrees. The total flux and therefore the direct-current voltage is increased in proportion to the area under one-half wave of the third harmonic flux (Fig. 420 (*b*)). If the excitation of 1 and 3 is decreased and that of 2 is increased the flux and the direct-current voltage are decreased as shown at (*c*). A variation of the direct-current voltage of 20 per cent. in either direction may be obtained.

The space required for the three-part pole is large and the converter must be made of large diameter to give room for it. Similar results can be obtained by using a two-part pole but the flux wave is not symmetrical and it is more difficult to maintain a suitable commutating field, but the machine has a smaller diameter and is less expensive.

354. Frequencies and Voltages.—Converters are built for 25 and 60 cycles and for voltages 250 and 600 volts. Some 25-cycle converters have been built for 1,200 and 1,500 volts, but

where such high direct-current voltages are required it is more usual to connect two converters in series or to use motor-generator sets.

The design of converters is similar to that of direct-current generators and no special difficulties are met in 25-cycle converters up to 600 volts but in 60-cycle converters for 600 volts and in 25-cycle converters for 1,200 volts, since the number of poles is fixed by the frequency and the speed, it is difficult to leave space enough between direct-current brushes for a sufficient number of commutator bars. The bars must be made narrow and the number of volts per bar is high and the danger of flash-over is great. However, by increasing the commutator peripheral speed to 5,500 ft. per minute the distance between neutral points is satisfactory and the number of volts per bar is reduced to a safe value. Sixty-cycle 600-volt converters are now giving perfect satisfaction in outputs up to 2,000 kw. but commutating poles are required on all above 500 kw.

355. Outputs and Efficiencies.—Six-hundred-volt, 25-cycle converters are built in sizes up to 4,000 kw. and range in efficiency from 95 to 96 per cent.; 600-volt 60-cycle converters are built up to 2,000 kw. and with efficiencies from 91 to 94 per cent. Converters for 250 volts are built up to 1,000 kw. with efficiencies of 93 to 95.5 per cent. for 25 cycles and 90.5 to 94 per cent. for 60 cycles.

356. Overload Capacity.—Commutating-pole converters will stand momentary overloads of 100 per cent. and in special cases of 200 per cent. The armature reaction is much smaller than in direct-current generators and therefore the interpoles do not require so much excitation and can be designed to take care of very heavy overloads without becoming saturated.

357. Dampers.—Synchronous converters should always be provided with damper windings to prevent hunting. A complete squirrel-cage winding is used with non-interpole converters but when interpoles are required separate grids are placed in the main pole faces. A short-circuited winding surrounding the interpole would delay the growth of flux in it and so interfere with commutation.

358. Starting.—Synchronous converters may be started: (1) from the alternating-current end, (2) from the direct-current end, and (3) by an auxiliary induction motor.

359. Alternating-current Self-starting.—If low alternating voltage is applied to the slip rings the converter will start as an

induction motor. The damper windings in the pole faces act as the secondary and the rotating armature as the primary. When it is nearly up to synchronous speed, it falls into step as explained in Art. 277. The field circuit is then closed and full voltage impressed. For small converters one low-voltage tap only is required but for the larger ratings two starting taps are brought out from the supply transformers.

While coming up to speed alternating voltages are induced in the field winding. Their magnitude depends on the ratio of field turns to armature turns and on the slip of the rotor behind the revolving field, and they disappear when synchronous speed is reached. To prevent dangerous induced voltages in the field winding it is usually broken up into a number of sections during starting by means of a field break up switch, or it may be short-circuited.

The converter may be brought up to speed very quickly in this way since it does not require to be synchronized, but there is no way of predetermining the polarity of the direct-current brushes. If the polarity is wrong the converter may be forced to slip a pole by reversing the field switch while the armature is still connected to the low-voltage taps. The switch must then be returned to its original position.

When starting from the alternating-current end a large current at low power-factor is drawn from the lines and it may be objectionable.

360. Direct-current Self-starting.—Converters may be started as shunt motors from the direct-current end if suitable power is available in the station but a longer time is required to put them in operation than with alternating-current starting and they must be synchronized. The starting current is low.

361. Starting by an Auxiliary Motor.—If an induction motor with a smaller number of poles than the converter, and consequently a higher synchronous speed, is mounted on the same shaft it may be used as a starting motor. This method requires synchronizing and thus takes considerable time but the starting current is small and there is no trouble with the polarity. The extra starting motor increases the cost of the equipment.

362. Brush Lifting Device.—When starting a converter as an induction motor the revolving armature flux induces voltages in the coils short-circuited by the direct-current brushes and sparking results but it is not usually serious. In commutating-pole

converters the sparking tends to be very much worse since the reluctance of the path through the short-circuited coil is reduced by the presence of the interpole iron and therefore the flux and the induced voltage are increased. To prevent serious sparking a brush-lifting device is employed which raises all the brushes except two narrow ones which are required to indicate the polarity.

363. Bucking or Flashing.—Bucking or flashing may be the result of poor commutating conditions causing bad sparking under the brushes, or of high voltage between adjacent bars; or of low surface resistance between adjacent brushes. These causes are usually all present to a greater or lesser degree when flashing takes place.

Commutating conditions in converters are normally better than in direct-current generators since the resultant armature reaction is small; but very heavy overloads may distort the field and cause sparking; or sparking may result from mechanical defects as wrong brush setting, poor contact, high mica, etc.

Comparatively high voltage between bars is the usual condition in rotary converters since the space between neutral points is limited especially for the higher frequencies and voltages.

Any increase of impressed voltage due to disturbances on the lines will increase the volts per bar or a sudden increase of current, due to a short-circuit, may distort the field and increase the voltage between certain bars. This tends to cause local sparking between these bars. A spark starting under the brush due to any cause may then be carried round to the next brush especially if the commutator is dirty. Such a flash-over, if severe, will short-circuit the converter and cause it to buck, fall out of synchronism and stop.

364. Parallel Operation.—When two or more converters are connected to the same direct-current bus-bars they should be supplied from separate banks of transformers to prevent large currents circulating between them due to differences of counter e.m.f. or to variations in the resistance of the direct-current brush contacts. The separate transformer banks permit slight corrections to be made in the voltages of the converters.

365. Inverted Converter.—Where a small alternating-current load is to be supplied from a direct-current system, a rotary converter may be used as an inverted converter to transform direct current to alternating current.

The ratios of the voltages are the same as under normal operating conditions but the ratios of the currents vary since it is not possible to eliminate or control the wattless components of the alternating current. These components depend on the character of the load and are not affected by varying the exciting current.

When changing from alternating current to direct current the speed of the converter is fixed by the frequency of the system and remains constant. When changing from direct current to alternating current the speed is not fixed but depends on the excitation and varies as the field strength varies. When the load is inductive the wattless lagging current demagnetizes the field and so raises the speed of the converter and the frequency of the alternating current. This may increase the lagging current and so raise the speed more until it gets beyond safe limits. The inverted converter has thus the two disadvantages: (1) that it tends to run at dangerous speeds, and (2) that it supplies a current of varying frequency. It must, therefore, be provided with some means of cutting off the load when the speed rises above a certain value or with some means of limiting the speed.

If the converter is excited by a direct-current generator mounted on the same shaft any increase in speed raises the exciter voltage at a higher rate and, therefore, the field of the converter is strengthened and the speed is limited.

Rotary converters to be used as inverted converters should be shunt-wound to secure as constant a speed and frequency as possible.

If two converters are operating in parallel and the alternating-current circuit breakers of one open, this converter will run inverted as a direct-current shunt motor and if it is under-excited the speed will rise. If it is in a position to supply alternating current to a reactive circuit the rise in speed may be serious. This condition may be prevented if the direct- and alternating-current circuit-breakers are interlocked or if the direct-current breakers are provided with a trip operated by a reverse-current relay.

366. Double-current Generator.—If mechanical power is supplied to drive a rotary converter it can be used as a double-current generator to supply direct current from the commutator and alternating current from the slip rings.

The two currents in this case flow in the same direction in the armature conductors and the losses are increased and the arma-

ture reaction is the sum of the reactions due to the two currents. The voltage regulation is, therefore, poorer than in the converter.

367. Three-wire Generator.—The three-wire direct-current generator is similar in construction to a single-phase or quarter-phase rotary converter. It is used to supply a three-wire system with from 220 to 280 volts between outer wires and from 110 to 140 volts between each of the outer wires and the neutral wire.

In order to obtain a point at a potential midway between the potentials of the direct-current brushes special transformers called compensators are used. They have a single winding tapped at the center and are connected by means of slip rings across points on the armature winding 180 electrical degrees apart (Fig. 421). The neutral wire of the system is connected to the central point of the compensator and its potential is maintained almost midway between the potentials of the outer wires. When more than one compensator is used the center points of all the compensators are joined together before being connected to the neutral wire (Fig. 422).

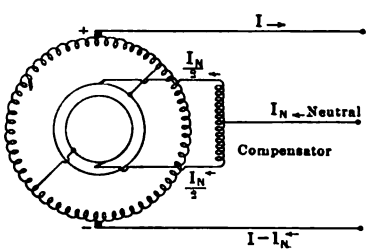


FIG. 421.—Three-wire generator with a single compensator.

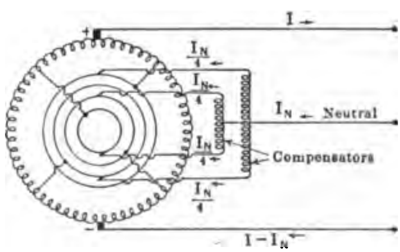


FIG. 422.—Three-wire generator with two compensators.

In some cases the compensators are connected directly to the armature windings and rotate with it and their neutral points are connected and brought out to a single slip ring.

The voltage between the terminals of the compensators is alternating and when the loads on the two sides of the three-wire system are equal and no current flows in the neutral wire the only current in the compensator is the very small exciting current.

When the loads are unequal the unbalanced current I_N flows in the neutral wire as shown in Fig. 421. On reaching the compensator it divides into two parts which flow through the winding and up to the positive brush by the path of least resistance. Since the current in the neutral wire is a direct-current the react-

ance of the compensator does not oppose it and the only voltage drop is that due to resistance.

The actual amount of current carried by the various sections of the armature winding is very difficult to calculate and it varies from instant to instant due to the change in the relative positions of the direct-current brushes and the compensator connections. The average current carried by each half of the compensator winding is $\frac{I_N}{2}$.

The unbalancing of the currents in the sections of the armature winding produces an unbalancing of the armature reactions and results in a slight unbalancing of the voltages between the neutral point and the brushes. With two or more compensators, Fig. 422, this unbalancing is reduced due to the more even distribution of the current.

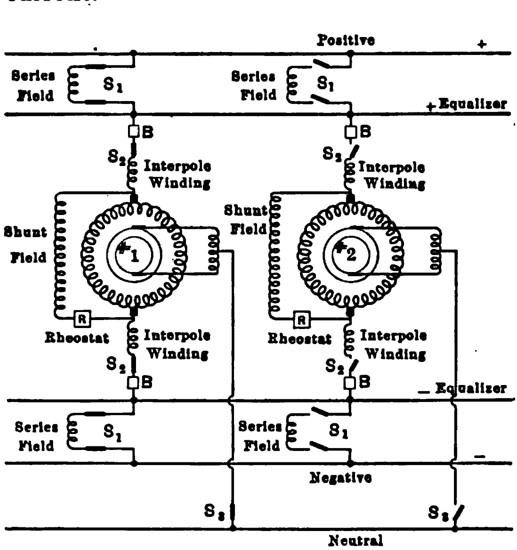


FIG. 423.—Parallel operation of three-wire generators.

Machines can be designed to give a regulation of 2 per cent. or less with an unbalanced load of 25 per cent.

With this system the voltages on the two sides cannot be regulated independently and the flexibility of the three-wire system supplied by two generators in series is lost, but there is a corresponding gain in space and cost of machines.

The capacity required in the compensators is small. With 25

per cent. unbalancing of the loads the required compensator capacity is less than 10 per cent. of the generator capacity.

368. Parallel Operation of Three-wire Generators.—Fig. 423 shows the diagram of connections for parallel operation of two compound-wound three-wire generators, which are provided with interpoles.

The series windings must be divided into two groups, one on either side of the system, and two equalizer connections are required. Half of each interpole winding must be connected on one side of the system and half on the other. In the case of the series-field windings it is sufficient to supply the south poles from one side and the north poles from the other.

To connect No. 2 in parallel with No. 1 bring it up to speed and close switches S_1, S_1, thus exciting its series windings, adjust the voltage by means of the shunt-field rheostat R, close S_2, S_2 and finally close the neutral switch S_3.

Circuit-breakers B, B with overload and reverse-current trip coils should be connected on each side to protect the machines.

369. Frequency Converters.—Frequency converters are used where power is transmitted at 25 cycles and is required by the consumer at 60 cycles or where two systems of different frequencies are to be linked together in a high-voltage network. The most usual form of frequency converter is a synchronous motor-generator set, a 25-cycle motor direct-connected to a 60-cycle alternator. The numbers of poles on the two machines must be in the ratio of 25 to 60. When a 10-pole motor is used the alternator must have 24 poles and the speed is fixed at 300 r.p.m.

When frequency converters are to be operated in parallel they must be synchronized on both the 25-cycle and 60-cycle ends. If the motor is synchronized first there is only one chance in five that the alternator is in synchronism, while if the alternator is synchronized first there is only one chance in twelve that the motor is in synchronism.

If the motor is synchronized and it is found that the generator is out of synchronism the circuit must be opened and the motor allowed to slip back a pair of poles at a time until the correct position is reached.

When linking up two systems by a frequency converter, the converter must be connected to one of them and then the second system brought up to the position of synchronism.

The induction frequency converter is discussed in Art. 415.

370. Mercury Vapor Rectifier.—A rectifier acts as an electric valve allowing current to flow through it in only one direction and it may therefore be used to rectify an alternating current, that is, to change it to a uni-directional current.

The mercury vapor rectifier, which is used very largely to convert alternating current to direct current for charging storage batteries, supplying arc lamps and many other purposes, is shown in Fig. 424(*a*). It consists of an exhausted bulb B which has two projections on its sides containing the positive terminals or anodes A and A' which are made of graphite, and two projections on the bottom containing mercury; C is the negative terminal or cathode and S is a third anode used only for starting. The large upper space in the bulb is the cooling chamber in which the mercury vapor, which has been heated by the passage of electricity, is condensed and from which it falls down into the cathode again.

The two anodes are connected to the terminals of the transformer TT' and the load circuit is connected between the center point of the transformer and the cathode C in series with the sustaining coil F.

The operation of the rectifier depends on the fact that current can pass through the tube in one direction only, from either of the anodes to the cathode, and it can only pass in this direction after an arc has been formed at the cathode in such a direction as to make the mercury negative. In the opposite direction the tube is a very good insulator and a difference of potential of thousands of volts would be required to produce a current. The starting arc is produced by impressing a voltage between the two mercury terminals C and S through the starting resistance r_S and tipping the tube until the mercury forms a bridge and closes the circuit. Current then passes and when the circuit is opened by raising the tube to the vertical position an arc is formed and the cathode is said to be excited. If at this instant either of the anodes is at a higher potential than the cathode, current will flow from it and will continue to flow so long as the difference of potential is greater than the counter e.m.f. of the rectifier. When the terminal T of the supply transformer is positive, current flows from it to A through the tube to C, through the sustaining coil F and load circuit to the terminal O. When the voltage reverses and T becomes negative, T' becomes positive and current flows from it to A' through the tube to C and through the coil F to the terminal O.

If there were no drop of voltage in the rectifier the current wave would be the same shape as the voltage wave and in the load circuit it would vary from zero to a positive maximum. There is, however, a drop of voltage of from 14 volts to 25 volts in the commercial rectifier, which remains approximately constant independent of the load, and if the sustaining coil were left out of the circuit the current through the bulb would drop to zero as soon as the potential of T had fallen below the counter e.m.f. of the rectifier and load circuit and would remain at zero until the potential of T' rose to a value greater than this counter e.m.f. In the meantime the cathode would have lost its excitation and

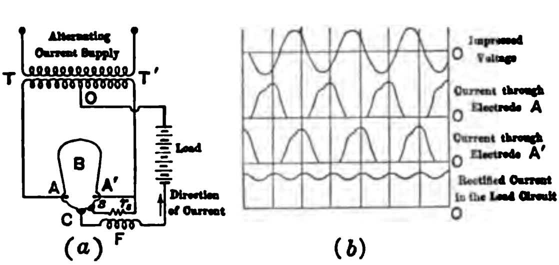


FIG. 424.—Single-phase mercury vapor rectifier.

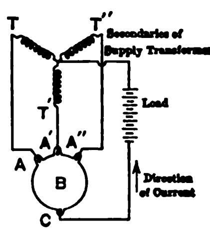


FIG. 425.—Three-phase mercury vapor rectifier.

the tube would have to be tipped again before any current could flow. To prevent the current in the tube from falling to zero and so to insure continuous operation the reactance coil F is introduced. Its action is as follows: While current is flowing from T energy is stored in the magnetic field of the coil and when the potential of T becomes too low to force the current against the counter e.m.f. of the converter the coil discharges its stored energy and maintains the current until the potential of T' rises and current flows from it to the load. The effect of the sustaining coil is therefore to spread out the two halves of the current wave so that they overlap and produce in the load circuit a direct current with only a slight pulsation (Fig. 424(b)).

371. Currents and Voltages.—The voltage is controlled by a regulating reactance connected in the alternating-current supply circuit and in the ordinary rectifiers the direct voltage ranges from 20 to 52 per cent. of the alternating voltage while the alternating current ranges from 40 to 66 per cent. of the direct current.

Rectifiers are designed for direct currents of 10, 20, 30 or 40

amp. and can be built to operate on any required voltage and any frequency.

372. Losses and Efficiency.—Since the counter e.m.f. of the rectifier is approximately constant independent of the load, the power losses vary directly as the current instead of as the square of the current. Neglecting the losses in the sustaining coil and regulator the efficiency of the rectifier is constant at all loads and is higher the higher the voltage. Values up to 80 per cent. are reached with rectifiers supplied from a 220-volt alternating-current circuit and delivering 110 volts direct current.

The power factor of the rectifier is high and under ordinary conditions may be assumed to be about 90 per cent.

373. Three-phase Rectifier.—The three-phase rectifier, Fig. 425, does not require any sustaining coil since the voltage waves of the three phases overlap and there is therefore no tendency for the load current to fall to zero and allow the cathode to lose its excitation. It must, however, be started in the same way as the single-phase rectifier.

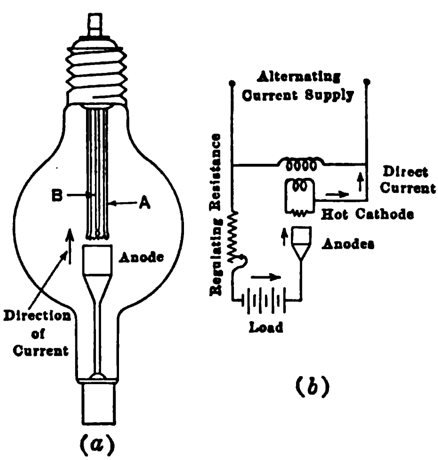


Fig. 426.—Hot-cathode half-wave rectifier.

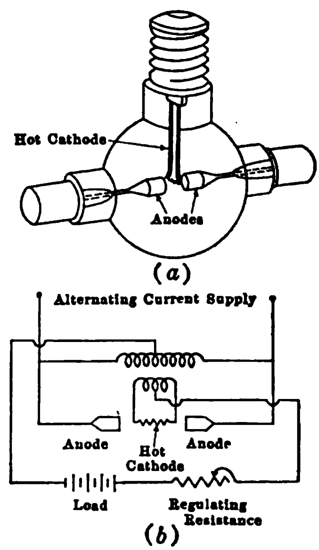


Fig. 427.—Hot-cathode full-wave rectifier.

374. Hot-cathode Argon-filled Rectifier.—Another type of rectifier suitable for low-voltage large-current circuits has been developed recently, which depends on the principle that current can flow from a cold to a hot electrode but not in the reverse

direction (Fig. 426). The rectifier consists of a glass bulb filled with argon gas with a graphite anode mounted on a heavy tungsten lead and two tungsten cathodes. One cathode *A* is used for starting and the other *B* for operating. Cathode *A* is similar to the filament of a tungsten lamp and is excited from a 40-watt transformer which raises it to a very high temperature and heats the operating cathode *B*. If the bulb is connected in series with a battery or other load across an alternating-current supply only one-half of each alternating wave can pass. If required, two half-wave rectifiers may be combined in one, Fig. 427, and both halves of the alternating wave passed through to the battery. Fig. 426(*b*) and Fig. 427(*b*) show the methods of connecting the half-wave and full-wave rectifier in the circuit. The current may be regulated by a resistance in series as shown or by means of an auto-transformer in the supply circuit.

In rectifiers for very small currents it is necessary to keep the exciting cathode in operation at all times but with larger currents it is only required for starting. When the exciting cathode is used continuously the loss in it and in the supply transformer is offset by a low energy loss in the arc between the electrodes. The arc drop in this case is from 4 to 8 volts while without the exciting electrode it is 10 to 14 volts.

The rectifier will start on voltages as low as 20 volts, and it is used principally for charging small storage batteries.

CHAPTER XII

INDUCTION MOTOR

375. Induction Motor.—Fig. 428 shows an induction motor of the squirrel-cage type. It consists of two main parts, the primary or stator and the secondary or rotor. The stator is exactly similar to the armature of a synchronous motor. The rotor which takes the place of the rotating field member of the synchronous motor is not excited by direct current but has currents induced in it by transformer action from the stator; thus

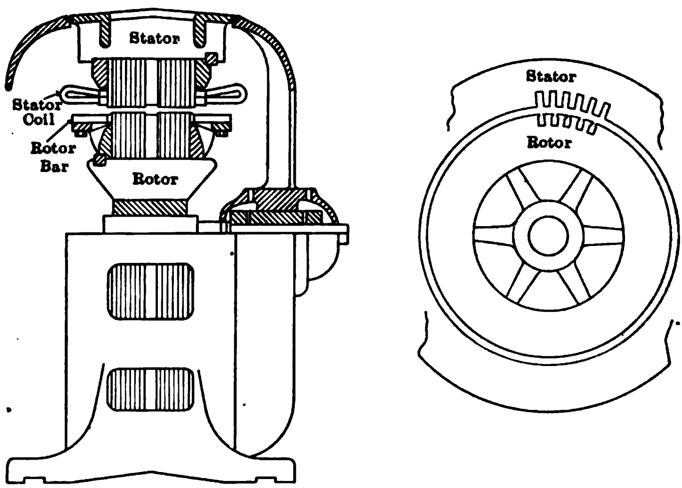


Fig. 428.—Induction motor.

the transfer of power from the stator to the rotor is similar to the transfer of power from the primary to the secondary in a transformer. The rotor is, however, free to move and there is an air gap in the magnetic circuit and therefore the magnetizing current is large, the leakage reactances are large and the power factor is low.

376. The Stator.—The primary or stator consists of a winding carried in slots on the inner face of a laminated iron core. The

winding is similar to an alternator or synchronous motor winding and the coils are connected in groups according to the number of phases and poles, one group per phase per pair of poles.

The stator is supplied with polyphase alternating currents and a revolving m.m.f. is produced similar to the m.m.f. of armature reaction in an alternator, which produces a magnetic field revolving at a constant speed called the synchronous speed of the motor.

In Fig. 429, (*a*) represents the stator winding of a two-pole, two-phase induction motor, (*b*) the currents supplied to the two

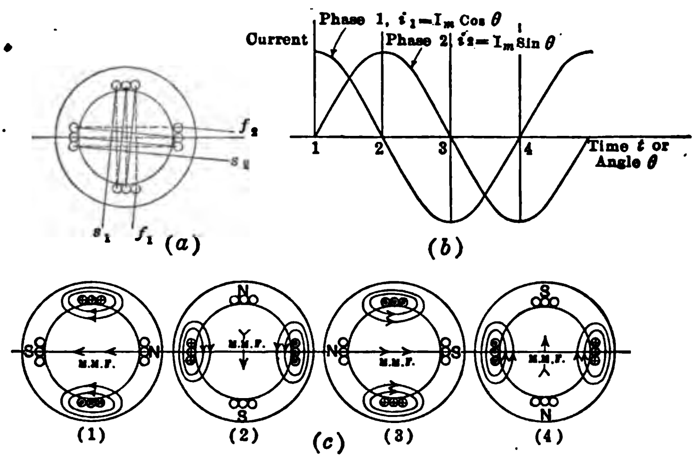


FIG. 429.—Revolving m.m.f. and flux in a two-pole, two-phase induction motor.

phases and (*c*) the fluxes produced by the resultant stator m.m.f. at the points (1), (2), (3) and (4) of the cycle.

The windings start at s_1 and s_2 and finish at f_1 and f_2 respectively. A positive current is one which enters at s_1 or s_2 and a negative current is one which enters at f_1 or f_2.

Referring to Fig. 429(*c*) it is seen that the north pole makes one complete revolution in the anti-clockwise direction while the current in phase 1 passes through one cycle.

Fig. 430(*a*) represents the stator of a two-pole three-phase induction motor, 430(*b*) the currents supplied and 430(*c*) the fluxes corresponding to the points (1), (2), (3) and (4) on the cycle. The north pole as before makes one revolution during one cycle.

Fig. 431(a) represents the stator of a four-pole, two-phase induction motor, (b) the currents supplied and (c) the fluxes produced.

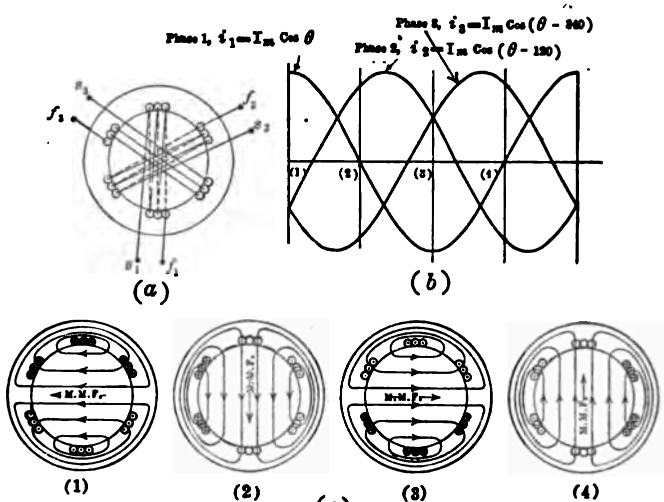


FIG. 430.—Revolving m.m.f. and flux in a two-pole, three-phase induction motor.

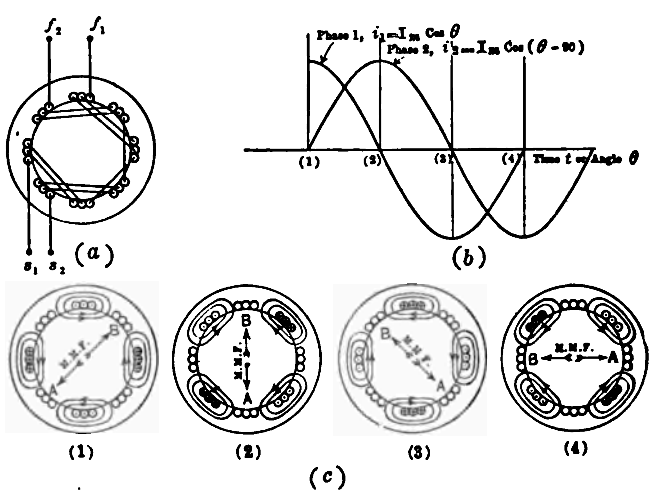


FIG. 431.—Revolving m.m.f. and flux in a four-pole, two-phase induction motor.

Fig. 432(a), (b) and (c) represent a similar set of conditions for a four-pole, three-phase stator.

In Fig. 431(c) and Fig. 432(c) it is seen that, while the current

goes through one cycle, the revolving field turns through the angle occupied by one pair of poles.

If the stator winding has p-poles, the revolving field turns through $\frac{360}{p/2}$ degrees, that is, through $\frac{2}{p}$ of one revolution during one cycle of the current.

If the frequency of the supply is f cycles per second the revolving field makes $\frac{2f}{p}$ r.p.s. or $\frac{120f}{p}$ r.p.m. The synchronous speed of an induction motor is, therefore,

$$N = \frac{120f}{p} \text{ r.p.m.} \tag{393}$$

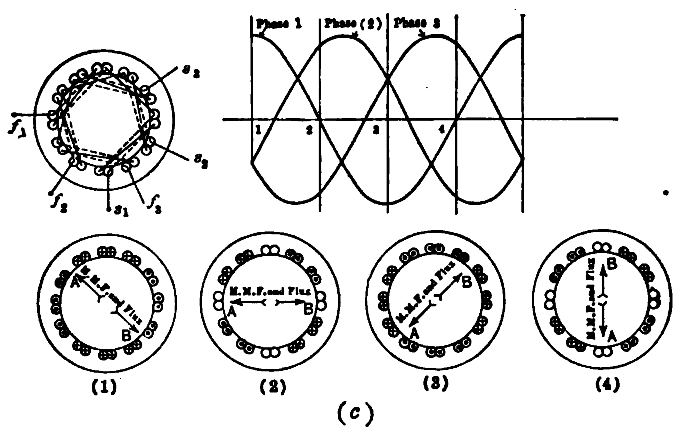


FIG. 432.—Revolving m.m.f. and flux in a four-pole, three-phase induction motor.

377. Revolving Magnetomotive Force and Flux of the Stator. —In Fig. 433, OX is the direction of the m.m.f. of phase 1 of the two-phase motor in Fig. 429 and its value at any instant is

$$m_1 = n_1 I_m \sin(\theta + 90) = n_1 I_m \cos\theta,$$

where n_1 is the number of turns per phase and $i_1 = I_m \sin(\theta + 90)$ is the current in phase 1.

At the same instant the m.m.f. of phase 2 is

$$m_2 = n_1 I_m \sin\theta, \text{ in direction } OY,$$

where

$$i_2 = I_m \sin\theta \text{ is the current in phase 2.}$$

The resultant m.m.f. of the two phases is

$$m = \sqrt{m_1^2 + m_2^2} = n_1 I_m \sqrt{\cos^2 \theta + \sin^2 \theta} = n_1 I_m$$

and makes an angle θ with the OX axis.

The resultant m.m.f. is, therefore, constant in value, being equal to the maximum m.m.f. of one phase, and it revolves at synchronous speed in the anti-clockwise direction.

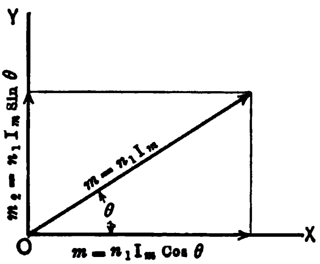


FIG. 433.

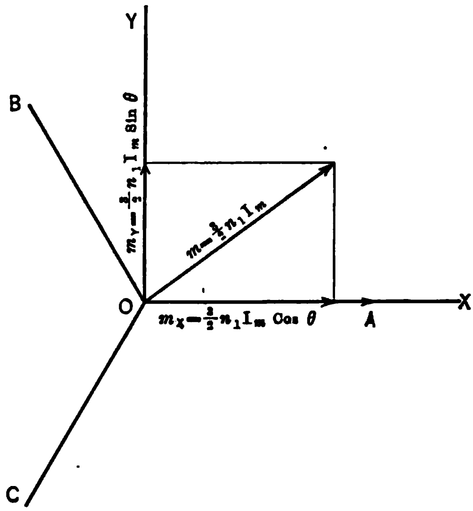


FIG. 434.

This constant m.m.f. acting on a path of constant reluctance produces a field of constant strength revolving with the m.m.f. and, therefore, revolving at synchronous speed relative to the winding of the stator. The flux linking with each phase of the stator is an alternating flux which reaches its maximum when the current in the phase is maximum and is therefore in phase with it.

Figs. 430(*a*), (*b*) and (*c*) represent respectively the winding of a three-phase, two-pole stator, the currents supplied and the m.m.fs. produced. Fig. 434 shows the m.m.fs. of the three phases as vectors.

The currents are:

$$\begin{aligned} i_1 &= I_m \cos \theta, \text{ in phase 1,} \\ i_2 &= I_m \cos (\theta - 120), \text{ in phase 2,} \\ i_3 &= I_m \cos (\theta - 240), \text{ in phase 3.} \end{aligned}$$

The m.m.f. of phase 1 is $n_1 I_m \cos \theta$ in direction OA.

The m.m.f. of phase 2 is $n_1 I_m \cos (\theta - 120)$ in direction OB.

The m.m.f. of phase 3 is $n_1 I_m \cos (\theta - 240)$ in direction OC.

The resultant m.m.f. in the horizontal direction is

$$m_X = n_1 I_m \cos \theta + n_1 I_m \cos (\theta - 120) \cos 120 + n_1 I_m \cos (\theta - 240) \cos 240$$

$$= n_1 I_m \left\{ \cos \theta - \frac{1}{2}(\cos \theta \cos 120 + \sin \theta \sin 120) - \frac{1}{2}(\cos \theta \cos 240 + \sin \theta \sin 240) \right\}$$

$$= n_1 I_m \frac{3}{2} \cos \theta = \frac{3}{2} n_1 I_m \cos \theta.$$

The resultant m.m.f. in the vertical direction is

$$m_Y = n_1 I_m \{\cos (\theta - 120) \sin 120 + \cos (\theta - 240) \sin 240\}$$

$$= n_1 I_m \left\{ \frac{\sqrt{3}}{2}(\cos \theta \cos 120 + \sin \theta \sin 120) - \frac{\sqrt{3}}{2}(\cos \theta \cos 240 + \sin \theta \sin 240) \right\}$$

$$= n_1 I_m \frac{\sqrt{3}}{2} \sqrt{3} \sin \theta = \frac{3}{2} n_1 I_m \sin \theta.$$

The resultant m.m.f. of the three phases is

$$m = \sqrt{m_X^2 + m_Y^2} = \tfrac{3}{2} n_1 I_m \sqrt{\cos^2 \theta + \sin^2 \theta} = \tfrac{3}{2} n_1 I_m$$

and makes an angle θ with the OX axis.

The resultant m.m.f. is, therefore, constant in value being equal to $\tfrac{3}{2}$ times the maximum m.m.f. of one phase and it revolves at synchronous speed.

This constant m.m.f. produces a field of constant strength revolving at synchronous speed. The revolving field links successively with the windings and generates e.m.fs. in them. The flux linking with any phase is maximum when the current in that phase is maximum and, therefore, the flux and current are in phase.

If a four-pole stator, Fig. 431, had been chosen instead of the two-pole stator, the m.m.fs. of the two-phase windings would have been combined at 45 degrees instead of 90 degrees and the resultant m.m.f. and flux would not remain constant but would pulsate four times during each revolution. The flux threading any phase would, however, still vary according to a sine law and would be in phase with the current in that phase.

To reverse the direction of rotation of a two-phase induction motor, it is necessary to reverse one phase only.

To reverse a three-phase motor any two leads may be interchanged.

378. The Rotor.—The secondary or rotor is made in two forms: (*a*) the wound rotor, and (*b*) the squirrel-cage rotor. The wound rotor consists of a laminated iron core with slots carrying the winding, which must have the same number of poles as the stator winding but may have a different number of phases. It is usually wound for three phases and the ends of the windings are brought out to slip rings so that resistances may be inserted in the windings for starting and the terminals short circuited under running conditions.

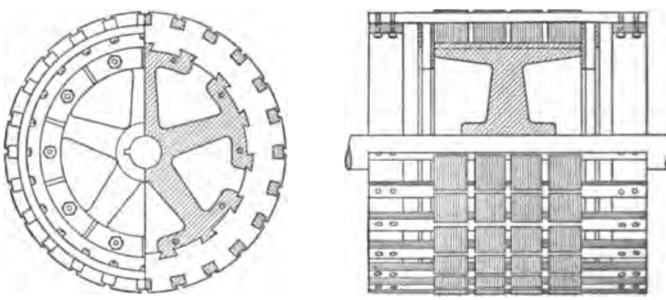


FIG. 435.—Squirrel-cage rotor.

The squirrel-cage rotor winding consists of a number of heavy copper bars short-circuited at the two ends by two heavy brass rings, Fig. 435. The construction is very rugged and there is nothing to get out of order.

When the rotor with its closed windings is placed in the revolving magnetic field produced by the stator currents, the flux cuts across the conductors on the rotor and generates e.m.fs. in them. Currents flow in the rotor equal to the e.m.fs. divided by the rotor impedances. These currents reacting on the magnetic field produce torque and the rotor revolves in the direction of the field. At no load the rotor runs almost as fast as the field and very small e.m.fs. and currents are induced in its conductors. When the motor is loaded the rotor lags behind the field in speed and large currents are induced to give the required torque.

379. Slip.—The difference between the synchronous speed or speed of the stator field and the speed of the rotor is called the slip and is expressed as a per cent. of synchronous speed.

A six-pole, 60-cycle motor has a synchronous speed

$$N = \frac{120 \times 60}{6} = 1{,}200 \text{ r.p.m.}$$

If the speed of the rotor at full load is 1,176 r.p.m., the slip is

$$s = \frac{1{,}200 - 1{,}176}{1{,}200} \times 100 \text{ per cent.} = 2 \text{ per cent.}$$

The slip at full load varies from 2 to 5 per cent. in motors designed for constant speed.

The rotor speed may be expressed as

$$S = (1 - s)\, N \text{ r.p.m.} \tag{394}$$

380. Magnetomotive Force of the Rotor.—The frequency of the e.m.fs. and currents induced in the rotor windings at a slip s is sf if f is the frequency of the e.m.fs. impressed on the stator.

The polyphase currents in the rotor windings produce a resultant m.m.f. revolving relative to the rotor at a speed $\frac{120sf}{p} = sN$ r.p.m. But the rotor itself is revolving at a speed $S = (1 - s)\, N$ r.p.m. and therefore the rotor m.m.f. is revolving at a speed $sN + (1 - s)\, N = N$, that is, at the same speed as the stator m.m.f. The two m.m.fs. are therefore stationary relative to each other and they are nearly opposite in phase as in the transformer.

381. Electromotive Force and Flux Diagram for the Induction Motor.

Let r_1 = stator resistance per phase.

L_1 = stator self-inductance per phase.

$x_1 = 2\pi f L_1$ = stator reactance per phase.

$Z_1 = \sqrt{r_1^2 + x_1^2}$ = stator impedance per phase.

r_2 = rotor resistance per phase.

L_2 = rotor self-inductance per phase.

$x_2 = 2\pi f L_2$ = rotor reactance per phase at standstill.

$sx_2 = 2\pi s f L_2$ = rotor reactance per phase at slip s.

$Z'_2 = \sqrt{r_2^2 + x_2^2}$ = rotor impedance per phase at standstill.

$Z_2 = \sqrt{r_2^2 + s^2 x_2^2}$ = rotor impedance per phase at slip s.

In Fig. 436

E_1 = the e.m.f. impressed on one phase of the stator.

I_1 = the current in one phase of the stator. $I_1 = I_M + I'$.

I_M = the magnetizing current in one phase of the stator.

I' = the load current in one phase of the stator.

I_2 = the current in one phase of the rotor; $I_2 = \frac{n_1}{n_2}I'$, where

n_1 = the turns per phase per pair of poles on the stator,

n_2 = the turns per phase per pair of poles on the rotor.

Φ_1 = the flux per pole linking one phase of the stator which would be produced by the current I_1 acting alone. $\Phi_1 = \Phi_{1g} + \Phi_{1L}$.

Φ_{1g} = the part of Φ_1 which crosses the gap and links with one phase of the rotor. $\Phi_{1g} = v_1\Phi_1$, where v_1 is a constant.

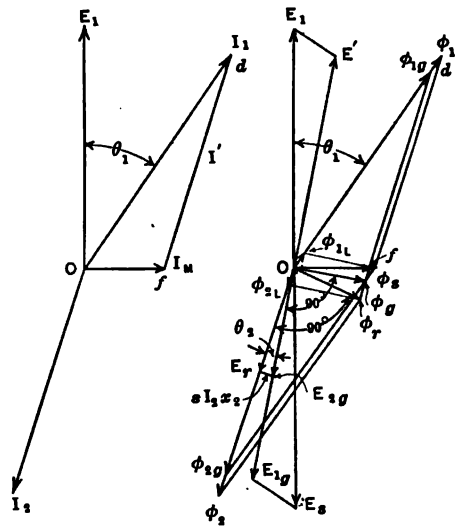


FIG. 436.—Currents, fluxes and e.m.fs. in an induction motor.

Φ_{1L} = the part of Φ_1 which does not cross the gap, but is the leakage flux of self-inductance of the stator.

Φ_2 = the flux per pole linking one phase of the rotor, which would be produced by the current I_2 acting alone. $\Phi_2 = \Phi_{2g} + \Phi_{2L}$.

Φ_{2g} = the part of Φ_2 which crosses the gap and links with one phase of the stator. $\Phi_{2g} = v_2\Phi_2$, where v_2 is a constant.

Φ_{2L} = the part of Φ_2 which does not cross the gap, but is the leakage flux of self-inductance of the rotor.

Φ_g = the actual flux per pole crossing the gap and linking with one phase of both stator and rotor; it is the resultant of Φ_{1g} and Φ_{2g}.

Φ_s = actual flux per pole linking one phase of the stator it; is the resultant of Φ_g and Φ_{1L}.

Φ_r = the actual flux per pole linking one phase of the rotor; it is the resultant of Φ_g and Φ_{2L}.

E_{1g} = the back e.m.f. generated in one phase of the stator by the flux Φ_g.

E_{1L} = the e.m.f. of self-inductance generated in one phase of the stator by the leakage flux Φ_{1L}. $E_{1L} = I_1x_1$.

E_s = the back e.m.f. generated in one phase of the stator by the flux Φ_s. E_s is the resultant of E_{1g} and E_{1L}.

E_1 = the e.m.f. impressed on one phase of the stator. It must be exactly equal and opposite to E_s if the stator resistance drop I_1r_1 is neglected. This drop is of the order of 2 per cent. at full load.

Since E_1 is constant, E_s and Φ_s must be constant.

E_{2g} = the e.m.f. generated in one phase of the rotor by the flux Φ_g. $E_{2g} = s\frac{n_2}{n_1}E_{1g} = sE_2$, where E_2 is the e.m.f. which would be generated in one phase of the rotor by the flux Φ_g at standstill.

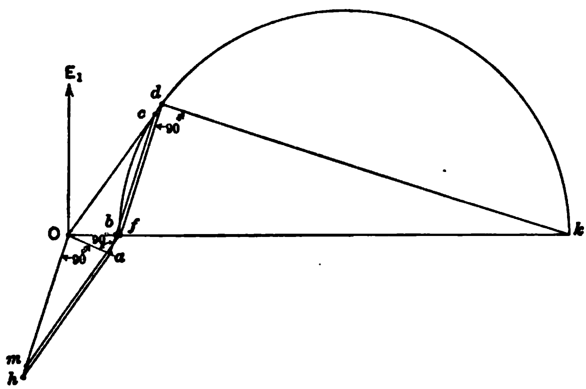


FIG. 437.

E_{2L} = the e.m.f. of self-inductance generated in one phase of the rotor by the leakage flux Φ_{2L}. $E_{2L} = sI_2x_2$.

E_r = the e.m.f. generated in one phase of the rotor by the flux Φ_r. E_r is the difference between E_{2g} and E_{2L} and is equal to I_2r_2. It is in phase with the rotor current I_2 and lags 90 degrees behind the flux Φ_r.

As the induction motor is loaded the end d of the vector Φ_1 follows a circle passing through f and having its center on of produced (Fig. 437).

382. Proof that the Locus is a Circle.—From d draw dk at right angles to fd to cut of produced in k. Then the semicircle fdk is the locus of d.

In the triangles aob and fdk

$$\angle oab = \angle fdk, \text{ being right angles,}$$
$$< oba = \angle dfk,$$

therefore

$$\frac{fk}{ob} = \frac{fd}{ab} = \frac{fd}{ac - cb} = \frac{fd}{oh - cb},$$

and

$$fk = \frac{ob \times fd}{oh - cb} = \frac{v_1 \cdot of \times v_2 \cdot oh}{oh - v_1 v_2 oh} = of \cdot \frac{v_1 v_2}{1 - v_1 v_2} = \text{a constant}$$

since of is constant.

Therefore the locus of d is a circle described on the diameter fk.

383. Magnetomotive Force Diagram.—Since the magnetic circuit of the machine is not saturated by the fluxes Φ_s and Φ_r, the flux diagram, Fig. 437, may be replaced by the m.m.f. diagram, Fig. 438.

$n_1 I_1$ = total m.m.f. of the stator per phase.
$n_1 I'$ = m.m.f. of the load component of stator current.
$n_1 I_M$ = m.m.f. of the magnetizing current.
$n_2 I_2$ = m.m.f. of the rotor per phase; it is equal and opposite to $n_1 I'$.

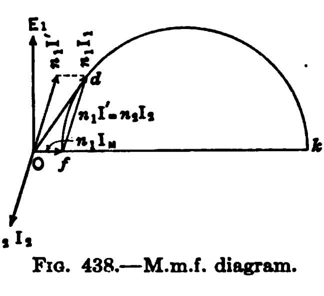


FIG. 438.—M.m.f. diagram.

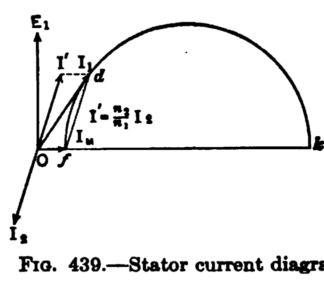


FIG. 439.—Stator current diagram.

384. Stator Current Diagram.—The m.m.f. diagram, Fig. 438, may be replaced by the stator current diagram, Fig. 439.

I_1 = total current in one phase of stator. $I_1 = I_M + I'$.
I_M = magnetizing current in one phase of stator.
I' = load component of current in one phase of stator.
I_2 = current in one phase of rotor. $I_2 = \frac{n_1}{n_2} I'$.

$fd = I' = \frac{n_2}{n_1} I_2 =$ load component of stator current per phase and represents the rotor current per phase.

To obtain an expression for the diameter of the circle, consider the special case where the motor is at standstill and the rotor and stator resistances are both zero.

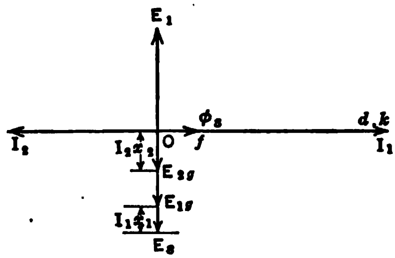


FIG. 440.—Induction motor at standstill.

Referring to Fig. 440

$$E_1 = E_s = E_{1g} + I_1 x_1,$$

and

$$E_{1g} = \frac{n_1}{n_2} E_{2g} = \frac{n_1}{n_2} I_2 x_2 = \left(\frac{n_1}{n_2}\right)^2 I' x_2;$$

therefore,

$$E_1 = I_1 x_1 + \left(\frac{n_1}{n_2}\right)^2 I' x_2$$

$$= I' \left(x_1 + \left(\frac{n_1}{n_2}\right)^2 x_2\right),$$

if the magnetizing current I_M is neglected in comparison with the load component of stator current I'.

This value of I' is represented by fk and is the diameter of the circle. Its value is

$$D = fk = I' = \frac{E_1}{x_1 + \left(\frac{n_1}{n_2}\right)^2 x_2} \tag{395}$$

The diameter of the circle, therefore, varies directly as the impressed e.m.f. and inversely as the sum of the reactances and therefore as the frequency of the supply.

Thus as the motor is loaded the end d of the vector representing the stator current follows a semicircle of diameter

$$\frac{E_1}{x_1 + \left(\frac{n_1}{n_2}\right)^2 x_2}.$$

385. Rotor Electromotive Force and Current.—The e.m.f. generated in one phase of the rotor at slip s by the flux Φ_g is

$$E_{2g} = sK\Phi_g, \text{ where } K \text{ is a constant,}$$

$= sE_2$, where $E_2 = K\Phi_g$ is the e.m.f. which would be generated in the rotor at standstill by the flux Φ_g. E_2 does not remain constant as the motor is loaded, since the flux Φ_g does not remain constant but decreases about 30 per cent. from no load to standstill when the rotor is locked and slip is unity. Up to full load Φ_g and E_2 may be considered to remain constant.

The impedance of the rotor at slip s is

$$Z_2 = \sqrt{r_2^2 + s^2x_2^2} \tag{369}$$

The rotor current is

$$I_2 = \frac{sE_2}{Z_2} = \frac{sE_2}{\sqrt{r_2^2 + s^2x_2^2}}. \tag{397}$$

The rotor power factor is

$$\cos\theta_2 = \frac{r_2}{\sqrt{r_2^2 + s^2x_2^2}}. \tag{398}$$

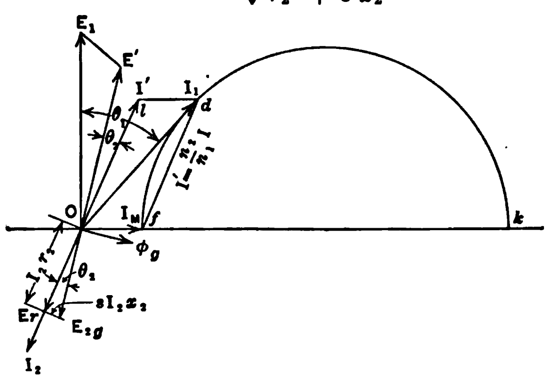


Fig. 441.

386. Rotor Input.—The power transferred from the stator to the rotor per phase is the product of the back voltage E', generated in the stator by the flux Φ_g, the load component of the stator current I' and the cosine of the angle between them; it is

$$p_r = E'I' \cos\theta_2 \text{ (see Fig. 441)} \tag{399}$$

but

$$E' = E_{1g} = \frac{n_1}{n_2}E_2;$$

and

$$I' = \frac{n_2}{n_1}I_2;$$

therefore,

$$p_r = E'I' \cos\theta_2 = \frac{n_1}{n_2} E_2 \frac{n_2}{n_1} I_2 \cos\theta_2 = E_2 I_2 \cos\theta_2. \quad (400)$$

Thus the power input to the rotor per phase is the product of the e.m.f. which would be generated in the rotor by the flux Φ_g at standstill and the power component of the rotor current.

The total power input to the n phases of the rotor is

$$P_r = nE_2 I_2 \cos\theta_2 \quad (401)$$

387. Rotor Copper Loss and Slip.—The power consumed by the rotor copper loss per phase is

$$I_2^2 r_2 = sE_2 I_2 \cos\theta_2, \quad (402)$$

and for the n phases it is

$$L_r = nI_2^2 r_2 = nsE_2 I_2 \cos\theta_2. \quad (403)$$

$$\text{Slip} = \text{the ratio } \frac{\text{rotor copper loss}}{\text{rotor input}} = \frac{L_r}{P_r} = \frac{snE_2 I_2 \cos\theta_2}{nE_2 I_2 \cos\theta_2} = s. \quad (404)$$

388. Rotor Output.—The rotor output per phase is

$$\begin{aligned} p = p_r - I_2^2 r_2 &= E_2 I_2 \cos\theta_2 - sE_2 I_2 \cos\theta_2 \\ &= (1 - s) E_2 I_2 \cos\theta_2 \end{aligned} \quad (405)$$

and the total rotor output is

$$P = np = n(1 - s) E_2 I_2 \cos\theta_2 \text{ watts} \quad (406)$$

$$= \frac{n(1 - s) E_2 I_2 \cos\theta_2}{746} \text{ h.p.} \quad (407)$$

From equations (401) and (406) the rotor output is

$$P = (1 - s) P_r = \frac{S}{N} P_r$$

and it is equal to the rotor input multiplied by the rotor speed in per cent. of synchronous speed.

389. Torque.—If T is the torque in lb.-ft. the output may be expressed as

$$P = \frac{2\pi ST}{33{,}000} \text{ h.p.}$$

Thus the torque is

$$T = \frac{n(1 - s) E_2 I_2 \cos\theta_2}{746} \times \frac{33{,}000}{2\pi S} = 7.05 \frac{nE_2 I_2 \cos\theta_2}{\frac{S}{1 - s}}$$

$$= 7.05 \frac{P_r}{N} = 7.05 \frac{\text{rotor input}}{\text{sync. speed}} \text{ lb.-ft.} \quad (408)$$

The torque of an induction motor is usually expressed not in pounds-feet but in synchronous watts, that is, in terms of the power which would be developed at synchronous speed.

The torque in synchronous watts is

$$T_{\text{sync. watts}} = P \times \frac{N}{S} = P_r \tag{409}$$

and it is equal to the power input to the rotor.

390. Rotor Efficiency.—Neglecting all losses except the rotor copper loss the rotor efficiency is

$$\eta_r = \frac{P}{P_r} 100 \text{ per cent.} = \frac{n(1 - s)\, E_2 I_2 \cos \theta_2}{n E_2 I_2 \cos \theta_2} 100 \text{ per cent.}$$

$$= (1 - s)\, 100 \text{ per cent.} = \frac{S}{N} 100 \text{ per cent.} \tag{410}$$

that is, the rotor efficiency is equal to the rotor speed in per cent. of synchronous speed and, therefore, the efficiency of an induction motor is always less than the speed in per cent. of synchronous speed.

391. Modification of Diagram.—When an induction motor is running without load, a current I_0 flows in each phase of the stator which has two components, Fig. 442, I_M the magnetizing current 90 degrees behind the impressed e.m.f. E_1, and I_P the power component in phase with E_1.

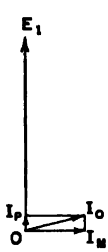


FIG. 442.

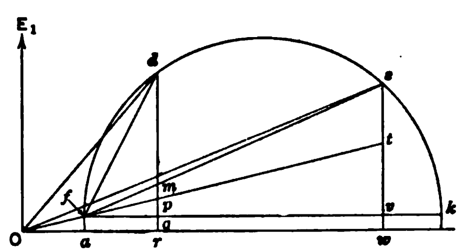


FIG. 443.—Circle diagram of an induction motor.

The product nE_1I_P (where n is the number of phases) is the power required to supply the no-load losses. These are the iron loss and a small copper loss in the stator and the friction and windage losses of the rotor. The iron loss in the rotor may be neglected since the rotor frequency is low.

The current required to supply the stator losses has no corresponding component in the rotor, but the power to overcome the friction and windage losses must be transferred from the stator to the rotor and therefore requires a current in the rotor.

As the motor is loaded and slows down the stator iron loss remains nearly constant, the friction and windage losses decrease and the rotor iron loss increases. At standstill the friction and windage losses are absent but the rotor iron loss is large since the rotor frequency is the same as the stator frequency. The iron friction and windage losses are therefore considered to remain constant and the small component of rotor current required to supply the friction and windage losses is neglected.

The diagram, Fig. 439, must therefore be changed to Fig. 443 by the addition of I_P the power component of the stator current per phase at no load.

The diameter of the circle is raised through the distance $af = I_P$ and of now represents not the magnetizing current I_M but the no-load current $I_0 = \sqrt{I_M^2 + I_P^2}$.

If os represents the stator current per phase at standstill and sw is its power component, then, since there is no output, the power input is consumed by the losses. Therefore, input = losses $= nE_1sw$; constant losses $= nE_1I_P = nE_1vw$ and copper losses $= nE_1sv = n(I'^2r_1 + I_2^2r_2)$.

The stator copper loss is taken as nI'^2r_1 because the stator copper loss at no load is included in the constant losses. It is therefore assumed that $I_1^2r_1 = I_0^2r_1 + I'^2r_1 +$ which is approximately correct up to full load.

Divide sv at t so that $st : tv = I_2^2r_2 : I'^2r_1$, then, stator-load copper loss $= nE_1tv$ and rotor copper loss $= nE_1st$.

Join ft and from d any point on the circle to the left of s draw $dmpqr$ perpendicular to the diameter fk. It is to be shown that mp is the stator current required to supply the rotor copper loss for a rotor current represented by fd and that pq is the current to supply the corresponding stator load copper loss.

The rotor copper loss is

$$nI_2^2r_2 = n\left(\frac{n_1}{n_2}I'\right)^2 r_2 = KI'^2 = K\overline{fd}^2 = K(\overline{fq}^2 + \overline{qd}^2)$$
$$= K \times fq(fq + qk) = K \times fq \times D = fq \times \text{ a constant,}$$

since K and D are both constants.

Therefore the rotor copper loss is proportional to fq; but

$$\frac{mp}{fq} = \frac{st}{fv}, \text{ or } \frac{mp}{(fd)^2} = \frac{st}{(fs)^2},$$

and since st represents the rotor copper loss for a current fs, mp represents the rotor copper loss for a current fd.

Similarly pq represents the stator copper loss for stator current fd.

392. Interpretation of Diagram.—At any value of stator current $od = I_1$, Fig. 443.

$$\begin{aligned} nE_1\,dr &= \text{stator input in watts,}\\ nE_1\,qr &= \text{constant losses,}\\ nE_1\,pq &= \text{stator copper loss,}\\ nE_1\,mp &= \text{rotor copper loss,}\\ nE_1\,dm &= \text{rotor output in watts} = \text{mechanical load,}\end{aligned}$$

$$\frac{dr}{od} = \text{power factor,}$$

$$\frac{dm}{dr} = \text{efficiency,}$$

$$\frac{mp}{dp} = \frac{\text{rotor copper loss}}{\text{rotor input}} = \text{slip,}$$

$$\frac{dm}{dp} = \frac{\text{rotor output}}{\text{rotor input}} = \frac{\text{actual speed}}{\text{synchronous speed}} = \frac{S}{N} = 1 - s.$$

The torque corresponding to output $nE_1\,dm$ is

$$T = \frac{nE_1\,dm}{746} \times \frac{33{,}000}{2\pi\,(\text{r.p.m.})} \text{ lb. at 1 ft. radius.}$$

At synchronous speed this torque would represent an output

$$nE_1\,dm \times \frac{N}{S} = nE_1\,dm \times \frac{dp}{dm} = nE_1\,dp \text{ watts} = \text{rotor input.}$$

The torque in synchronous watts is equal to the watts input to the rotor $= nE_1\,dp$.

At standstill the torque in synchronous watts is $nE_1 st$ and represents the starting torque of the motor.

The maximum value of torque in synchronous watts is $nE_1 \times$ maximum value of dp.

The maximum output in watts is $nE_1 \times$ maximum value of dm. For average 25-cycle motors, starting torque is one and one-half to two and one-half times full-load torque; starting current is six to eight times full-load current; and maximum running torque is

two and one-half to three and one-half times full-load torque. For 60-cycle motors, starting torque is one to one and one-half times full-load torque; starting current is five to six times full-load current; and maximum running torque is two to two and one-half times full-load torque.

393. Construction of Diagram from Test for a Three-phase Motor.—1. Run the motor light at rated voltage and rated frequency. Read impressed voltage, current and watts input E_1, I_0 and W_0.

$$I_0 = of \text{ on the diagram.}$$

$$\frac{W_0}{\sqrt{3}E_1} = I_P = af = wv \text{ on the diagram.}$$

2. Lock the rotor and impress reduced voltage and raise it until twice full-load current flows in the stator. Read impressed voltage, current, watts input and torque, E_L, I_L, W_L and T_L. To get the value of locked current at rated voltage, raise the values of I_L in the ratio $E_1 : E_L$. To get the values of locked watts and locked torque at rated voltage, raise the values of W_L and T_L in the ratio $E_1^2 : E_L^2$. To get accurate results for the circle diagram it is better to reduce the values of watts and torque to terms of power current per phase. This is done by dividing the values of W_L and T_L by $\sqrt{3}E_L$.

Plot on a base of impressed voltage,

$$(1)\ I_L, \quad (2)\ \frac{W_L}{\sqrt{3}E_L}, \quad (3)\ \frac{T_L}{\sqrt{3}E_L}.$$

These three loci should be straight lines passing through the origin and can be produced until they cut the ordinate at the rated voltage of the motor.

The following results are obtained.

Value of I_L at rated voltage $= os$ on the diagram.

Value of $\dfrac{W_L}{\sqrt{3}E_L}$ at rated voltage $= sw$ on the diagram.

3. Measure the resistance of the stator per phase $= r_1$. Then the stator copper loss locked at rated voltage is $3\,\overline{os}^2\,r_1$ watts and $\dfrac{3\,\overline{os}^2\,r_1}{\sqrt{3}E_1} = vt$ on the diagram.

The rotor copper loss, locked at rated voltage, is $\sqrt{3}E_1 \times st$ and is known since $st = sw - wv - vt$.

The rotor copper loss also represents the starting torque in synchronous watts and therefore if T_1 is the value of T_L at rated voltage

$$\frac{T_1 \times 2\pi \times (\text{sync. speed r.p.m.})}{33{,}000} \text{ should equal } \frac{\sqrt{3}E_1 \times st}{746}.$$

The circle diagram can be drawn in from the values obtained above.

The motor has been assumed to be Y-connected and the voltage E_1 is the line voltage.

394. Test of an Induction Motor.—The following readings were obtained from tests on a 30-hp., 440-volt, 60-cycle, three-phase star-connected induction motor, having eight poles and a synchronous speed of 900 r.p.m.

Resistance of stator between terminals at normal operating temperature = 0.50 ohms.

Resistance of stator per phase = r_1 = 0.25 ohms.

RUNNING SATURATION OR NO-LOAD TEST

Impressed e.m.f. = E	Exciting current = I_0	Watts input = W_0
550	23.0	2,920
480	15.8	1,750
440	14.0	1,475
405	12.5	1,350
380	10.9	1,250
290	7.8	890

LOCKED TEST

Impressed e.m.f. = E_L	Stator current = I_L	Watts input = W_L	Starting torque = T_L lb.-ft.	$\frac{W_L}{\sqrt{3}\,E_L}$	$\frac{T_L}{\sqrt{3}E_L}$
307.5	121.0	32,100	145	60.3	0.271
276.5	108.5	25,400	120	53.2	0.251
257.5	100.0	21,000	99	47.3	0.244
233.5	88.0	16,500	84	40.9	0.207
201.5	74.0	11,550	60	33.2	0.170
164.0	59.0	7,350	35	25.9	0.124
440.0	170.0	61,600	297.5	81.0	0.390

These values are plotted in Fig. 444 against impressed e.m.f. and the values at 440 volts are obtained by producing the curves. The loci of I_L, $\frac{W_L}{\sqrt{3}E_L}$ and $\frac{T_L}{\sqrt{3}E_L}$ are straight lines passing through the origin.

With line voltage $E = 440$, the impressed voltage per phase $= E_1 = \frac{E}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.$

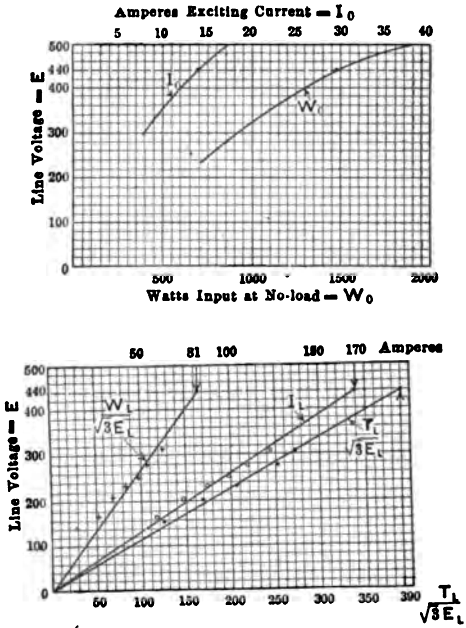


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FIG. 444.

At No Load.

Stator current or exciting current $= I_0 = 14.0$ amp. $= of$ (Fig. 445).

Watts input $= W_0 = 1{,}475 =$ iron loss + friction and windage losses and these are assumed to be constant.

$$\text{Power factor} = \cos\theta_0 = \frac{W_0}{\sqrt{3}EI_0} = \frac{1{,}475}{\sqrt{3} \times 440 \times 14} = 0.137$$

$= 13.7$ per cent.

In-phase current $= I_P = I_0 \cos\theta_0 = 1.92$ amp. $= fa = vw.$

Magnetizing current $= I_M = \sqrt{I_0^2 - I_P^2} = \sqrt{14^2 - 1.92^2} = 13.8$ amp. $= oa.$

Rotor Locked.

Stator current $= I_L = 170$ amp. $= os$.

Watts input $= W_L = 61{,}600 =$ constant losses + stator copper loss + rotor copper loss.

Power factor $= \cos\theta_L = \dfrac{W_L}{\sqrt{3}EI_L} = \dfrac{61{,}600}{\sqrt{3} \times 440 \times 170} = 0.476$

$= 47.6$ per cent.

In-phase component of stator current $= I_L \cos\theta_L = 81$ amp. $= sw$.

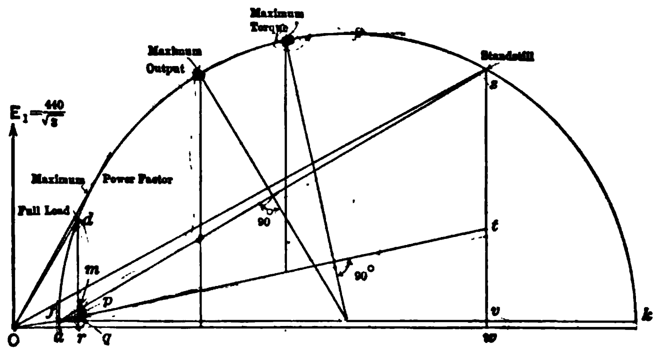


FIG. 445.

Quadrature component of stator current $= I_L \sin\theta_L = 149$ amp. $= ow$.

Stator copper loss $= 3I^2_L r_1 = 3 \times 170^2 \times 0.25 = 21{,}700$ watts, represented by the current $tv = \dfrac{21{,}700}{\sqrt{3} \times 440} = 28.4$ amp.

Rotor copper loss $= 61{,}600 - 1{,}475 - 21{,}700 = 38{,}425$, represented by the current $st = \dfrac{38{,}425}{\sqrt{3} \times 440} = 50.5$ amp.

The points o, f, a, s, t, v and w are therefore fixed and the circle may be drawn in, as shown in Fig. 445.

The starting torque at 440 volts $= 0.39 \times \sqrt{3} \times 440 = 297.5$ lb.-ft. and the corresponding output at synchronous speed $= \dfrac{2\pi NT}{33{,}000} = \dfrac{2 \times 3.14 \times 900 \times 297.5}{33{,}000} = 51$ synchronous hp. $= 51 \times 746 = 38{,}046$ synchronous watts $=$ rotor input $=$ rotor copper loss and should agree with the value 38,425 above. This serves as a check on the circle.

For full-load output of 30 hp. the vertical intercept between the circle and the line fs must be the required in-phase current $= \frac{30 \times 746}{\sqrt{3} \times 440} = 29.3$ amp. $= dm$. The full-load point d is thus fixed and the corresponding stator current $= I_1 = od = 40$ amp., power factor $= \cos\theta_1 = \frac{dr}{od} = 0.8625 = 86.25$ per cent., efficiency $\eta = \frac{dm}{dr} 100$ per cent. $= 84.3$ per cent., speed $= S = \frac{dm}{dp} \times$ sync. speed $= 0.9275 \times 900 = 835$ r.p.m., and the slip $= s = \frac{mp}{dp} = 0.0725 = 7.25$ per cent.

At one-half load, $I_1 = 23$ amp., $\cos\theta_1 = 75$ per cent., $\eta = 84.0$ per cent., $S = 870$ r.p.m., $s = 3.3$ per cent.; at one and one-half load, $I_1 = 65$ amp., $\cos\theta_1 = 85.5$ per cent., $\eta = 77.5$ per cent., $S = 779$ r.p.m., $s = 13.4$ per cent.

Maximum torque or pull-out torque is represented by the maximum intercept between the circle and the line $ft = 73.6$ amp. $= 73.6 \times \sqrt{3} \times 440 = 56{,}000$ sync. watts $= 440.0$ lb.-ft. The slip for maximum torque is 36.4 per cent.

Maximum output or stalling load is represented by the maximum intercept between the circle and the line $fs = 57.6$ amp. $= 57.6 \times \sqrt{3} \times 440 = 44{,}000$ watts $= 58.9$ hp. and the corresponding slip is 24.4 per cent.

395. Analysis by Rectangular Coördinates.—Using the method of rectangular coördinates called the symbolic method the performance characteristics of an induction motor can be determined if the constants of the motor are known.

Let

$Y = g - jb$ = stator exciting admittance per phase at rated voltage.

$Z_1 = r_1 + jx_1$ = stator impedance per phase.

$Z_2 = r_2 + jsx_2$ = rotor impedance per phase at slip s.

Assume that the ratio of turns is $n_1 : n_2 = 1 : 1$ and take as real axis of coördinates the e.m.f. generated in the stator by the flux of mutual inductance. The quantities used refer to one phase of the stator and the corresponding phase of the rotor.

E' = e.m.f. generated in the stator.

$E_2 = E'$ = e.m.f. generated in the rotor at standstill.

sE_2 = e.m.f. generated in the rotor at slip s.

These three e.m.fs. lie along the real axis.

The following equations show the relations between the various e.m.fs. and currents at slip s.

Rotor current:

$$\dot{I}_2 = \frac{sE_2}{r_2+jsx_2} = E_2\left(\frac{sr_2}{r_2^2+s^2x_2^2} - j\frac{s^2x_2}{r_2^2+s^2x_2^2}\right) = E'(a_1 - ja_2) \tag{411}$$

where

$$a_1 = \frac{sr_2}{r_2^2 + s^2x_2^2} \text{ and } a_2 = \frac{s^2x_2}{r_2^2 + s^2x_2^2}. \tag{412}$$

Stator load current:

$$\dot{I}' = \dot{I}_2 = E'(a_1 - ja_2). \tag{413}$$

Stator exciting current:

$$\dot{I}_0 = E'\dot{Y} = E'(g - jb.) \tag{414}$$

Total stator current:

$$\dot{I}_1 = \dot{I}' + \dot{I}_0 = E'\{(a_1 + g) - j(a_2 + b)\} = E'(b_1 - jb_2) \tag{415}$$

where $b_1 = a_1 + g$ and $b_2 = a_2 + b$ (416)

E.m.f. impressed on the stator:

$$\begin{aligned} \dot{E}_1 &= E' + \dot{I}_1\dot{Z}_1 = E' + E'(b_1 - jb_2)(r_1 + jx_1) \\ &= E'\{(1 + b_1r_1 + b_2x_1) + j(b_1x_1 - b_2r_1) = E'(c_1 + jc_2) \end{aligned} \tag{417}$$

where $c_1 = 1 + b_1r_1 + b_2x_1$ and $c_2 = b_1x_1 - b_2r_1$ (418)

The constant absolute value of the impressed e.m.f. is

$$E_1 = E'\sqrt{c_1^2 + c_2^2}. \tag{419}$$

Thus the e.m.f. generated in the stator is

$$E' = \frac{E_1}{\sqrt{c_1^2 + c_2^2}}. \tag{420}$$

Substituting this value for E' in the equations above the absolute values of the various quantities as slip s can be obtained.

Rotor current:

$$I_2 = I' = E'\sqrt{a_1^2 + a_2^2} = E_1\frac{\sqrt{a_1^2 + a_2^2}}{\sqrt{c_1^2 + c_2^2}} \tag{421}$$

Exciting current:

$$I_0 = E'\sqrt{g^2 + b^2} = E_1\frac{\sqrt{g^2 + b^2}}{\sqrt{c_1^2 + c_2^2}}. \tag{422}$$

Stator current:

$$I_1 = E'\sqrt{b_1^2 + b_2^2} = E_1\frac{\sqrt{b_1^2 + b_2^2}}{\sqrt{c_1^2 + c_2^2}}. \tag{423}$$

The torque in synchronous watts, was found in Art. 389 to be equal to the rotor input in watts, which is

$$P_r = nE_2I_2 \cos \theta_2 = nE' \times E'a_1 = nE'^2a_1 = nE_1^2 \frac{a_1}{c_1^2 + c_2^2} \quad (424)$$

where n is the number of phases.

The torque in pounds-feet is from Art. 389.

$$T = 7.05 \frac{Pr}{N} = \frac{7.05\, nE_1^2}{N} \frac{a_1}{c_1^2 + c_2^2}. \quad (425)$$

The rotor output is

$$P = (1 - s)\, Pr = nE_1^2 \frac{a_1 (1 - s)}{c_1^2 + c_2^2}. \quad (426)$$

The stator power factor is $\cos \theta_1$, where θ_1 is the angle of lag of the current I_1 behind the impressed e.m.f. E_1. $\theta_1 = \theta' + \theta''$, $\theta' = \cos^{-1} \frac{c_1}{\sqrt{c_1^2 + c_2^2}}$ = the angle by which E_1 leads E' and $\theta'' = \cos^{-1} \frac{b_1}{\sqrt{b_1^2 + b_2^2}}$ = the angle of lag of I_1 behind E'; therefore

$$\cos \theta_1 = \cos (\theta' + \theta'') = \cos \theta' \cos \theta'' - \sin \theta' \sin \theta''$$
$$= \frac{c_1b_1 - c_2b_2}{\sqrt{c_1^2+c_2^2}\ \sqrt{b_1^2+b_2^2}}. \quad (427)$$

The power input to the stator is

$$P_1 = nE_1I_1 \cos \theta_1 = nE_1 \times E_1 \frac{\sqrt{b_1^2 + b_2^2}}{\sqrt{c_1^2 + c_2^2}} \times \frac{c_1b_1 - c_2b_2}{\sqrt{c_1^2+c_2^2}\ \sqrt{b_1^2+b_2^2}}$$
$$= nE_1^2 \frac{c_1b_1 - c_2b_2}{c_1^2 + c_2^2}. \quad (428)$$

Using these equations the various quantities can be calculated in terms of the slip and the characteristic curves of the motor plotted.

To obtain expressions for the maximum torque and output equations (425) and (426) may be differentiated with respect to s but the work is very complicated. Much simpler expressions, which are quite accurate enough, may be obtained by neglecting the exciting current and deriving new equations for torque and output on this basis, as follows:

Stator load current:

$$I' = I_2 = \frac{sE'}{r_2 + jsx_2} = E' \left(\frac{sr_2}{r_2^2 + s^2x_2^2} - j \frac{s^2x_2}{r_2^2 + s^2x_2^2} \right) = E' (a_1 - ja_2).$$

Stator impressed e.m.f.:

$$\dot{E}_1 = \dot{I}'\dot{Z}_1 + \dot{E}' = \dot{I}'(r_1 + jx_1) + \dot{I}'\left(\frac{r_2 + jsx_2}{s}\right) = \dot{I}'\left\{\left(r_1 + \frac{r_2}{s}\right) + j\,(x_1 + x_2)\right\} \tag{429}$$

and

$$\dot{I}' = \frac{\dot{E}_1}{\left(r_1 + \frac{r_2}{s}\right) + j\,(x_1 + x_2)} = \dot{E}_1\left\{\frac{r_1 + \frac{r_2}{s}}{\left(r_1 + \frac{r_2}{s}\right)^2 + (x_1 + x_2)^2} - j\,\frac{x_1 + x_2}{\left(r_1 + \frac{r_2}{s}\right)^2 + (x_1 + x_2)^2}\right\} \tag{430}$$

and

$$I' = \frac{E_1}{\sqrt{\left(r_1 + \frac{r_2}{s}\right)^2 + (x_1 + x_2)^2}}. \tag{431}$$

Power input to the stator is

$$P_1 = nE_1 \times E_1 \frac{r_1 + \frac{r_2}{s}}{\left(r_1 + \frac{r_2}{s}\right)^2 + (x_1 + x_2)^2} = \frac{nE_1^2\left(r_1 + \frac{r_2}{s}\right)}{\left(r_1 + \frac{r_2}{s}\right)^2 + (x_1 + x_2)^2} \tag{432}$$

Power input to the rotor is

$$P_r = P_1 - \text{stator copper loss} = P_1 - nI'^2 r_1 = \frac{nE_1^2 \frac{r_2}{s}}{\left(r_1 + \frac{r_2}{s}\right)^2 + (x_1 + x_2)^2} \tag{433}$$

and this is equal to the torque in synchronous watts.

Torque in pounds-feet is

$$T = 7.05\frac{P_r}{N} = \frac{7.05}{N}\,\frac{nE_1^2 \frac{r_2}{s}}{\left(r_1 + \frac{r_2}{s}\right)^2 + (x_1 + x_2)^2} \tag{434}$$

Rotor output is

$$P = (1 - s)P_r = \frac{nE_1^2 \frac{r_2}{s}(1 - s)}{\left(r_1 + \frac{r_2}{s}\right)^2 + (x_1 + x_2)^2}. \tag{435}$$

To obtain an expression for the maximum torque, differentiate (434) with respect to s:

$$\frac{dT}{ds} = \frac{7.05}{N} nE_1^2 r_2 \frac{d}{ds}\left(\frac{1}{s\left\{\left(r_1 + \frac{r_2}{s}\right)^2 + (x_1 + x_2)^2\right\}}\right)$$

$$= \frac{7.05}{N} nE_1^2 r_2$$

$$\left[\frac{-\left\{\left(r_1 + \frac{r_2}{s}\right)^2 + (x_1 + x_2)^2\right\} + s\left\{2\left(r_1 + \frac{r_2}{s}\right)\left(-\frac{r_2}{s^2}\right)\right\}}{s^2\left\{\left(r_1 + \frac{r_2}{s}\right)^2 + (x_1 + x_2)^2\right\}^2}\right] = 0 \text{ for max.}$$

Therefore

$$\left(r_1 + \frac{r_2}{s}\right)^2 + (x_1 + x_2)^2 - \frac{2r_2}{s}\left(r_1 + \frac{r_2}{s}\right) = 0$$

and from this the slip at maximum torque is

$$s_{\text{max. torque}} = \frac{r_2}{\sqrt{r_1^2 + (x_1 + x_2)^2}}. \tag{436}$$

Substituting this value of s in (434) the maximum torque is found

$$T_{\text{max.}} = \frac{7.05}{N} \frac{nE_1^2\sqrt{r_1^2 + (x_1 + x_2)^2}}{(r_1 + \sqrt{r_1^2 + (x_1 + x_2)^2})^2 + (x_1 + x_2)^2}$$

$$= \frac{7.05}{N} \frac{nE_1^2}{2[r_1 + \sqrt{r_1^2 + (x_1 + x_2)^2}]}. \tag{437}$$

Stator current for maximum torque is found by substituting the value of slip in 436 in equation (431):

$$I_{\text{max. torque}} = \frac{E_1}{\sqrt{(r_1 + \sqrt{r_1^2 + (x_1 + x_2)^2})^2 + (x_1 + x_2)^2}} \tag{438}$$

and it is independent of the rotor resistance r_2.

Starting torque may be found by substituting $s = 1$ in (434):

$$T_{\text{starting}} = \frac{7.05}{N} \frac{nE_1^2 r_2}{(r_1 + r_2)^2 + (x_1 + x_2)^2} = \frac{7.05}{N} n(I_2^2 r_2) \tag{439}$$

The starting current is found by substituting $s = 1$ in (431):

$$I_{\text{starting}} = \frac{E_1}{\sqrt{(r_1 + r_2)^2 + (x_1 + x_2)^2}}. \tag{440}$$

The rotor resistance corresponding to maximum starting torque is found by substituting $s = 1$ in (436). It is:

$$r_{2\,(\text{for max. starting torque})} = \sqrt{r_1^2 + (x_1 + x_2)^2}. \tag{441}$$

The slip corresponding to maximum output may be found by differentiating P with respect to s and equating to zero; the operation is rather long and the result only will be given here:

$$s_{\text{max. output}} = \frac{r_2}{r_2 + \sqrt{(r_1 + r_2)^2 + (x_1 + x_2)^2}}. \tag{442}$$

Substituting this value of s in equation (435) the maximum output or stalling load is found:

$$P_{\text{max.}} = \frac{nE_1^2}{2\{(r_1 + r_2) + \sqrt{(r_1 + r_2)^2 + (x_1 + x_2)^2}\}}. \tag{443}$$

These equations are quite accurate except when the exciting current is very large. When calculating the stator power factor and the stator current up to full load the exciting current cannot be neglected.

From equations (437) and (443) it is seen that both the maximum torque and the maximum output are proportional to the square of the impressed e.m.f. and therefore any decrease in the line e.m.f. will seriously affect the motor characteristics.

396. Characteristics of an Induction Motor by the Symbolic Method.—Before applying this method the constants g, b, r_1, r_2, x_1 and x_2 must be determined. Taking as an example the motor of Art. 394, the constants may be found as follows:

Impressed e.m.f. per phase $= E_1 = \dfrac{440}{\sqrt{3}} = 254$ volts.

Stator resistance per phase $= r_1 = 0.25$ ohms.

Stator current per phase at no load $= I_0 = 14$ amp.

Watts input at no load $= W_0 = 1{,}475$.

Power factor at no load $= \cos\theta_0 = \dfrac{W_0}{\sqrt{3}EI_0} = \dfrac{1{,}475}{\sqrt{3} \times 440 \times 14}$ $= 0.137 = 13.7$ per cent.

Stator exciting admittance per phase $= Y = \dfrac{I_0}{E_1} = \dfrac{14}{254} = 55.2 \times 10^{-3}$.

Stator exciting conductance per phase $= g = Y\cos\theta_0 = 7.6 \times 10^{-3}$.

Stator exciting susceptance per phase $= b = \sqrt{Y^2 - g^2} = 54.0 \times 10^{-3}$.

Stator current per phase locked $= I_L = 170$ amp.

Watts input locked $= W_L = 61{,}600$.

Power factor locked $= \cos\theta_L = \dfrac{W_L}{\sqrt{3}EI_L} = \dfrac{61{,}600}{\sqrt{3} \times 440 \times 170} =$ $0.476 = 47.6$ per cent.

Equivalent impedance per phase locked $= Z_L = \dfrac{E_1}{I_L} = \dfrac{254}{170} =$ 1.49 ohms.

Equivalent resistance per phase locked $= R_L = Z_L \cos\theta_L =$ 0.71 ohms.

Equivalent reactance per phase locked $= X_L = \sqrt{Z_L^2 - R_L^2} =$ 1.31 ohms.

Rotor resistance per phase referred to the stator $= r_2 = R_L - r_1 = 0.71 - 0.25 = 0.46$ ohms.

Assuming that the rotor reactance per phase referred to the stator is equal to the stator reactance per phase:

$$x_1 = x_2 = \frac{X_L}{2} = \frac{1.31}{2} = 0.655 \text{ ohms.}$$

The characteristic curves may be obtained by substituting any values of slip in the equations in Art. 395 using the constants found above.

It will be sufficient to check the results of this method at one or two values of slip against the results obtained from the circle diagram.

When

$$s = 0.05:$$

$$a_1 = \frac{sr_2}{r_2^2 + s^2x_2^2} = 0.095, \qquad a_2 = \frac{s^2x_2}{r_2^2 + s^2x_2^2} = 0.00764,$$

$$b_1 = a_1 + g = 0.1026, \qquad b_2 = a_2 + b = 0.0626,$$

$$c_1 = 1 + b_1r_1 + b_2x_1 = 1.066, \qquad c_2 = b_1x_1 - b_2r_1 = 0.0514.$$

$$\text{Stator current} = I_1 = E_1\sqrt{\frac{b_1^2 + b_2^2}{c_1^2 + c_2^2}} = 28.6 \text{ amp.}$$

The check values from the circle diagram will be taken at this value of stator current.

$$\text{Power factor} = \cos\theta_1 = \frac{c_1b_1 - c_2b_2}{\sqrt{(c_1^2 + c_2^2)(b_1^2 + b_2^2)}} = 0.825 =$$

82.5 per cent. (circle gives 81.8 per cent.)

$$\text{Output} = P = nE_1^2\frac{(1 - s)a_1}{c_1^2 + c_2^2} = 15{,}300 \text{ watts} = 20.5 \text{ hp.}$$

(the circle diagram gives 15,225 watts = 20.4 hp.).

$$\text{Efficiency} = \eta = \frac{\text{Output}}{\text{Input}} = \frac{P}{\sqrt{3}EI_1 \cos \theta_1} = 0.842 = 84.2 \text{ per cent. (circle gives 85 per cent.)}$$

$$\text{Torque} = T = \frac{7.05\, nE_1^2}{N} \cdot \frac{a_1}{c_1^2 + c_2^2} = 126.5 \text{ lb.-ft. (circle gives 125.5).}$$

Slip at 28.6 amp. from the circle is 4.55 per cent.

When $s = 0.10$:

$a_1 = 0.212$, $a_2 = 0.0302$, $b_1 = 0.22$, $b_2 = 0.085$, $c_1 = 1.11$, $c_2 = 0.123$. $I_1 = 53.4$ amp., $\cos \theta_1 = 88.4$ per cent., $P =$ 29,250 watts, $\eta = 81.3$ per cent., $T = 255$ lb.-ft.

The values obtained from the circle diagram for a stator current $I_1 = 53.4$ are $\cos \theta_1 = 88.1$ per cent., $P = 29{,}000$ watts, $\eta = 80.9$ per cent., $T = 252$ lb.-ft., $s = 0.10$.

At standstill when $s = 1$:

$a_1 = 0.716$, $a_2 = 1.02$, $b_1 = 0.724$, $b_2 = 1.075$, $c_1 = 1.885$, $c_2 = -0.2185$, $I_1 = 168$ amp. and $T = 300$ lb.-ft.; the corresponding values from the circle diagram are $I_1 = 170$ and $T = 297.5$ lb.-ft.

Slip for maximum torque, equation (431), is

$$s_{\text{max. torque}} = \frac{r_2}{\sqrt{r_1^2 + (x_1 + x_2)^2}} = 0.345 = 34.5 \text{ per cent.}$$

Maximum torque, equation (437), is

$$T_{\text{max.}} = \frac{7.05}{N} \frac{nE_1^2}{2[r_1 + \sqrt{r_1^2 + (x_1 + x_2)^2}]} = 480 \text{ lb.-ft.}$$

From the circle diagram, $T_{\text{max.}} = 440$ lb.-ft. and the corresponding slip $= 36.4$ per cent.

Slip for maximum output, equation (442), is

$$s_{\text{max. output}} = \frac{r_2}{r_2 + \sqrt{(r_1 + r_2)^2 + (x_1 + x_2)^2}} = 23.6 \text{ per cent.}$$

Maximum output, equation (443), is

$$P_{\text{max.}} = \frac{nE_1^2}{2[(r_1 + r_2) + \sqrt{(r_1 + r_2)^2 + (x_1 + x_2)^2}]} = 44{,}000 \text{ watts} = 58.9 \text{ hp.}$$

From the circle diagram, $P_{\text{max.}} = 44{,}000$ watts $= 58.9$ hp. and the corresponding slip is 24.4 per cent.

From the calculations above it is seen that the two methods check very satisfactorily.

397. Methods of Starting.—Except in the case of small machines, induction motors should not be started by connecting them directly to the mains, since the large starting current at low power factor disturbs the voltage regulation of the system.

Two methods of reducing the starting current are in use. (1) The voltage impressed on the stator is reduced by using an induction starter which is simply an auto-transformer with one or more taps (Fig. 374). (2) Resistance is inserted in series with the rotor windings.

1. When the impressed voltage is reduced, the starting current is reduced in proportion to it, but the starting torque is reduced as the square of the voltage.

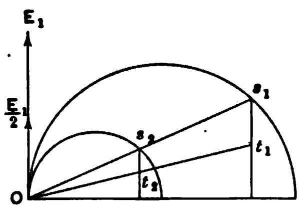


FIG. 446.—Starting on reduced voltage.

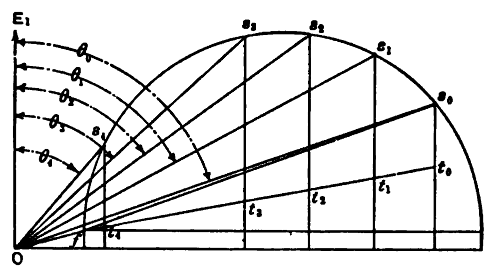


FIG. 447.—Starting torque with various rotor resistances.

In Fig. 446 $E_1.s_1t_1$ represents the starting torque of the motor at full voltage and os_1 represents the starting current, neglecting the exciting current, and $\frac{E_1}{2} \cdot s_2t_2$ represents the starting torque at half voltage and os_2 represents the starting current.

Since $os_2 = \frac{os_1}{2}$ the starting current is reduced to one-half its value at full voltage, but the starting torque is reduced to one-quarter. The power factor is not changed.

Thus starting with reduced voltage gives very small starting torque and low power factor.

A squirrel-cage rotor may be used.

2. When resistance is inserted in the rotor windings the starting current is reduced and is brought more nearly in phase and the starting torque is increased.

In Fig. 447 s_0t_0 represents the starting torque when the rotor circuits are closed without any starting resistance.

s_1t_1 is the starting torque when resistance R_1 is inserted.

s_2t_2 is the starting torque when resistance $R_2 > R_1$ is inserted.

s_3t_3 is the maximum possible starting torque and is obtained by inserting a resistance $R_3 > R_2$; it is the same as the maximum running torque of the motor. $R_3 + r_2 = \sqrt{r_1^2 + (x_1 + x_2)^2}$, equation (441). os_0, os_1, os_2 and os_3 are the corresponding stator currents and $\cos\theta_0$, $\cos\theta_1$, etc., are the power factors at start.

The curves in Fig. 448 are the "speed-torque" characteristics for the motor operating with the various resistances in the rotor. The maximum torque is the same in all cases but it is reached at different speeds. One current curve holds in all cases.

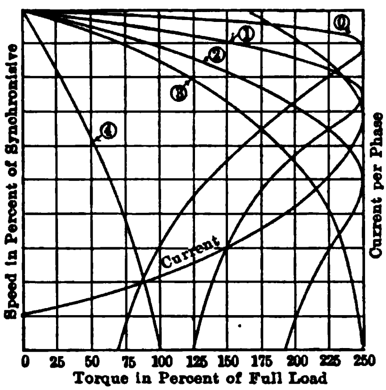


FIG. 448.—Speed-torque characteristics of an induction motor with various rotor resistances.

If a resistance $R_4 > R_3$ is inserted in the rotor windings the starting current is further reduced and the power factor is improved but the starting torque is decreased. R_4 may be made of such value that the starting torque s_4t_4 is equal to full-load torque and the starting current os_4 is equal to full-load current. Curve (4) is the speed-torque characteristic for this case.

Thus by inserting resistance in the rotor any starting torque up to the maximum running torque or "pull-out" torque may be obtained. The starting current is reduced and the power factor is improved.

In starting a heavy load resistance R_3 is used and the motor gives its maximum torque at start. The resistance is then cut out gradually as the speed increases and the motor operates with short-circuited rotor with characteristics as shown in curve (0).

If the load to be started is not very great and a large starting current at low power factor is objectionable, resistance R_4 is used and the motor starts with full-load torque and draws full-load current.

This second method of starting requires a wound rotor with slip rings and large starting resistances which is much more expensive than a squirrel-cage rotor.

For the same line current, resistance starting gives about four times the torque given at reduced voltage.

398. Applications.—The constant-speed or squirrel-cage induction motor takes the place of the direct-current shunt motor and has very similar characteristics. It is of much more simple and rugged construction than the shunt motor and the wear and danger due to sparking are entirely eliminated.

It should be used where fairly constant power is required for long periods, where good speed regulation is required, where starting is infrequent and only average starting torque is necessary, where the motor is exposed to dust or to inflammable materials or is not easily inspected. It is suitable for driving line shafting, for high- and low-speed centrifugal pumps, blowers, fans, etc. It must be started on reduced voltage except for the smallest sizes.

The variable-speed induction motor has a wound rotor with its terminals connected to slip rings so that resistance may be introduced to vary the speed or to give a large starting torque.

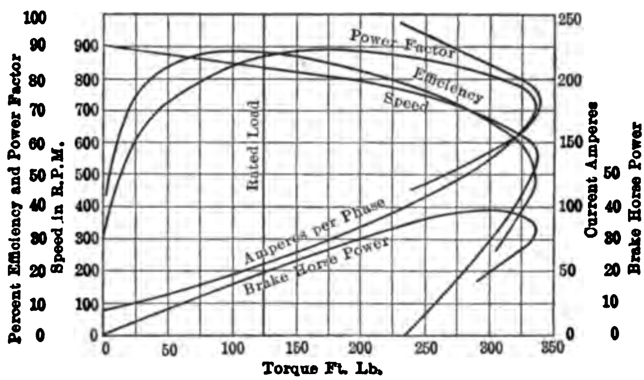


FIG. 449.—Characteristic curves of a three-phase, 60-cycle, 220-volt, 20 horsepower induction motor.

It should be used where frequent starts under load are necessary, or where the motor is large enough to have a bad effect on the regulation of the system due to the large starting current at low power factor, as for cranes, elevators, hoists, etc.

The squirrel-cage motor with a comparatively high-resistance rotor can be used where fairly large starting torque is required and where a wound-rotor motor is not advisable, as in cement mills, etc.

Fig. 449 shows the characteristic curves of a three-phase, 60-cycle, 220-volt, 20-hp. induction motor with a squirrel-cage rotor.

399. Speed Control of Induction Motors.—The induction motor is inherently a constant-speed motor like the direct-current shunt motor but its speed may be varied in four principal ways: (1) By variation of the impressed voltage, (2) by inserting resistance in the rotor windings, (3) by changing the number of poles and (4) by cascade control or concatenation.

400. Voltage Control.—Since the torque for a given slip is proportional to the square of the impressed voltage, the speed may be varied through a small range by variation of the voltage. This may be accomplished (*a*) by introducing resistance in series with the stator windings or (*b*) by using a variable ratio transformer or compensator.

Control by series resistance is very simple but is inefficient while control by a compensator is more expensive but gives higher efficiency. Both methods to be effective require a motor having a high-resistance rotor with its resulting large slip. A squirrel-cage rotor may be used.

401. Rotor Resistance Control.—The speed of an induction motor may be varied by using a wound rotor with slip rings and connecting external resistances into the circuit. With suitable resistances any required speed-torque curve may be obtained (Art. 397). This method of control corresponds to the use of resistance in series with the armature of a direct-current shunt motor. It is very inefficient and the speed changes with load.

402. Pole-changing.—By the use of special windings the number of poles on an induction motor may be changed and therefore also its synchronous speed. Two or at most three synchronous speeds may be obtained by this means. Take for example an eight-pole motor with a double-layer winding of two-thirds coil pitch; its synchronous speed may be doubled by reconnecting for four poles with a coil pitch of one and one-third. A squirrel-cage rotor is generally used but if intermediate speeds are required a wound rotor must be used and its poles changed at the same time as those on the stator. Resistance may then be inserted in the rotor circuits. Such a motor is very expensive.

403. Cascade Control or Concatenation.—When operating in cascade two similar induction motors with wound rotors are rigidly connected to the same shaft. The stator of the first motor is connected to the line; the stator of the second motor is connected to the rotor winding of the first motor and receives

power from it; the rotor of the second motor is closed through starting resistances (Fig. 450).

The frequency of the e.m.fs. generated in the rotor of an induction motor is sf, where f is the frequency of the supply and s is the slip. Thus the frequency impressed on the stator of the second motor is the frequency of slip of the first motor. The speed of the two motors is always the same and thus at no load $(1 - s)f = sf$ and $s = 0.5$.

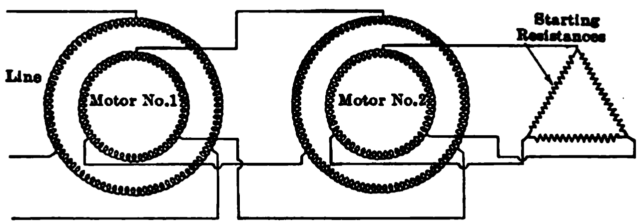


FIG. 450.—Cascade control of induction motors.

Therefore, two similar motors connected in cascade tend to approach a speed of half synchronous speed at no load and fall below this speed under load. Speeds below half synchronous speed are obtained by inserting resistance in the rotor windings of the second motor. For speeds above half synchronous speed the stator of the second motor must be connected to the line and the rotor of the first motor closed through resistances.

This method of control is used for some three-phase traction systems and is very similar to the series parallel control of direct-current series motors. The induction motor does not, however, tend to increase its speed indefinitely. If it operates above synchronous speed, it becomes a generator and feeds back power to the line acting as a brake.

If the two motors have different numbers of poles, the no-load speed when connected in cascade will not be half synchronous speed but the speed of the set will be that of a motor having as many poles as the two motors combined. For example, if on a 60-cycle system a 12-pole motor is connected in cascade with a 4-pole motor connected to run in the same direction the speed will be that of a motor with $12 + 4 = 16$ poles, that is, $\frac{120 \times 60}{16}$ $= 450$ r.p.m. When the two motors tend to run in the same direction they are in direct concatenation. If the 4-pole motor is connected to run in the opposite direction from the 12-pole

motor, the resulting speed will be that corresponding to 12 − 4 = 8 poles. This is called differential concatenation.

With this combination of motors, four speeds may be obtained, 450 r.p.m. with direct concatenation, $\frac{120 \times 60}{12} = 600$ r.p.m. using only the 12-pole motor, $\frac{120 \times 60}{12-4} = 900$ r.p.m. with differential concatenation and finally $\frac{120 \times 60}{4} = 1{,}800$ r.p-m., using only the 4-pole motor. Intermediate speeds can be obtained inserting resistances in the rotors.

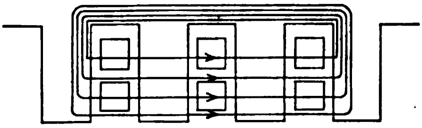
When connected in cascade the impedance of the second motor is added to the rotor of the first and its exciting current is supplied through the first motor resulting in very low power factor. With two similar motors in cascade each will operate at about half voltage and since the output varies as the square of the voltage the combined output of the set will be about half that of each motor separately. The losses will be those of two motors and therefore the efficiency will be very low. The maximum or breakdown output will be reduced in the same proportion.

404. Exciting Current.—The exciting current or no-load current of an induction motor is larger than that of a transformer and is of the order of 25 per cent. of full-load current. The power component supplies the iron losses of the stator and the friction and windage losses of the rotor. The quadrature component or magnetizing current is large because the magnetic circuit contains an air gap and the iron on both sides is cut away to form the slots for the rotor and stator windings. The large magnetizing current lowers the power factor of the motor particularly at light loads and is therefore objectionable. To reduce it the air gap is made as short as possible, being determined largely by the necessity for mechanical clearance. The slots may be partly closed to increase the effective gap area; the rotor slots are often completely closed. Closing the slots, however, adds to the reactance of the motor.

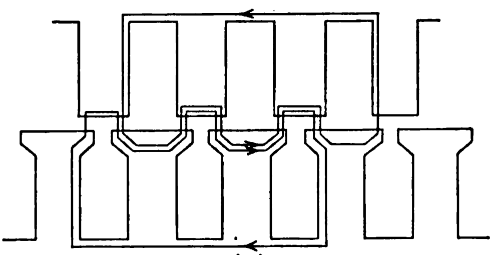
Low-speed motors have larger magnetizing currents than high-speed motors and therefore their power factors tend to be lower.

Unless the teeth of the stator and rotor are properly designed the reluctance of the magnetic circuit of the motor will vary and the flux will vary. If the stator and rotor have the same number of teeth, then in position (*b*), Fig. 451, the reluctance will be a

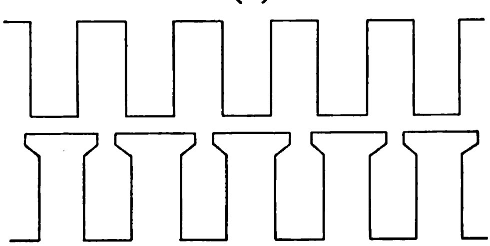
maximum and the flux a minimum while in (*c*) the flux will be maximum. The rotor will tend to lock in the position of maximum flux (*a*) and the torque will be reduced. Such pulsations of flux may be prevented by designing the rotor and stator with different numbers of teeth and making the width of the rotor teeth a multiple of the stator slot pitch.



(*a*) Slot Leakage Flux



(*b*)



(*c*) Zig-Zag Leakage Flux

FIG. 451.

405. Leakage Reactances.—The diameter of the circle, in the diagram for the induction motor, is inversely proportional to the sum of the reactances, and it is therefore necessary for the designer to determine these quantities very accurately and they should ordinarily be kept as low as possible.

The leakage fields are: (1) that about the end connections, (2) the slot leakage, (3) the zig-zag or tooth-tip leakage and (4) the belt leakage.

The end-connection flux may be reduced by using fractional-pitch windings but this results in increased magnetizing current.

The slot leakage flux, Fig. 451, is directly proportional to the slot depth and is inversely proportional to the slot width. Deep narrow slots, therefore, give large slot leakage. When the slots are partly closed, to decrease the magnetizing current, the slot leakage is very much increased. The use of magnetic wedges in the slots gives the same result.

The zig-zag leakage flux, Fig. 451(*b*), corresponds to the tooth-tip leakage flux in alternators and is usually quite large since the gap is short. It also depends on the relative position of the rotor and stator teeth. In position (*b*), Fig. 451, the leakage flux is maximum and in (*c*) it is minimum. Such a large pulsation of the flux would have very serious results. The rotor would tend to lock in the position of maximum flux especially when being started at reduced voltage with large current. The rapid pulsation of the flux might also produce an objectionable noise.

If the numbers of teeth on the stator and rotor have no common divisor, only one tooth will be in the position of maximum flux at any time and the tendency to lock will be negligible. If less than 20 per cent. of the teeth are in the position of maximum flux at any time the tendency to lock will not be serious. This condition can easily be fulfilled in a squirrel-cage rotor but with wound rotors the number of slots on the rotor and stator must be divisible by the number of poles and phases and conditions are not so good. However, wound-rotor machines are usually started with resistance in the rotor circuits and the starting current and the zig-zag leakage are therefore small.

The belt-leakage flux is due to the fact that the stator phases are not fixed in position relative to the rotor phases as in the transformer and increased leakage results when they are not directly opposite.

When the sum of the reactances of a motor is small, the diameter of the circle is large and the pull-out torque is large but the starting current is also high.

406. Stator and Rotor Resistances.—The stator resistance r_1 causes a loss of power and should be kept as small as possible. The rotor resistance r_2 likewise causes a loss of power but it may be necessary to make it comparatively large in order to get suitable characteristics.

The slip is proportional to the rotor copper loss and therefore

for low slip the rotor resistance must be low. However, by increasing the rotor resistance the starting torque is increased and the starting current is reduced and where large starting torque is required a comparatively high rotor resistance is necessary.

With wound rotors external resistance may be introduced at start and then cut out, but with the squirrel-cage rotor the resistance cannot be varied.

If the starting torque of a squirrel-cage rotor is found to be too small, it may be increased by slotting the rotor end rings and so increasing the resistance.

If the bars of a squirrel-cage rotor are made very deep, good starting torque can be obtained without reducing the efficiency. At start the frequency of the rotor current is high and the slot leakage flux is large, the lower parts of the bars have a high reactance and carry very little current. Due to the unequal current distribution the effective rotor resistance is increased and the I^2r loss is increased directly increasing the starting torque. Near synchronous speed the rotor current and frequency both become low and the current is uniformly distributed and the effective rotor resistance approaches its true value. The efficiency is therefore not reduced but due to the high slot leakage the reactance is increased and the maximum output is decreased.

407. Effect of Change of Voltage and Frequency.—If the voltage impressed on an induction motor is decreased 10 per cent. while the frequency is maintained constant, the flux is decreased 10 per cent. and the exciting current is decreased more than 10 per cent.; the iron loss is decreased nearly 20 per cent.; the diameter of the circle is decreased 10 per cent. For a given output the current is increased 10 per cent. and the copper loss is increased 20 per cent.; the slip is increased about 20 per cent.; the speed is reduced. The efficiency is decreased slightly. The power factor at light load is increased due to the decreased exciting current but above full load it is decreased due to the increased current and decreased circle diameter. The maximum output and maximum torque are decreased 20 per cent.

If the frequency is increased 10 per cent. while the voltage is kept constant, the flux is decreased 10 per cent.; exciting current is decreased more than 10 per cent.; the iron loss remains about as before; the diameter of the circle is decreased 10 per cent. For the same output the current is as before; the copper loss and slip are as before but the speed is higher by 10 per cent. The effi-

ciency is as before. The power factor is improved at light loads. Maximum output and torque are decreased less than 10 per cent.

A 10 per cent. increase in voltage would be pretty well offset by a 10 per cent. decrease in frequency but the increased losses and lower speed would cause the motor to heat up.

408. Single-phase Induction Motor.[1]—The stator of a single-phase induction motor has a single winding with any number of pairs of poles.

The rotor is either of the squirrel-cage type or is wound with the same number of poles as the stator but with any number of phases.

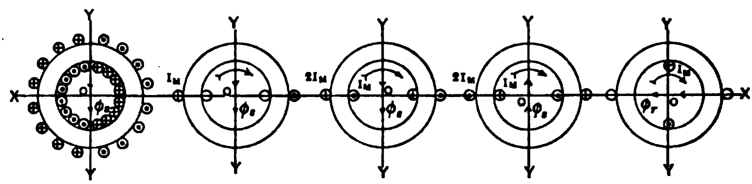


FIG. 452. FIG. 453. FIG. 454. FIG. 455. FIG. 456.

FIGS. 452 TO 456.—Single-phase induction motor.

Fig. 452 shows the relative directions of the currents in the two windings at standstill. The stator carries a current I_1 which consists of two components, I' the load component and I_M the magnetizing current. The rotor carries a current I_2 opposite in phase to I' and equal to it in m.m.f. If the ratio of turns is assumed to be $n_1 : n_2 = 1 : 1$; then $I_2 = I'$. The motor at standstill is a transformer with a short-circuited secondary.

The flux which crosses the air gap and links with both stator and rotor is produced by the stator exciting current. It is always directed along the line YOY. There is no component of flux in the horizontal direction XOX and therefore no torque is exerted tending to turn the rotor in either direction. Thus the rotating field which is produced in the polyphase induction motor does not exist in the single-phase motor at standstill. The single-phase induction motor, therefore, has no starting torque. If, however, it is started in either direction it will develop torque and will accelerate and come up approximately to synchronous speed at no load.

Fig. 453 represents the motor with the rotor open-circuited and, therefore, without current in its windings. The stator carries only the magnetizing current.

Fig. 454 represents conditions at synchronous speed at the

instant when the stator magnetizing current is maximum. The stator flux is then maximum downwards.

The rotor conductors moving at synchronous speed cut the stator flux and an e.m.f. is generated in them proportional to the product of flux and speed. Since the flux is alternating the e.m.f. generated is of double frequency and produces a current of double frequency in the closed-rotor winding. The current produces a flux the rate of change of which through the rotor windings generates in them an e.m.f. equal and opposite to the e.m.f. generated by rotation. This flux must, therefore, be of the same value as the stator flux and it is in phase with the rotor current.

The rotor current goes through two complete cycles during one revolution. In Fig. 454 it is maximum and is opposed to the stator current, but the e.m.f. impressed on the stator is constant and the stator flux is constant, and, therefore, a current must flow in the stator to balance the m.m.f. of the rotor current I'_M. Since the ratio of turns has been taken as 1 : 1 the increase in stator current is I'_M and the total stator current at synchronous speed is $I_M + I'_M$. In the position shown the rotor flux is not produced because the rotor m.m.f. is opposed by an equal and opposite m.m.f. on the stator.

Fig. 455 represents conditions after the rotor has turned through one-half a revolution and the stator current has passed through one-half cycle. The rotor current is in the same direction as before and has completed one cycle.

Fig. 456 represents conditions midway between Fig. 454 and Fig. 455. The stator current is zero and the rotor current is maximum and exerts a m.m.f. in the horizontal direction. There is no stator m.m.f. opposing it and a flux is produced of the same value as the stator flux in Fig. 454 or Fig. 455. Since the reluctance of the path for the horizontal flux is the same as that for the vertical stator flux, the rotor magnetizing current I'_M must be equal to the stator magnetizing current at standstill I_M, and, therefore, at synchronous speed the stator magnetizing current is $2I_M$ and is double its value at standstill.

Thus at synchronous speed there is a resultant m.m.f. of constant value revolving at synchronous speed and the magnetic field of the single-phase motor is identical with that of the polyphase motor, Fig. 457. The m.m.f. to produce the vertical field is supplied by the true stator magnetizing current, while the m.m.f. to produce the horizontal field is provided by an equal stator

magnetizing current, in phase with the true stator magnetizing current, which induces in the rotor the rotor magnetizing current.

When the rotor runs at synchronous speed its conductors do not cut this revolving flux and the only current in the rotor is the double-frequency magnetizing current.

When the rotor runs at a slip s below synchronous speed the rotor conductors cut the flux and currents are produced in them and torque is developed just as in the case of the polyphase motor.

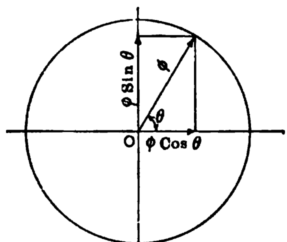


FIG. 457.—Revolving field of a single-phase induction motor at synchronous speed.

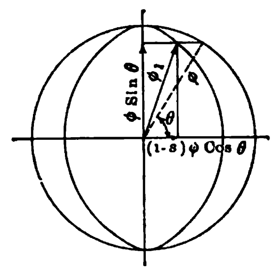


FIG. 458.—Revolving field of a single-phase induction motor at slip s.

409. Horizontal Field at Slip s.—When the rotor runs at a speed $S = (1 - s) \times$ synchronous speed, the e.m.f. generated in it due to cutting the stator flux is less than at synchronous speed in the ratio $1 - s : 1$ and the horizontal flux and the rotor magnetizing current are less in the same ratio.

The stator current is $I_M + (1 - s) I_M + I'$ and the rotor current is $(1 - s) I_M + I_2$. The frequency of the rotor magnetizing current is $(2 - s)f$ and the frequency of the rotor load current is sf, where f is the frequency of the e.m.f. impressed on the stator.

The revolving field at slip s is not constant in value but has the horizontal axis shorter than the vertical in the ratio $1 - s : 1$, Fig. 458. The field follows an elliptical instead of a circular locus.

The torque which is proportional to the product of the rotor load current and the horizontal field is less than that produced in the polyphase motor in the ratio $1 - s : 1$.

410. Starting Single-phase Induction Motors.—In order to obtain the torque required to start a single-phase induction motor a component of flux in quadrature in time and in space with the stator flux must be produced at standstill. It has been shown that when once the motor is started the rotor produces the required quadrature flux and thus the torque to carry the load.

Two principal methods are employed to produce the quadrature flux at standstill: (1) phase splitting and (2) shading coils.

1. If the two stator windings of a two-phase induction motor are connected to a single-phase supply, phase 1 directly and phase 2 through a suitable resistance or condensive reactance, the flux produced by phase 2 will have a component in quadrature in time with phase 1 and will thus give the required starting torque, Fig. 459. When the motor has come up to half speed, the starting winding is cut out and the motor runs as a single-phase motor on phase 1. This method of starting is called phase splitting. The second winding need not have as many turns as the first but it should be placed at 90 electrical degrees to it.

Fig. 459(*b*) shows a three-phase motor connected to a single-phase circuit. Two phases are used for normal operation and the third phase is used for starting only.

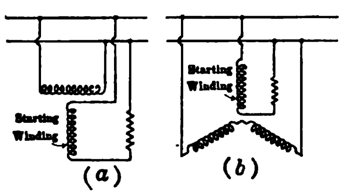


FIG. 459.

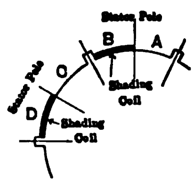


FIG. 460.

2. The shading coils, Fig. 460, are short-circuited coils surrounding part of each pole of the stator. Currents are induced in them and oppose the increase and decrease of the flux in the parts of the poles which they inclose. Thus the north pole in section *A* will reach its maximum value before it is maximum in section *B*. When the north pole has decreased to zero in *B* it will be increasing in section *C* and thus there is a rotation of the magnetic field and torque is produced. When the motor is started the short-circuited coils may be opened and they will then be idle and will not cause any power loss.

411. Comparison of Single-phase and Polyphase Motors.—Take the case of a two-phase motor operating on a single-phase circuit using only one phase of the stator winding.

The slip single-phase is less than two-phase since the whole rotor corresponds to one phase of the stator and thus the rotor current and rotor copper loss are decreased.

The efficiency is lower because the output decreases more than

the losses. For a given impressed e.m.f. and frequency the iron and friction losses remain practically constant.

The power factor is lower because the magnetizing current is approximately doubled.

A given motor wound single-phase can be operated at higher densities than when wound polyphase since the losses are less and its ventilation is the same, and in this way its output may be made from 65 to 75 per cent. of its output polyphase.

The torque at any speed can be increased by introducing resistance into the rotor windings, but this changes the maximum torque since the torque is proportional to $1 - s$.

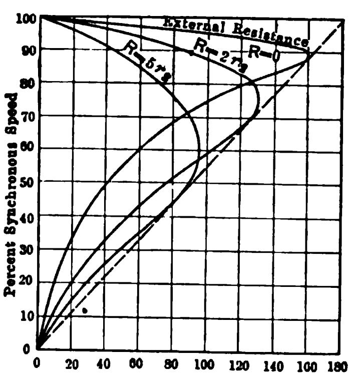


FIG. 461.—Speed-torque curves of a single-phase induction motor.

Fig. 461 shows typical speed-torque curves of a single-phase induction motor with various external resistances connected in the rotor windings.

A single-phase induction motor is usually either a two-phase or three-phase motor operated on a single-phase circuit using only part of the stator winding.

If one phase of a two-phase motor is opened at light load, the magnetizing current of the other phase is doubled and the motor runs as a single-phase motor. If two phases of a three-phase motor are opened the motor runs as a single-phase motor with the magnetizing current in the third phase trebled. In both cases the flux distribution and flux densities remain approximately the same as before.

412. Induction Generator.—If the stator of an induction motor is connected to the supply lines and its rotor is driven above synchronous speed, the machine will develop electrical power and supply it to the system.

The stator flux is not affected by the increase in the speed of the rotor, but revolves in the same direction as when the machine operates as a motor. The slip is, however, reversed and the e.m.fs. and currents induced in the rotor are reversed. Thus the direction of torque and power is reversed and the mechanical power supplied to drive the rotor is transformed into electrical power and supplied over the lines to the load.

The power transferred from the rotor to the stator depends on the slip just as in the induction motor. Using the same notation as in Art. 386, the power transferred to the stator is

$$
\begin{aligned}
P &= nE_2I_2 \cos \theta_2 \\
&= nE_2 \frac{sE_2}{\sqrt{r_2^2 + s^2x_2^2}} \cdot \frac{r_2}{\sqrt{r_2^2 + s^2x_2^2}} \\
&= \frac{snE_2^2r_2}{r_2^2 + s^2x_2^2}.
\end{aligned}
$$

Thus to increase the power delivered by the generator its speed must be increased. If therefore an induction generator is connected to a prime mover of variable speed, it will supply power almost in proportion to the increase of its speed above synchronous speed.

The characteristics of an induction generator may be determined from a circle diagram (Fig. 462). The diagram is constructed as for a motor but since the slip is negative the complete circle is required. Points below the line *oa* represent negative slip and therefore power output as a generator.

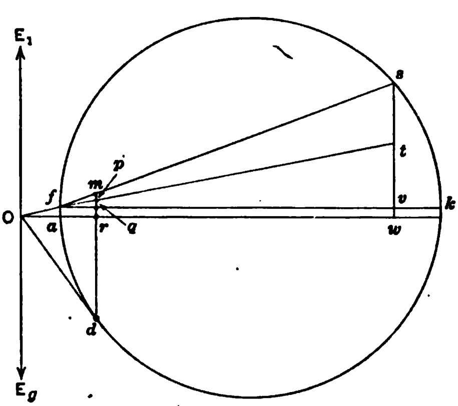


FIG. 462.—Circle diagram for an induction generator.

Assuming the generator to be three-phase star-connected, the voltage per phase is E_G and the voltage between terminals is $E = \sqrt{3}E_G$.

For the point d:

Input $= \sqrt{3}E \times dm$ watts = mechanical power supplied.

Rotor copper loss $= \sqrt{3}E \times pm$ watts.

$$
\begin{aligned}
&\text{Output to stator} &&= \sqrt{3}E \times dp \text{ watts.}\\
&\text{Stator copper loss} &&= \sqrt{3}E \times qp \text{ watts.}\\
&\text{Stator core loss} &&= \sqrt{3}E \times rq \text{ watts.}\\
&\text{Output} &&= \sqrt{3}E \times dr \text{ watts.}\\
&\text{Efficiency} &&= \frac{dr}{dm}\, 100 \text{ per cent.}
\end{aligned}
$$

The magnetizing current *oa* at no load and *or* under load must be supplied by a synchronous machine operating in parallel with the induction generator (Fig. 463). The voltage and the frequency of the system are also fixed by this synchronous generator and are not affected by the induction generator.

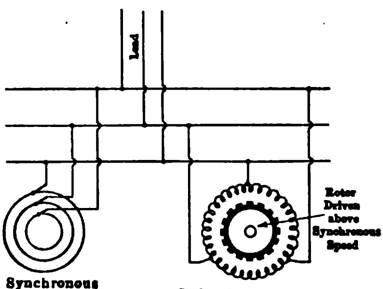


FIG. 463.—Induction generator.

The induction generator has two very serious disadvantages; it is not self-exciting, and it cannot supply reactive currents to an inductive load. It must, therefore, be operated in parallel with an alternator of sufficient capacity to supply both the magnetizing current for the induction generator and the reactive currents required by the load. The exciting kilovolt-amperes at no load may be as low as 12 per cent. of the full-load rating but under load it will never be less than 25 per cent. If, however, the output of the induction generator is small in comparison with the capacity of the synchronous machines on the system this will not have serious effects.

The induction generator has some good points: (*a*) it does not require to be synchronized; (*b*) there is no danger of trouble due to hunting, since it does not operate at constant speed; (*c*) in case of short-circuits it loses its excitation and does not tend to supply power to the fault.

In construction an induction generator is similar to an induction motor but the exciting current must be kept as low as possible by making the air gap short. Since no starting torque is required, the rotor may be made of very low resistance and is usually of the squirrel-cage type.

413. Asynchronous Phase Modifier.—The asynchronous phase modifier or phase advancer is a machine for use with induction motors to improve their power factor; it fills the place of the ex-

citer for a synchronous motor. A number of different types have been designed but only the simplest one will be discussed here. It consists of a direct-current drum armature with a commutator and three sets of brushes per pair of poles displaced at 120 degrees. The stator is merely a frame with laminations but without slots or windings except in the larger sizes where a compensating winding may be required to take care of commutation. For small sizes the stator may be omitted if the armature windings are placed in totally closed slots. The iron above the slots serves to complete the magnetic circuit.

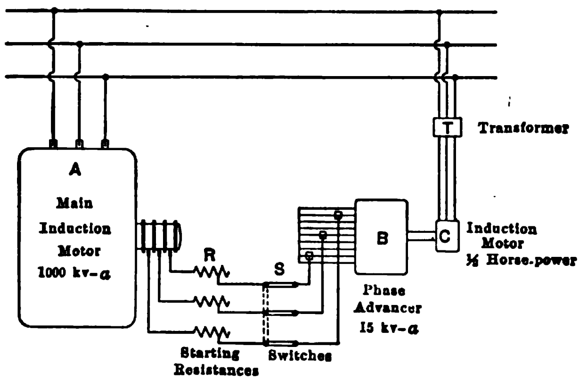


FIG. 464.—Asynchronous-phase modifier.

Referring to Fig. 464, A is the main motor, which must have a wound rotor and starting resistances R. The switch S is used to disconnect the phase advancer and close the rotor circuits while starting up. When full speed is reached, the switch S is opened, connecting the terminals of the rotor to the three brushes on the phase advancer B. If the armature is at rest it acts as a three-phase reactance and produces a revolving field of the frequency of slip. When the armature is driven in the direction of the revolving field and at the same speed the reactance becomes zero. If the speed is then increased, the reactance is reversed and acts as a capacity reactance causing the rotor current I_2 to lead the rotor induced e.m.f. E_{2g}, and so improves the stator power factor.

The change in phase is illustrated in Fig. 465. (a) is the diagram without the phase advancer and (b) represents the case where the phase advancer is driven at such a speed that the stator current I_1 is in phase with the impressed e.m.f. E_1. The

rotor current I_2 has then two components, I'_2 the component exerting a m.m.f. equal and opposite to that of the stator current I_1 and I'_M, the rotor magnetizing current filling the place of the ordinary stator magnetizing current. The rotor reactance drop is shown to be reversed and is approximately $sI_2(x_B - x_2)$ where x_B is the condensive reactance of the phase advancer winding at the given speed. By driving the phase advancer at a still higher speed the primary power factor may be made leading.

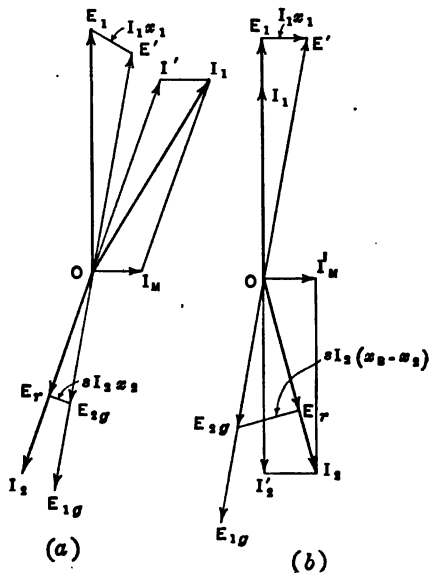


FIG. 465.

The capacity of the phase advancer is very small, being equal to the required change in the stator reactive volt amperes multiplied by the slip. This may be understood from the following: A stator e.m.f. E_{1g} corresponds to a rotor e.m.f. $E_{2g} = sE_2 = s\,\frac{n_2}{n_1}E_{1g}$. A stator current I' corresponds to a rotor current $I_2 = \frac{n_1}{n_2}I'$. And, therefore, the stator volt-amperes $E_{1g}I'$ correspond to the rotor volt-amperes $E_{2g}I_2 = sE_{1g}I'$.

The phase advancer is driven by a small high-speed, squirrel-cage induction motor C connected to the supply lines directly or through a transformer. It supplies the friction windage and iron losses of the phase advancer but the copper loss is supplied from the rotor of the main motor.

The capacity of the main induction motor is very largely increased by the elimination of the reactive currents from the stator. The supply lines are also relieved of a large component of current.

To raise the power factor of a 1,000-kva. induction motor from 86.6 per cent. to 100 per cent., the reactive kilovolt-amperes required in the stator $= 1{,}000\sqrt{1 - 0.866^2} = 1{,}000 \times 0.50 = 500$ kva. Assuming a slip of 3 per cent. the capacity of the phase advancer is only $0.03 \times 500 = 15$ kva. A ½-hp. driving motor would supply the losses. The power developed by the induction motor is increased from $1{,}000 \times 0.866 = 866.0$ kw. to 1,000 kw., that is, by about 15 per cent.

To reduce the required phase advancer capacity the slip of the motor should be made as small as possible by decreasing its rotor resistance.

If the equipment of a shop consists of one large motor and a number of smaller ones, a phase advancer may be applied to the large motor to make its power factor leading and so supply the reactive currents for all the motors.

414. Phase Converter.—To change from one polyphase system to another, as from two-phase to three-phase or three-phase to six-phase, stationary transformers should be used; but when a change from single-phase to two-phase or three-phase is required, transformers are no longer satisfactory since, due to the pulsating nature of single-phase power, energy storage must take place and this can best be accomplished by means of the momentum of a revolving machine.

To change from single-phase to two-phase an ordinary two-phase induction motor with a squirrel-cage rotor may be used as a phase converter. The connections are shown in Fig. 466(*a*). One phase *A* of the stator winding is connected to the single-phase supply and the converter runs as a single-phase induction motor and near synchronous speed a revolving field is produced of approximately constant value (Art. 409). This field cuts the second stator phase *B* and generates in it a voltage in quadrature with the supply voltage and with approximately the same effective value at no load, Fig. 466(*b*).

One phase of the two-phase system receives its power directly from the single-phase supply and the second phase is supplied from the phase *B* of the converter. Under load the voltage of phase 1 remains constant but that of 2 decreases due to increased

slip and to the impedance drop in the converter and it falls back in phase to E'_2, Fig. 466(c). This unbalance of the two-phase system may be corrected for a given load by changing the connections as indicated by the broken lines in Fig. 466(a). The voltage across phase A under load is increased by changing the transformer tap, thus raising the voltage generated in B and in addition a component oa of the supply voltage is connected in series with B. The voltage of phase 2 is thus changed from E'_2 back to E_2.

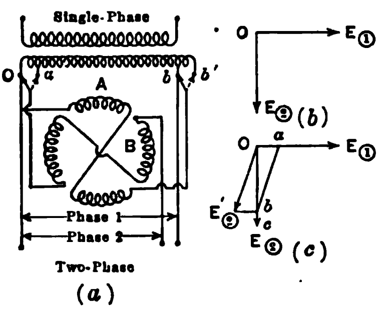


FIG. 466.

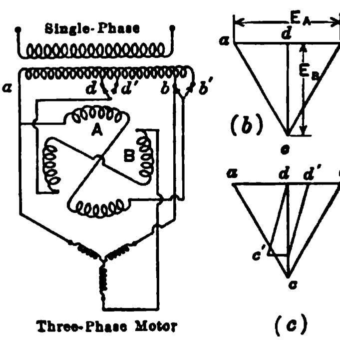


FIG. 467.

FIGS. 466 AND 467.—Phase converters.

When changing from single-phase to three-phase a similar phase converter may be used but phase B must be wound for only 87 per cent. of the voltage of phase A. Fig. 467(a) shows the connections required for operating a three-phase induction motor from a single-phase circuit and the equilateral triangle abc represents the balanced three-phase voltages at light load. Under load the voltage E_B decreases and falls back in phase and the point c moves to c'. To balance the system the connection for phase B must be changed from d to d' and the voltage impressed on A must be increased by changing to tap b'. This, however, gives a balance only at one particular load.

In both cases the momentum of the rotor stores and returns energy and makes possible the change from pulsating single-phase power to constant polyphase power.

By using a phase converter three-phase induction motors can be operated from a single-phase supply circuit for the propulsion of electric locomotives. A single converter may be used to supply a number of induction motors and it may be designed as a high-speed machine and will therefore be light and cheap; and

since it exerts very little torque the shaft may be made small. To reduce the losses and slip the rotor should be made of very low resistance. The weight of the phase converter may be only about 25 or 30 per cent. of the combined weight of the motors which it supplies.

The function of the phase converter is reversible and when the motors are running above synchronous speed they become generators supplying single-phase power back to the system and give a large and uniform braking effect.

415. Induction Frequency Converter.—The induction frequency converter may be used instead of the synchronous-motor generator set to convert power from one frequency to another or to link up two systems of different frequencies. It consists of an induction motor with a wound rotor driven by a synchronous motor connected to the 25-cycle supply lines. The stator of the induction motor is also connected to the supply and produces a revolving field. At standstill the frequency of the e.m.fs. generated in the rotor windings is 25 cycles. When the rotor is driven backward at synchronous speed the frequency is 50 cycles and when driven at 140 per cent. of synchronous speed it is 60 cycles. If a receiver circuit is connected to the rotor slip rings 60-cycle power can be supplied to it. Twenty-five-sixtieths or five-twelfths of the power output is supplied to the rotor by transformer action from the stator and the remaining seven-twelfths is supplied by the synchronous motor as mechanical power.

The principal disadvantage of the induction frequency converter is its poor voltage regulation. Due to the presence of the air gap in the magnetic circuit the reactances are large and the e.m.fs. consumed by the reactances are large.

The exciting current required by the induction motor may be provided by over-exciting the fields of the synchronous motor and making it draw a leading current. In this way the power factor of the set may be made unity, but the increased current in the synchronous-motor windings increases the copper losses and the heating.

Since the rotor of an induction motor may be wound for any number of phases irrespective of the number of stator phases the induction frequency converter may be used to change the number of phases as well as the frequency. The same result may of course be obtained with the synchronous-motor generator set.

CHAPTER XIII

ALTERNATING-CURRENT COMMUTATOR MOTORS

416. Motor Characteristics.—Direct-current motors are of three types, shunt, compound and series. The methods of varying the speed of such motors were discussed in Arts. 161 and 172.

Of the alternating-current motors, the synchronous motor runs at a constant speed at all loads and this speed cannot be varied. The induction motor with a low-resistance rotor also runs approximately at constant speed and this can be decreased only by using a wound rotor with slip rings and inserting resistance in the rotor windings to increase the slip. This is analogous to the introduction of resistance in series with the armature of a direct-current motor and is inefficient. There is no way of increasing the speed above synchronous speed corresponding to the field control in a shunt motor.

The single-phase synchronous motor and induction motor cannot start under load but the polyphase motors exert large starting torque. The current required, however, is greater than for a similar direct-current motor since it is not in phase with the impressed voltage and the torque per ampere is therefore less.

The induction motor with a comparatively high-resistance rotor has characteristics very similar to those of the direct-current compound motor. The speed falls off under load and the starting torque is good.

To design an alternating-current motor with characteristics similar to the direct-current series motor a commutator and brushes must be added. A large number of motors of this kind have been developed and the principles of their operation are discussed below.

By adding a commutator and brushes to the rotor of a single-phase induction motor the power factor may be compensated and the speed may be made adjustable from values above synchronous speed down to half synchronous speed.

417. Alternating-current Series Motor.—The alternating-current series motor is very similar to the direct-current series motor and can be operated on direct-current with increased efficiency and output.

If a direct-current series motor is connected to an alternating-current supply circuit it will rotate since the currents in the field and armature reverse together and therefore the torque is always in one direction, but it will be very inefficient and will spark very badly.

With alternating current flowing in the field winding an alternating magnetic flux is set up through the magnetic circuit and causes very large losses due to hysteresis and eddy currents. To reduce these to a minimum the whole magnetic circuit of an alternating-current series motor must be laminated. The field circuit must be very heavily insulated to prevent short-circuits between turns which would burn out the motor on account of the large induced currents.

The relation between the e.m.fs. and current in the direct-current series motor is given by the equation

$$E = \mathcal{E} + I(r_a + r_f), \tag{444}$$

where E = impressed e.m.f.,
$\mathcal{E}$ = counter e.m.f. generated by rotation,
I = current,
r_a = resistance of the armature,
r_f = resistance of the field.

In the alternating-current series motor the alternating flux sets up large e.m.fs. of inductance in both the field and armature windings, which consume components of the impressed e.m.f. in quadrature ahead of the current. If L_f is the inductance of the field and L_a the inductance of the armature, their reactances are $x_f = 2\pi f L_f$ and $x_a = 2\pi f L_a$, respectively, where f is the frequency of the impressed e.m.f.

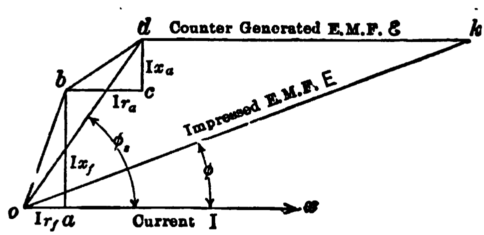


FIG. 468.—Vector diagrams of a single-phase series motor.

Fig. 468 shows the vector diagram for the motor.

$ox = I$ = current in field and armature.
$oa = Ir_f$ = e.m.f. consumed by the resistance of the field.

$ab = Ix_f$ = e.m.f. consumed by the reactance of the field.
$bc = Ir_a$ = e.m.f. consumed by the resistance of the armature.
$cd = Ix_a$ = e.m.f. consumed by the reactance of the armature.
$dk = \mathcal{E}$ = e.m.f. generated in the armature due to rotation, in phase with the field flux and, therefore, in phase with the current, neglecting the hysteretic lag.
$ok = E$ = impressed e.m.f.
$\cos kox = \cos \phi$ = load power factor.
$\cos dox = \cos \phi_s$ = power factor at start.

Taking the current as the real axis the relation between the current and the impressed e.m.f. can be expressed in rectangular coördinates as

$$\dot{E} = \dot{\mathcal{E}} + I(r_a + r_f) + jI(x_a + x_f) \tag{445}$$

and taking absolute values

$$E = \sqrt{\{\mathcal{E} + I(r_a + r_f)\}^2 + \{I(x_a + x_f)\}^2}. \tag{446}$$

At standstill

$$E = I\sqrt{(r_a + r_f)^2 + (x_a + x_f)^2}. \tag{447}$$

and the current is

$$I = \frac{E}{\sqrt{(r_a + r_f)^2 + (x_a + x_f)^2}}. \tag{448}$$

Full voltage can usually be impressed on the motor at standstill without causing any injury since the current is limited by the large impedance.

The power factor under running conditions is

$$\cos \phi = \frac{\mathcal{E} + I(r_a + r_f)}{\sqrt{\{\mathcal{E} + I(r_a + r_f)\}^2 + I(x_a + x_f)\}^2}} \tag{449}$$

but $\mathcal{E} = kn\Phi$, where Φ is the maximum value of the flux per pole, n is the motor speed in revolutions per second and k is a constant depending on the number of turns in the armature winding and on the shape of the flux wave. The flux Φ is almost proportional to the current I and the generated e.m.f. may be expressed as

$$\mathcal{E} = k'nI.$$

Substituting this value for $\mathcal{E}$ in equation and eliminating I

$$\cos \phi = \frac{k'n + r_a + r_f}{\sqrt{(k'n + r_a + r_f)^2 + (x_a + x_f)^2}}; \tag{450}$$

the power factor, therefore, increases with increasing speed and approaches unity. At low speed and at standstill it is low on

account of the reactances in the field and armature and for satisfactory operation it is necessary to make these reactances as low as possible.

418. Design for Minimum Reactance.—The inductance of any coil is proportional to the square of the number of turns and is inversely proportional to the reluctance of the magnetic circuit through it. To reduce the inductance L_f of the field winding it is designed with a small number of turns but this reduces the field m.m.f. and in order to obtain the required flux the reluctance of the magnetic circuit must be made very low. For this purpose large sections of high permeability are used, the slots are partially closed and the air gap is made as short as possible.

The reactance of the winding is proportional to the product of the inductance and the frequency and therefore the frequency should be low. Motors are usually designed for 25 cycles since that is the lowest standard frequency, but they will operate on 15 cycles or on direct current with a much improved efficiency and power factor and a larger output. The frequency of the supply does not affect the speed of the motor directly, but it does indirectly since the reactance drop decreases with the frequency and, therefore, the speed for a given current increases.

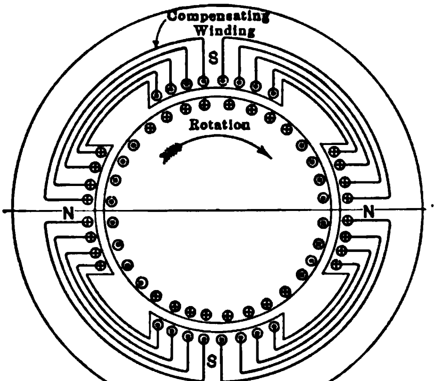


FIG. 469.—Four-pole, single-phase, series motor with compensating winding.

419. Compensating Windings.—The armature inductance and reactance cannot be decreased by reducing the number of turns on the armature since, for a given impressed voltage, that would increase the speed of the motor, and, further, since the field is made comparatively weak the armature must be made corre-

spondingly strong in ampere-turns in order to produce the required torque.

The armature m.m.f. as in direct-current machines is cross-magnetizing and distorts the main field and so weakens it and interferes with commutation. The flux produced by it is alternating and induces in the armature a back e.m.f. of armature inductance. This flux of armature reaction or armature inductance may be reduced as in the direct current generator by the use of a compensating winding. The compensating winding is placed in slots in the pole faces as shown in Fig. 469. It is distributed over the whole periphery of the armature and exerts a m.m.f. opposing the armature m.m.f. and so limiting the cross flux to a very small value and reducing the armature inductance and reactance in the same proportion.

The m.m.f. of the compensating winding can be produced in two ways illustrated in Fig. 470 and Fig. 471. The first is called inductive compensation and the second conductive compensation.

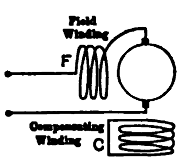


Fig. 470.—Inductively compensated series motor.

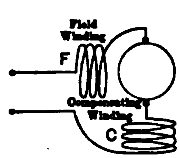


Fig. 471.—Conductively compensated series motor.

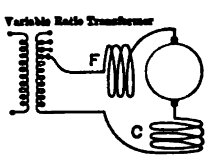


Fig. 472.—Series motor with variable voltage supply.

1. In the inductively compensated series motor the compensating m.m.f. is produced by short-circuiting the compensating coil. It then acts as the closed secondary of a transformer of which the armature is the primary. The m.m.f. of the compensating winding is almost equal to the m.m.f. of the armature but can never be greater than it and, therefore, over-compensation is not possible. The combined reactance of the armature and compensating winding corresponds to the reactance of a transformer on short-circuit. The mutual flux is almost destroyed but the leakage fluxes remain.

2. In the conductively compensated motor the compensating coil is connected in series with the field and armature and the amount of compensation can be varied. When the m.m.fs. of the two windings are equal there is no mutual flux and the combined reactance is a minimum. When the m.m.f. of the compensating winding is stronger than that of the armature the armature reaction flux is reversed but the reactance of the compensating

winding is increased and so part of the advantage is lost, but the flux due to over-compensation assists commutation of the load current in the same way that interpoles do and is thus a great advantage.

A conductively compensated motor can be operated on direct current but an inductively compensated motor cannot since the compensating winding would not be effective and sparking would occur.

420. Commutation.—Satisfactory commutation is very much more difficult to obtain in the alternating-current series motor than in the direct-current motor because, as may be seen in Fig. 473, the short-circuited coil is in the position of the short-circuited secondary of a transformer with the main field as primary and tends to have as many ampere-turns induced in it as there are on

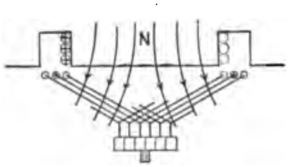


FIG. 473.

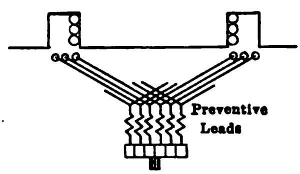


FIG. 474.

a pair of field poles. This large short-circuit current interferes with commutation and must be reduced as far as possible. For this purpose high-resistance leads, called preventive leads, are connected between the coils and the commutator bars, as shown in Fig. 474, and narrow carbon brushes of high contact resistance are used. The short-circuit current must pass through two resistance leads in series and is thus greatly reduced while the load current is carried by two or more in multiple. The resistance of one of the leads must be very much higher than that of an armature coil in order to reduce the current sufficiently.

There are losses in the leads due to the resultant of the two currents flowing in them, but by increasing the resistance up to a certain point the short-circuit current is reduced and the combined loss is reduced.

The resistance leads are not made of large enough capacity to carry the current continuously but under running conditions any one lead is in circuit for only a very short time. If the motor is stalled with power on the leads are likely to be destroyed.

The torque of the motor is very much improved by the use of

resistance leads since without them the large short-circuit current would weaken the main field and decrease the torque.

Fig. 475 shows the characteristic curves of a 150 hp., single-phase series motor. The torque and speed curves are very much the same shape as those of the direct-current series motor. The power factor approaches unity at light load when the speed is high as explained above, but at full load it is still very good, reaching 90 per cent. in some cases. At start and at low speeds it is low because the reactance of the motor is constant.

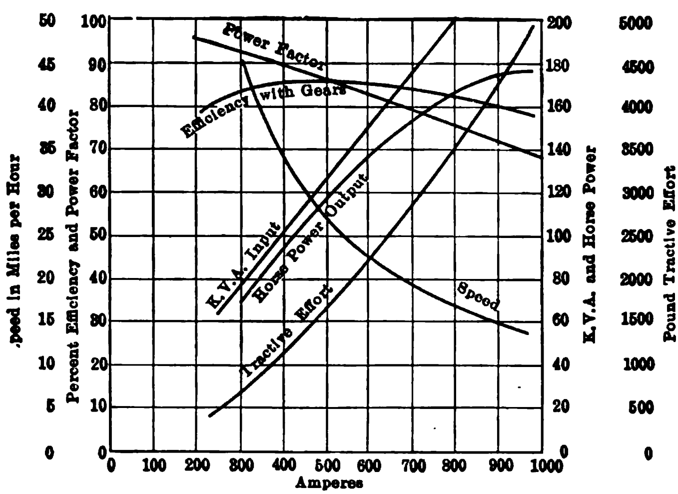


FIG. 475.—Characteristic curves of a 25-cycle, 250-volt, 150-horse-power, single-phase series motor.

Efficiencies up to 85 per cent. can be obtained but the motors must be designed more liberally than the corresponding direct-current motors and are therefore heavier and more expensive.

On account of unsatisfactory commutation alternating-current series motors are only built for voltages of 250 volts and under.

421. Speed Control.—The speed of the series motor can be controlled by supplying the motor through a transformer with a number of secondary taps (Fig. 472). By changing the taps the speed of the motor for any load can be adjusted through a wide range. The alternating-current series motor has in this respect an advantage over the direct-current series motor, since the change of impressed voltage is not accompanied by rheostatic losses.

422. Polyphase Commutator Motor.—A three-phase commutator motor with a series characteristic is shown in Fig. 476. The armature is similar to that in the single-phase motor but the commutator has three sets of brushes per pair of poles, spaced at 120 electrical degrees. The three-phase stator winding is usually connected to the line through transformers and the other terminals are connected to the three brushes. The machine is a combination of three single-phase series motors in one and has similar characteristics. If three compensating windings are added to the stator, at right angles to the three exciting windings and connected in series with them, the power factor will be improved but the cost will be increased. The speed for any load can be adjusted by moving the brushes relative to the exciting windings but the 120-degree displacement must be maintained.

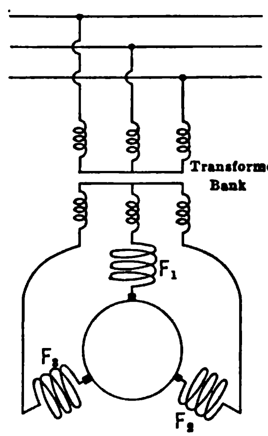


Fig. 476.—Three-phase commutator motor.

423. Repulsion Motor.—In construction the repulsion motor resembles the single-phase series motor with conductive compensation. The armature is, however, not connected in series with the field but is short-circuited and receives its current by induction.

The principle of its operation can be understood by reference to Figs. 477 to 479. In Fig. 477 the armature is shown short-circuited with the brushes in line with the field poles. Current is

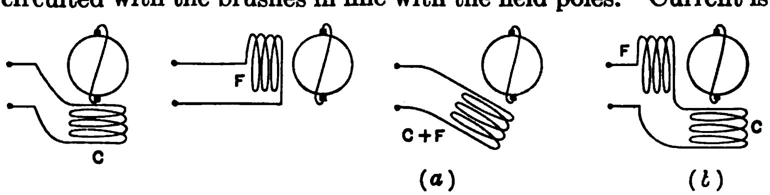


Fig. 477. Fig. 478. Fig. 479.

Figs. 477 to 479.—Repulsion motor.

induced in it as in the secondary of a transformer and is very large but the torque exerted in each direction is the same and thus the resultant torque is zero. In Fig. 478, with the brushes turned through 90 degrees there is no current induced in the armature and therefore no torque. In order that the motor may

exert torque the brushes must be placed in some intermediate position Fig. 479(*a*). The same result is accomplished by placing a second winding at right angles to the main field winding. This is shown as the compensating coil *C* in Fig. 479(*b*) and is carried in slots in the pole faces as in the series motor. The brushes are placed in line with this coil and the armature receives its current by induction from it. Torque is produced which is proportional to the product of the armature current induced by the compensating coil and the flux produced by the main field, but it is necessary to show that the current and flux are in time phase with one another.

If voltage is impressed on the motor at rest, current flows in both coils *C* and *F*. There is a large drop of voltage across *F* since its reactance is high, but only a very small drop across *C* since its reactance is low, due to the presence of the short-circuited armature winding and thus at standstill a large flux passes through *F* and a small flux through *C*.

The flux in *F* is in time phase with the field current; the current in the armature is in phase opposition to the field current and therefore reaches its maximum at the same instant as the flux in *F*, and the torque which is proportional to their product retains its sign as they reverse together.

When the armature rotates an e.m.f. is generated between the brushes by the armature conductors cutting the flux from *F*. This e.m.f. is at every instant proportional to the product of the flux and the speed and is in phase with the flux and is therefore 90 degrees behind the e.m.f. across *F*. The armature now acts at the primary of a transformer with the compensating coil as secondary and it produces a flux which transfers the speed e.m.f. to the compensating coil and the coil *C* therefore consumes a large component of the impressed e.m.f.

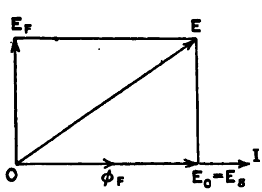


FIG. 480.

In Fig. 480,

I is the line current which flows in the coils F and C.

Φ_F is the flux in F, which is in phase with the current.

E_F is the component of impressed e.m.f. across the terminals of F.

E_S is the e.m.f. generated in the armature by rotation and transferred to the compensating coil.

E_C is the component of impressed e.m.f. across the terminals of C; it is equal to E_S if the coil C has the same number of turns as the armature, and it is in phase with it.

$E = \sqrt{E_F^2 + E_C^2}$ is the constant line voltage impressed on the motor.

The e.m.f. consumed by the impedance of the armature and compensating winding is neglected. As the speed increases the e.m.f. E_C increases and E_F decreases, the flux in the main field F decreases and the current and torque decrease.

At start, when the drop across C is small, the current is large and the main field F is very strong. The repulsion motor, therefore, gives a good starting torque. The field at start will be decreased to a certain extent by the current in the short-circuited coil undergoing commutation, as in the series motor.

424. Commutation.—In the single-phase series motor and the repulsion motor there are two currents to be commutated: (1) the load current and (2) the short-circuit current produced in the coil under the brush by the alternating flux of the main field.

1. To reverse the load current a m.m.f. is required opposing the m.m.f. of armature reaction and strong enough to produce a flux in the opposite direction to the armature reaction flux. Such a flux can be produced by interpoles placed between the main poles and excited by a winding in series with the main field or it can be produced by a compensating winding. The conductively compensating winding is the only one which can give perfect commutation since its m.m.f. can be made stronger than the armature m.m.f. Commutation is assisted by the use of high-resistance carbon brushes.

2. To eliminate the short-circuit current in the coil under the brush an e.m.f. must be generated in the coil equal and opposite to the e.m.f. producing the short-circuit current. The neutralizing e.m.f. cannot be generated by the alternation of a magnetic flux through the coil, since that would require a flux equal and opposite to the field flux and would destroy the field of the motor. The required e.m.f. can, however, be generated by the rotation of the armature through a commutating field of the proper intensity and position, but the field must be in quadrature with the main field in both time and space. In the repulsion motor under running conditions such a field is produced in the compensating coil C. The intensity of the field varies with the speed of the

motor. Near synchronous speed the e.m.f. is entirely neutralized and the current is wiped out. Below synchronous speed the current is reduced and above synchronous speed another current is produced as objectionable as before and commutation becomes bad again. At standstill no neutralizing e.m.f. is produced.

In the single-phase series motor there is no field in quadrature with the main field in time and so no neutralizing e.m.f. can be produced, but the short-circuit current is reduced by using high-resistance preventive leads as explained in Art. 420.

Thus near synchronous speed the commutation of the repulsion motor is better than that of the series motor.

Since preventive leads are not used in the repulsion motor the short-circuit current in it at start will be greater than in the series motor and will weaken the main field and decrease the starting torque.

While running the short-circuit current is not so great, since it cannot reach its maximum value on account of the self-inductance of the coil.

The repulsion motor cannot be operated more than 40 per cent. above synchronous speed on account of commutation troubles.

425. Compensated Repulsion Motor.—The main field winding, *F*, or the torque field of the repulsion motor may be omitted if two extra brushes are placed on the commutator at right angles to the main brushes as shown in Fig. 481. The armature winding

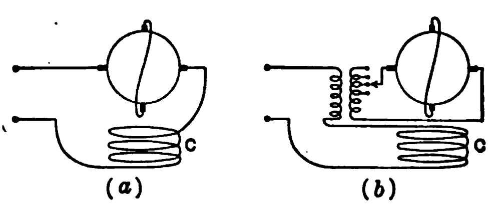


FIG. 481.—Compensated repulsion motor.

is thus made to carry the exciting current and the total reactance of the motor is decreased and the power factor improved. It is not usually desirable to design the armature with the proper number of turns for excitation and the motor Fig. 481(*a*) may be replaced by that in (*b*), in which the exciting brushes are supplied from taps on the secondary of a transformer with its primary in series with the compensating coil *C*. With this arrangement the speed may be controlled by changing the tap to

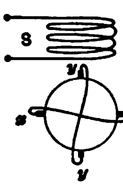
which the exciting brush is connected. These motors have the serious objection that four sets of brushes are required per pair of poles.

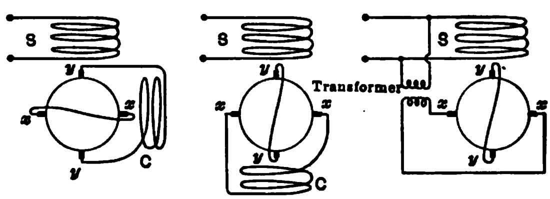
A large number of motors, differing in certain details from the simple series and repulsion motors described here, have been designed and have good operating characteristics, but a discussion of them is beyond the scope of this book.

426. Alternating-current Commutator Motors with Shunt Characteristics.—The single-phase induction motor has a speed characteristic similar to a shunt motor but the speed cannot be adjusted and the motor has no starting torque.

If the squirrel-cage rotor is replaced by a direct-current armature with two sets of short-circuited brushes at right angles as shown in Fig. 482 the characteristics of the motor are not changed but it is possible to improve its power factor and to adjust its speed.

When the motor is at rest there is no e.m.f. between the xx brushes and no current flows and therefore there is no flux along the xx axis. There is flux along the yy axis and current flows





(a) (b) (c)

FIG. 482. FIG. 483.—Single-phase commutator motor with power-factor compensation.

through the yy brushes. If the motor is started in either direction the conductors cut the yy flux and an e.m.f. is generated between the xx brushes in phase with the yy flux and proportional to the speed. This e.m.f. causes a current to flow in the xx brushes which produces a torque field along the xx axis and thus torque is developed as in the single-phase induction motor (Art. 408).

There are three ways of improving the power factor of the motor: (1) By introducing in the circuit of the yy brushes a stator winding in the xx axis, Fig. 483(a); (2) by introducing in the circuit of the xx brushes a stator winding in the yy axis,

Fig. 483(*b*); and (3) introducing in the circuit of the *xx* brushes an e.m.f. in phase with the line voltage, Fig. 483(*c*).

There are likewise three methods of controlling or adjusting the speed: (1) By introducing in the circuit of the *xx* brushes a stator winding in the *xx* axis, Fig. 484(*a*); (2) by introducing in the circuit of the *xx* brushes a variable inductance, Fig. 484(*b*); and (3) by introducing in the circuit of the *yy* brushes an e.m.f. in phase with the line voltage, Fig. 484(*c*).

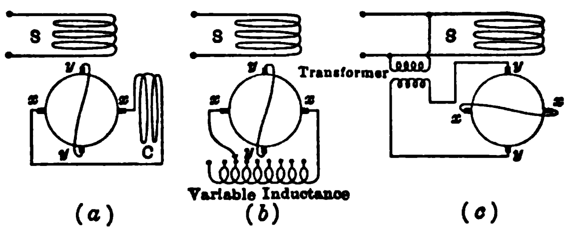


FIG. 484.—Single-phase commutator motor with speed adjustment.

427. Single-phase Induction Motor with Repulsion Starting.—The repulsion motor in Fig. 479(*a*) may be changed into a single-phase induction motor by short-circuiting the commutator. When the brushes of such a motor are set at the proper angle to the stator winding a good starting torque is produced. A centrifugal governor may be set to short-circuit all the segments of the commutator and to raise the brushes when a certain speed is reached and the motor will then operate as an induction motor at approximately constant speed.

CHAPTER XIV

TRANSMISSION SYSTEMS

428. Transmission Line.—The transmission line carries the electrical energy from the generating station to the receiving station or substation, where it is either transformed into mechanical energy or distributed to the customers throughout the district.

The most important characteristics of a transmission line are: (1) reliability, (2) regulation and (3) efficiency.

1. To insure reliability of service lines should, wherever possible, be installed in duplicate and all the necessary protective devices applied.

2. For good regulation the reactance of the line should be as small as possible and therefore the frequency should be low. The capacity of a line draws a leading current, which partially counteracts the drop in voltage due to reactance and so improves the regulation.

3. The power losses in a line are the resistance loss, which varies as the square of the current, and the comparatively small losses due to leakage over the insulators and to the formation of corona around the conductors.

To reduce the power loss the resistance of the line should be made as low as possible. This can be done by increasing the cross-section of the conductors, but the increased cost of the material required soon overcomes the saving due to the increase in efficiency.

For a given loss and a given voltage between lines power can be transmitted with a smaller amount of conducting material three-phase than either single-phase or two-phase.

429. Relative Amounts of Conducting Material for Single-, Two- and Three-phase Transmission Lines.

Let P = power input to the line in watts.
p = per cent. loss of power in the line resistance due to full-load current.
I = full-load current.

$\cos\theta$ = power factor.
r = resistance of each conductor.
n = number of conductors in the system.

The loss in the line is

$$\frac{pP}{100} = nI^2r,$$

and the resistance of each conductor is

$$r = \frac{pP}{100\,nI^2}.$$

For the same voltage E between conductors the current is

$$I = \frac{P}{E\cos\theta},\ \text{single-phase},$$

$$I = \frac{P}{2E\cos\theta},\ \text{two-phase},$$

$$I = \frac{P}{\sqrt{3}E\cos\theta},\ \text{three-phase},$$

and the resistance per conductor is

$$r = \frac{pP \times E^2\cos^2\theta}{100 \times 2P^2} = 0.005\frac{pE^2\cos^2\theta}{P},\ \text{single-phase},$$

$$r = \frac{pP \times 4E^2\cos^2\theta}{100 \times 4P^2} = 0.01\frac{pE^2\cos^2\theta}{P},\ \text{two-phase},$$

$$r = \frac{pP \times 3E^2\cos^2\theta}{100 \times 3\,P^2} = 0.01\frac{pE^2\cos^2\theta}{P},\ \text{three-phase}.$$

Since the single-phase line has only two conductors while the two-phase line has four the amount of copper required for both is the same. The three-phase line consists of three conductors of the same section as the two-phase conductors and, therefore, the amount of copper required for a three-phase line is only 75 per cent. of that required for a two-phase or single-phase line with the same per cent. power loss and the same maximum voltage between lines.

430. Reactance.—The inductance of a line per mile of conductor is (equation 163)

$$L = \left(0.74\log_{10}\frac{D}{R} + 0.0805\right)10^{-3}\ \text{henrys}$$

where D is the distance between conductors,
and R is the radius of the conductors.

The reactance of the line per mile of conductor is

$$X = 2\pi fL\ \text{ohms}.$$

The reactance could be decreased by decreasing the distance between the conductors or by increasing the radius, but these quantities are fixed by other considerations than the reactance and reactance drop.

431. Capacity.—The capacity of a line per mile of conductor between the conductor and neutral is

$$C = \frac{38.8}{\log_{10}\frac{D - R}{R}} 10^{-9} \text{ farads (equation 38).}$$

This value applies for each conductor of a single-phase or polyphase line. If the conductors of a three-phase line are suspended in one plane instead of in the form of an equilateral triangle the capacity of the central conductor is slightly greater than that of the others, but since all lines are transposed the total capacity of each of the three is the same and is given with sufficient accuracy by the formula above if the distance D is taken as the shortest distance between conductors.

The capacity reactance per mile of conductor is

$$X_c = \frac{1}{2\pi fC} \text{ ohms,}$$

and the charging current per mile of conductor is

$$I_c = \frac{e}{X_c} = 2\pi fCe,$$

where e is the voltage between conductor and neutral.

For transmission lines up to 50,000 volts the capacity is very small and its effect on the regulation may be neglected. If, however, any part of the transmission is carried out through underground cables, the capacity may be very largely increased and may not be negligible. Above 50,000 volts the capacity of the line must be considered in calculating the regulation. For lines up to 100 miles in length and for voltages up to 100,000 volts the capacity of each conductor may be considered as a condenser connected at the center of the line between conductor and neutral. If more accurate results are necessary the fact that both the reactance and the capacity of the line are distributed over the whole length must be taken into account.

Due to the presence of the charging current in a line the current flowing into the receiving circuit may be very much larger than the current entering the line at the generating station.

432. Voltage and Frequency.—Voltages up to 150,000 volts are now in use for the transmission of large amounts of power over long distances. The voltage employed in any given system is as a general rule approximately one thousand times the length of the line in miles.

Power is usually generated and transmitted at either 25 cycles or 60 cycles. With 25 cycles the reactance drop in the line is less

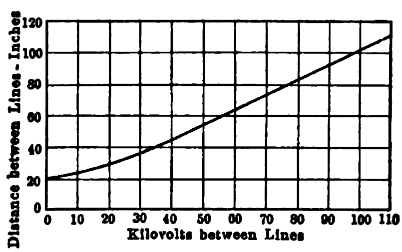


FIG. 485.—Spacing of conductors.

than with 60 cycles and therefore the voltage regulation is better. In the case of very long high-voltage lines the increased charging current at the higher frequency may counteract the larger reactance drop of voltage. Where power is required for lighting 60 cycles is necessary unless frequency chargers are installed.

433. Spacing of Conductors.—The distance between the conductors of a transmission line depends both on the voltage and also on certain points in the mechanical design, such as the material of the conductor, length of span and the amount of sag allowed. The curve in Fig. 485 gives approximately the relation between the spacing of the conductors and the voltage.

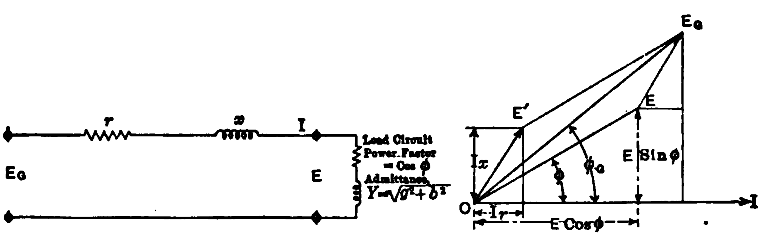


FIG. 486.—Single-phase transmission line.

434. Single-phase Transmission Line.—1. A single-phase transmission line, Fig. 486, delivers 5,100 kw. to a receiver circuit at

60,000 volts. If the power factor of the load is 85 per cent., find the generator voltage.

$$r = \text{resistance of the line} = 20 \text{ ohms}.$$
$$x = \text{reactance of the line} = 50 \text{ ohms}.$$

The power delivered to the receiver circuit is

$$P = EI \cos \phi = 5{,}100{,}000 \text{ watts},$$

where $E = 60{,}000$ is the receiver voltage

and $\cos \phi = 0.85$ is the power factor;

the current is therefore

$$I = \frac{P}{E \cos \phi} = \frac{5{,}100{,}000}{60{,}000 \times 0.85} = 100 \text{ amp}.$$

The vector diagram is drawn with the current $OI = I$ as horizontal.

The receiver e.m.f. $OE = E$ leads the current by an angle ϕ and has two components

$$OE_1 = E_1 = E \cos \phi \text{ in phase with } I \text{ and}$$
$$OE_2 = E_2 = E \sin \phi \text{ in quadrature ahead of } I.$$

The voltage consumed in the resistance of the line is Ir in phase with I; the voltage consumed in the reactance of the line is Ix in quadrature ahead of I.

The component of the generator e.m.f. in phase with I is

$$E_1 + Ir = E \cos \phi + Ir$$

and the component in quadrature ahead of I is

$$E_2 + Ix = E \sin \phi + Ix,$$

and therefore the generator e.m.f. is

$$E_G = \sqrt{(E \cos \phi + Ir)^2 + (E \sin \phi + Ix)^2},$$

or substituting the numerical values

$$E_G = \sqrt{(60{,}000 \times 0.85 + 100 \times 20)^2 + (60{,}000 \times 0.52 + 100 \times 50)^2}$$
$$= 64{,}000 \text{ volts}.$$

The e.m.f. consumed in the line is

$$I\sqrt{r^2 + x^2} = 100\sqrt{20^2 + 50^2} = 5{,}400 \text{ volts}.$$

The loss of power in the line is

$$I^2 r = 100^2 \times 20^2 = 200{,}000 \text{ watts}.$$
$$= 200 \text{ kw}.$$

The power factor at the generator is

$$\cos \phi_G = \frac{E \cos \phi + Ir}{E_G} = \frac{53{,}000}{64{,}000} = 0.828 = 82.8 \text{ per cent}.$$

Using rectangular coördinates and taking the current as axis, the e.m.f. at the receiver terminals is

$$\underset{\cdot}{E} = E\cos\phi + jE\sin\phi,$$

the e.m.f. consumed in the impedance of the line is

$$\underset{\cdot}{E}' = Ir + jIx,$$

and the generator e.m.f. is

$$E_G = \underset{\cdot}{E} + \underset{\cdot}{E}' = (E\cos\phi + Ir) + j(E\sin\phi + Ix),$$

and its absolute value is

$$E_G = \sqrt{(E\cos\phi + Ir)^2 + (E\sin\phi + Ix)^2}.$$

The capacity of the line has been neglected in this example.

2. A transmission line of impedance $\underset{\cdot}{Z} = r + jx$ delivers power to a receiver circuit of admittance $\underset{\cdot}{Y} = g - jb$ at a constant voltage E. If the capacity of the line is assumed to be concentrated at the center determine the charging current of the line, the total current delivered by the generator and the terminal voltage of the generator.

The condensive reactance of the line is

$$x_c = \frac{1}{2\pi fC}\text{ ohms},$$

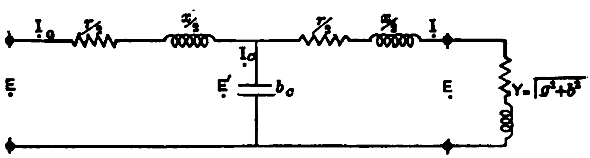


FIG. 487.—Single-phase transmission line with capacity.

where f is the frequency of the impressed e.m.f. and C is the capacity of the line in farads; the condensive susceptance of the line is

$$y_c = b_c = \frac{1}{x_c}$$

and it is represented as a condenser connected across the line (Fig. 487).

The current in the receiver circuit is

$$\underset{\cdot}{I} = \underset{\cdot}{E}(g - jb),$$

and the e.m.f. at the center of the line is

$$\underset{\cdot}{E}' = \underset{\cdot}{E} + \underset{\cdot}{I}\left(\frac{r + jx}{2}\right)$$

$$= \underset{\cdot}{E}\left\{1 + \frac{r + jx}{2}(g - jb)\right\}\cdot$$

The charging current of the line is

$$\begin{aligned} I_c &= E'jb_c \\ &= jb_cE\left\{1 + \frac{r + jx}{2}(g - jb)\right\}, \end{aligned} \tag{451}$$

and the current from the generator is

$$I_G = I + I_c = E\left[g - jb + jb_c\left\{1 + \frac{r + jx}{2}(g - jb)\right\}\right]. \tag{452}$$

The terminal e.m.f. of the generator is

$$\begin{aligned} E_G &= E' + I_c\left(\frac{r + jx}{2}\right), \\ &= E\left\{1 + \frac{r + jx}{2}(g - jb) + \frac{r + jx}{2}(g - jb) + jb_c\left(\frac{r + jx}{2}\right)\right. \\ &\qquad\left. + jb_c\frac{(r + jx)^2(g - jb)}{4}\right\} \\ &= E\left\{1 + (r + jx)\left(g - jb + j\frac{b_c}{2}\right) + j\frac{b_c}{4}(r + jx)^2(g - jb)\right\}. \end{aligned} \tag{453}$$

For lines of small capacity the last term may be neglected and equation (453) reduces to

$$E_G = E\left\{1 + (r + jx)\left(g - jb + j\frac{b_c}{2}\right)\right\}. \tag{454}$$

This is equivalent to replacing the capacity susceptance b_c at the center of the line by a condenser of susceptance $\frac{b_c}{2}$ at the receiver end of the line.

3. A single-phase transmission line delivers 10,000 kw. at 100,000 volts to a receiver circuit of 85 per cent. power factor; find the voltage, current and power factor at the generating end of the line, the impedance drop and power loss in the line and the charging current. Find also the generator voltage required to give a receiver voltage of 100,000 volts at no load.

Length of line = 100 miles.
Size of wire = No. 000 B. & S. copper.
Diameter of wire = $2R$ = 0.41 in.
Distance between wires = D = 100 in.
Frequency = 60 cycles per second.

The inductance or coefficient of self-induction of each wire of the line is, by equation (163),

$$L_1 = \left(0.74\log_{10}\frac{D}{R} + 0.0805\right)10^{-3} \text{ henrys per mile,}$$

and therefore the inductance of the line consisting of two wires is

$$L = 200L_1 = 200\left(0.74 \log_{10}\frac{100}{0.215} + 0.0805\right)10^{-3} = 0.41 \text{ henrys};$$

the inductive reactance of the line is

$$\begin{aligned} x &= 2\pi fL \\ &= 2 \times 3.14 \times 60 \times 0.41 = 154.5 \text{ ohms.} \end{aligned}$$

The capacity of each wire to neutral is, by equation (38),

$$C_1 = \frac{38.8}{\log_{10}\dfrac{D-R}{R}}10^{-9} \text{ farads per mile,}$$

and the capacity between wires is

$$C_2 = \frac{C_1}{2} = \frac{19.4}{\log_{10}\dfrac{D-R}{R}}10^{-9} \text{ farads per mile of line;}$$

therefore, the capacity of the line is

$$\begin{aligned} C = 100C_2 &= 100 \times \frac{19.4}{\log_{10}\dfrac{99.8}{0.215}}10^{-9} \\ &= 0.73\ 10^{-6} \text{ farads.} \end{aligned}$$

The condensive reactance is

$$\begin{aligned} x_c &= \frac{1}{2\pi fC} \\ &= \frac{1}{2 \times 3.14 \times 60 \times 0.73 \times 10^{-6}} = 3640 \text{ ohms,} \end{aligned}$$

and the condensive susceptance of the line is

$$b_c = \frac{1}{x_c} = \frac{1}{3{,}640} = 0.000275.$$

The resistance of the line at 20°C. is

$$\begin{aligned} r &= \rho\frac{l}{cir.\ mils} \\ &= 10.4\frac{200 \times 5{,}280}{(410)^2} = 65.4 \text{ ohms.} \end{aligned}$$

The load delivered to the receiver is

$$P = EI\cos\phi = 10{,}000{,}000 \text{ watts.}$$

but $\quad E = 100{,}000$ and $\cos\phi = 0.85$;

therefore, the current in the receiver circuit is

$$I = \frac{P}{E\cos\phi} = \frac{10{,}000{,}000}{100{,}000 \times 0.85} = 117 \text{ amp.};$$

the power component is

$$I_P = I \cos \phi = 117 \times 0.85 = 100 \text{ amp.};$$

the quadrature component is

$$I_W = I \sin \phi = \sqrt{I^2 - I_P^2} = \sqrt{117^2 - 100^2} = 60 \text{ amp.}$$

The admittance of the receiver is

$$Y = \frac{I}{E} = \frac{117}{100{,}000} = 0.00117;$$

the conductance is

$$g = \frac{I \cos \phi}{E} = \frac{100}{100{,}000} = 0.001;$$

and the susceptance is

$$b = \frac{I \sin \phi}{E} = \frac{60}{100{,}000} = 0.0006.$$

The generator voltage is, by equation (453),

$$\dot{E}_G = \dot{E} \left\{ 1 + (r + jx)\left(g - jb + j\frac{b_c}{2}\right) + j\frac{b_c}{4}(r + jx)^2(g - jb) \right\},$$

and substituting the values obtained above

$$\dot{E}_G = \dot{E} \left\{ 1 + (65.4 + 154.5j)\left(0.001 - 0.0006j + \frac{0.000275}{2}j\right) \right.$$
$$\left. + \frac{0.000275}{4} j\,(65.4 + 154.5j)^2\,(0.001 - 0.0006j) \right\},$$

and simplifying

$$\dot{E}_G = \dot{E}(1.137 + 0.114j),$$

and the absolute value is

$$E_G = E\sqrt{(1.137)^2 + (0.114)^2}$$
$$= 100{,}000 \times 1.142 = 114{,}200 \text{ volts.}$$

The current from the generator is, by equation (542),

$$\dot{I}_G = \dot{E}\left[g - jb + jb_c \left\{ 1 + \frac{r + jx}{2}(g - jb) \right\}\right]$$
$$= \dot{E}\left[0.001 - 0.0006j + 0.000275j \right.$$
$$\left. \left\{ 1 + \frac{65.4 + 154.5j}{2}(0.001 - 0.0006j) \right\}\right]$$
$$= \dot{E}(0.000984 - 0.00029j),$$

and its absolute value is

$$I_G = 100{,}000\sqrt{(0.000984)^2 + (0.00029)^2} = 102.6 \text{ amp.}$$

The charging current of the line is, by equation (541),

$$\dot{I}_c = jb_c\dot{E}\left\{1 + \frac{r + jx}{2}(g - jb)\right\}$$
$$= \dot{E}\,(-\,0.0000158 + 0.00031j),$$

and its absolute value is

$$I_c = 100{,}000\sqrt{(0.0000158)^2 + (0.00031)^2} = 31 \text{ amp.}$$
$$= 26.5 \text{ per cent. of the receiver current.}$$

At no load $E = 100{,}000$, $g = 0$, $b = 0$ and the generator voltage is

$$\dot{E}_G = \dot{E}\left\{1 + (r + jx)\left(j\frac{b_c}{2}\right)\right\}$$

and its absolute value is

$$E_G = 100{,}000\sqrt{(0.9786)^2 + (0.0098)^2} = 97{,}860 \text{ volts.}$$

The power factor at the generator may be found by reference to the diagram in Fig. 488.

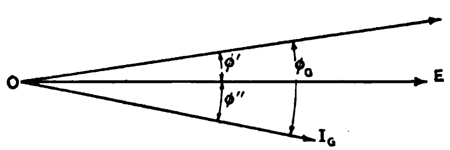


FIG. 488.

E_G leads E by an angle ϕ', where

$$\tan\phi' = \frac{11{,}400}{114{,}200} = 0.10;$$

I_G lags behind E by an angle ϕ'', where

$$\tan\phi'' = \frac{0.29}{98.4} = 0.0296;$$

and I_G lags behind E_G by an angle $\phi_G = \phi' + \phi''$;

$$\tan\phi_G = \tan(\phi' + \phi'') = \frac{\tan\phi' + \tan\phi''}{1 - \tan\phi'\tan\phi''} = 0.132,$$

and $$\phi_G = 7^\circ\ 30';$$

the power factor at the generator is

$$\cos\phi_G = \cos 7^\circ\ 30' = 0.99$$
$$= 99 \text{ per cent.}$$

The impedance drop in the line is found very approximately as

$$E_Z = I\sqrt{r^2 + x^2}$$
$$= 117\sqrt{(65.4)^2 + (154.5)^2} = 19{,}650 \text{ volts}$$
$$= 19.6 \text{ per cent. of the receiver voltage.}$$

The power loss in the line is found approximately as

$$
\begin{aligned}
P_L &= I_G^2 \frac{r}{2} + I^2 \frac{r}{2} \\
&= (102.6)^2 34 + (117)^2 34 \\
&= 826{,}000 \text{ watts} \\
&= 826 \text{ kw.} \\
&= 4.13 \text{ per cent. of the output.}
\end{aligned}
$$

435. Three-phase Transmission Line.—A three-phase, transmission line delivers 30,000 kva. at 100,000 volts, 60 cycles to a receiving circuit of 88 per cent. power factor; determine the voltage and current and power factor at the generating station, the charging current and charging kilovolt-amperes of the line and the efficiency of the transmission. Determine also the rise in voltage at the terminals of the receiving circuit if full load is suddenly removed.

Length of line 100 miles.

Conductor = No. 0000 B. & S. copper of 97 per cent. conductivity.

Diameter of conductor = $2R$ = 0.46 in.

Distance between conductors = 100 in.

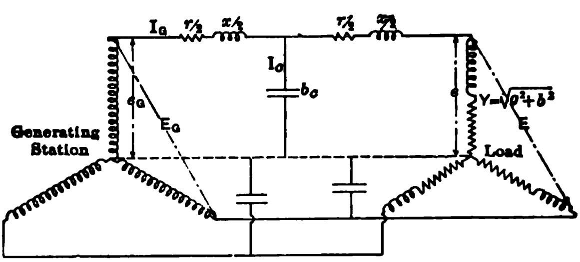


FIG. 489.—Three-phase transmission line.

In Fig. 489

E = voltage between lines at the receiving end = 100,000.

e = voltage between lines and neutral $= \dfrac{100{,}000}{\sqrt{3}} = 57{,}600.$

E_G = voltage between lines at the generating station.

e_G = voltage between lines and neutral $= \dfrac{E_G}{\sqrt{3}}.$

I = load current in each conductor.

I_c = charging current in each conductor.

I_G = current per conductor at the generating station.

$Y = \sqrt{g^2 + b^2}$ = admittance of each phase of the receiving circuit.

$Z = \sqrt{r^2 + x^2}$ = impedance of each line including the step-up and step-down transformers.

b_c = capacity susceptance of each conductor to neutral, which is assumed to be concentrated at the center of the line.

Resistance of a single conductor of 97 per cent. conductivity at 20°C., 100 miles long $= r' = 0.2667 \times 100 = 26.67$ ohms (article 86).

$$\text{Load current} = I = \frac{30{,}000{,}000}{\sqrt{3} \times 100{,}000} = 173 \text{ amp.}$$

Resistance drop in each conductor $= Ir' = 173 \times 26.67 =$

$$4{,}620 \text{ volts} = \frac{4{,}620 \times 100}{\dfrac{100{,}000}{\sqrt{3}}} = 8 \text{ per cent.}$$

Assuming a resistance drop in step-up and step-down transformers of 0.5 per cent. each, the total resistance drop per line is $8 + 2(0.5) = 9$ per cent.

r = equivalent resistance of each line $= 26.67 \times \frac{9}{8} = 30$ ohms.

Inductance of each conductor is

$$L = (0.74 \log_{10} \frac{100}{0.23} + 0.0805)10^{-3} \times 100 = 0.203 \text{ henry.}$$

Inductive reactance of each conductor $= x' = 2\pi f L = 2 \times 3.14 \times 60 \times 0.203 = 77$ ohms.

Reactance drop per conductor $= Ix' = 173 \times 77 = 13{,}321$

$$\text{volts} = \frac{13{,}321 \times 100}{\dfrac{100{,}000}{\sqrt{3}}} = 23.1 \text{ per cent.}$$

Reactance drop in each transformer = 3.45 per cent. (assumed).
Total reactance drop per line $= 23.1 + 2(3.45) = 30$ per cent.

$$x = \text{equivalent reactance of each line} = \frac{77 \times 30}{23.1} = 100 \text{ ohms.}$$

Capacity of each conductor to neutral is

$$C = \frac{38.8}{\log_{10} \dfrac{100 - 0.23}{0.23}} \times 10^{-9} \times 100 = 1.475 \times 10^{-6} \text{ farads.}$$

Capacity reactance of each conductor to neutral $= x_c = \dfrac{10^6}{2 \times 3.14 \times 60 \times 1.475} = 1{,}800$ ohms.

b_c = capacity susceptance per line $= \dfrac{1}{x_c} = 0.000555.$

Y = load admittance per phase $= \dfrac{I}{e} = \dfrac{173}{\dfrac{100{,}000}{\sqrt{3}}} = 0.003.$

$\cos\theta$ = load power factor $= 0.88.$

g = load conductance per phase $= \dfrac{I\cos\theta}{e} = Y\cos\theta = 0.003 \times 0.88 = 0.00264.$

b = load susceptance per phase $= \dfrac{I\sin\theta}{e} = Y\sin\theta = 0.003 \times 0.475 = 0.001425.$

Voltage between lines and neutral at the generating station at full load is

$$\dot{e}_G = \dot{e}\left\{1 + (r + jx)\left(g - jb + j\frac{b_c}{2}\right)\right\} \qquad \text{equation (454).}$$

$$= \dot{e}\left[1 + (30 + 100j)\left\{0.00264 - j(0.001425 - 0.0002775)\right\}\right]$$

$$= \dot{e}(1.1942 + 0.23j)$$

and its absolute value is

$$e_G = \frac{100{,}000}{\sqrt{3}}\sqrt{\overline{1.1942}^2 + \overline{0.23}^2} = 70{,}000 \text{ volts}$$

and it leads the terminal e.m.f. e by angle $\theta' = \tan^{-1}\dfrac{0.230}{1.1942} = \tan^{-1} 0.1925 = 10.9$ deg.

E_G = voltage between lines $= \sqrt{3}e_G = \sqrt{3} \times 70{,}000 = 121{,}600$ volts.

The current per conductor at the generating station is

$$\dot{I}_G = \dot{e}\left[g - jb + jb_c\left\{1 + \frac{r + jx}{2}\left(g - jb\right)\right\}\right]$$

$$= \dot{e}\left[0.00264 - 0.001425j + 0.000555j \times \left\{1 + \frac{30 + 100j}{2}\left(0.00264 - 0.001425j\right)\right\}\right]$$

$$= \dot{e}\left[0.00258 - 0.00081j\right].$$

and its absolute value is

$$I_G = \frac{100{,}000}{\sqrt{3}}\sqrt{\overline{0.00258}^2 + \overline{0.00081}^2} = 161 \text{ amp.}$$

It lags behind the terminal e.m.f. e by angle $\theta'' = \tan^{-1}\frac{0.00081}{0.00258}$ $= \tan^{-1} 0.314 = 17.45$ degrees.

I_G lags behind e_G by angle $\theta_G = \theta' + \theta'' = 10.9 + 17.45 = 28.35$ degrees and the power factor at the generator is $\cos\theta_G = \cos 28.35$ degrees $= 0.88 = 88$ per cent.

The charging current per line may be taken as the product of the capacity susceptance b_c and the average of the two voltages e_G and e, thus,

$$I_c = \frac{e_G + e}{2} b_c = \frac{70{,}000 + 57{,}600}{2} \times 0.000555$$
$$= 63{,}800 \times 0.000555 = 35.4 \text{ amp.}$$

and the charging kilovolt-amperes for the system

$$= \frac{3\left(\frac{e_G + e}{2}\right) I_c}{1{,}000} = \frac{3 \times 63{,}800 \times 35.4}{1{,}000} = 6{,}800 \text{ kva.}$$

The copper losses in the three lines and the step-up and step-down transformers may be taken as $3(I_G^2 + I^2)\frac{r}{2} = 3(161^2 + 173^2)15 = 2{,}520{,}000$ watts $= 2{,}520$ kw. $= \frac{2{,}502}{30{,}000 \times 0.88} \times 100$ $= 9.5$ per cent.

Assuming the iron losses in the transformers to total 1 per cent. the total loss is 10.5 per cent. and the efficiency of the transmission from the generator busbars to the low-voltage busbars in the receiving station is 89.5 per cent.

If the load is suddenly removed, the rise in the receiver voltage may be found from the following relations:

$g = 0$, $b = 0$, e_G is assumed to be maintained constant, and

$$\dot{e}_G = \dot{e}\left\{1 + (r + jx)\left(j\frac{b_c}{2}\right)\right\} = \dot{e}\{1 + (30 + 100j)(0.0002775j)\}$$
$$= \dot{e}(0.97225 - 0.008j)$$

and taking absolute values

$$70{,}000 = e_G = e\sqrt{(0.97225)^2 + (0.008)^2} = e \times 0.9723$$

and $$e = \frac{e_G}{0.9723} = \frac{70{,}000}{0.9723} = 72{,}000 \text{ volts}$$

and $$E = \sqrt{3}e = \sqrt{3} \times 72{,}000 = 124{,}700 \text{ volts.}$$

The regulation is $\frac{124{,}700 - 100{,}000}{100{,}000} \times 100$ per cent. $= 24.7$ per cent.

436. Application of a Synchronous Phase Modifier to a Transmission System.—In the transmission system worked out in the last article, the regulation is poor and the efficiency is low. These can both be improved by installing in the receiving station a synchronous machine to operate as a phase modifier. It should be over-excited and draw full leading kilovolt-ampères at periods of full load to keep the receiver voltage up and should be under-excited and draw lagging kilovolt-ampères at periods of light load to keep the voltage down. With such an equipment the voltage at the generating station may be maintained at a suitable constant value and the receiver voltage may likewise be maintained constant at some lower value by means of an automatic voltage regulator excited from the constant receiver voltage and operating on a shunt to the field rheostat of the phase modifier.

The equation $\underset{\cdot}{e}_G = \underset{\cdot}{e}\left\{1 + (r + jx)\left(g - jb + j\frac{b_c}{2}\right)\right\}$ was derived on the assumption that the capacity susceptance was connected at the center of the line, but the same result is obtained if one-half of this susceptance is connected at the receiving end. When the synchronous phase modifier is added, Fig. 490(a), the equation becomes

$$\underset{\cdot}{e}_G = \underset{\cdot}{e}\left\{1 + (r + jx)\left(g - jb + j\frac{b_c}{2} \pm jb_s\right)\right\} \qquad (455)$$

where b_s is the susceptance per phase of the phase modifier; b_s is positive when drawing a leading current and negative when drawing a lagging current.

The rating of the phase modifier required for any system can best be found by a graphical construction and equation (455) used only as a check.

1. If it is desired to operate the system of Art. 435 with a constant voltage E = 100,000 volts and a constant generating station voltage E_G = 110,000 volts, the rating of the required phase modifier may be found as follows:

In Fig. 490

Oa = receiver voltage to neutral = $e = \dfrac{100{,}000}{\sqrt{3}}$ = 57,600 volts.

ab = receiver current per phase = I = 173 amp. at full load.

Cos θ = load power factor = 0.88.

ac = resistance drop per line due to the load current = Ir = 173 × 30 = 5,200 volts.

cd = reactance drop per line due to the load current $= Ix =$ $173 \times 100 = 17{,}300$ volts.

ad = impedance drop per line $= I\sqrt{r^2 + x^2} = 173\sqrt{30^2 + 100^2}$ $= 173 \times 104.4 = 18{,}060$ volts leading the current by angle $\alpha =$ $\tan^{-1}\dfrac{x}{r} = \tan^{-1}\dfrac{100}{30}$.

Assuming the line capacity to be represented by the susceptance $\dfrac{b_c}{2}$ at the receiver, Fig. 490 (a), the charging current flowing over the line is $\dot{e} \times \dfrac{b_c}{2} = 57{,}600 \times 0.0002775 = 16$ amp. $= ah$.

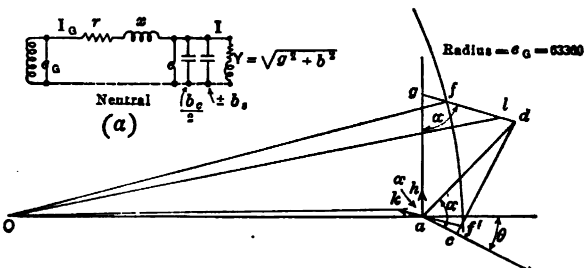


FIG. 490.

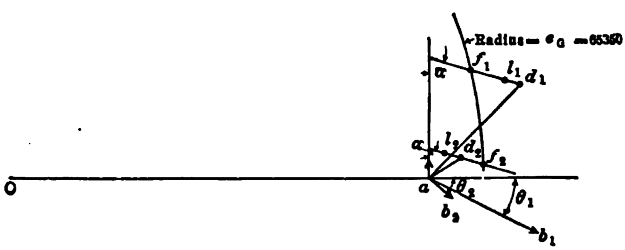


FIG. 491.

ak = impedance drop due to current $ah = 16\sqrt{30^2 + 100^2} =$ 1,670 volts. If dg is drawn through d making angle $\alpha = \tan^{-1}\dfrac{x}{r}$ with the charging current line and dl is cut off equal to ak, then Ol is the voltage to neutral required at the generating station, without the phase modifier = 70,000 volts from the diagram. This value was also obtained in Art. 435.

About O draw a circle of radius $e_G = \dfrac{E_G}{\sqrt{3}} = \dfrac{110{,}000}{\sqrt{3}} = 63{,}360$ to cut dg at f; then the line lf represents the impedance drop per line produced by the leading current flowing to the phase modifier.

Since ad is the impedance drop due to the load current the line lf will represent the kilovolt-ampère rating of the phase modifier to the same scale that the line ad represents the load kilovolt-ampères of 30,000. The rating of the phase modifier is therefore $\frac{lf}{ad} \times 30{,}000 = 12{,}500$ kilovolt-ampères.

At no load the voltage required at the generating station, without the phase modifier, is represented by the line Ok but since e_G is maintained constant at 63,360 volts, the phase modifier must be under-excited and draw a lagging current to cause an impedance drop represented by kf', where f' is the point of intersection of the e_G circle with the line kaf' making angle α with ag. kf' is approximately equal to lf and the phase modifier will be carrying full lagging current.

It is not usual to maintain constant voltage over such a wide range and the rating of the phase modifier may be reduced considerably especially if a greater difference between e_G and e is allowable.

2. If it is required to maintain a constant receiver voltage, $E = 100{,}000$ from full load of 30,000 kva. at 88 per cent. power factor to one-fourth load of 7,500 kva. at 75 per cent. power factor, determine the rating of the phase modifier and the most suitable voltage at the generating station.

The point l_1 in Fig. 491, corresponds to point l in Fig. 490 and is found in the same way; and angle $\theta_1 = \theta = \cos^{-1} 0.88$.

At one-fourth load of 7,500 kva. at 75 per cent. power factor.

$$ab_2 = \frac{ab_1}{4} = \frac{173}{4} = 43.25 \text{ amp.} = \text{load current.}$$

$$\text{Cos } \theta_2 = 0.75 = \text{load power factor.}$$

$$\text{Angle } d_2ab_2 = \text{angle } d_1ab_1 = \alpha = \tan^{-1}\frac{x}{r}.$$

Points d_2 and l_2 are found in the same way as d and l, Fig. 490.

To enable the phase modifier to be used to its full capacity leading at full load and full capacity lagging at one-fourth load, the generator voltage must have the value represented by the radius of a circle drawn about O, which will make the two intercepts f_1l_1 and f_2l_2 equal. The required value of e_G is 65,350 volts and $E_G = \sqrt{3} \times 65{,}350 = 113{,}000$ volts. The capacity of the phase modifier is $\frac{f_1l_1}{ad_1} \times 30{,}000 = 9{,}100$ kva. and its susceptance per phase at full load is $b_s = \frac{9{,}100{,}000}{3 \times e^2} = 0.00091$.

Referring to Fig. 490(*a*) the current in each line at full load is

$$I_G = e\left\{g - jb + j\frac{b_c}{2} + jb_s\right\}$$
$$= e\{0.00264 - j(0.001425 - 0.0002775 - 0.00091)\}$$
$$= e\,(0.00264 - 0.00025j)$$

and $\quad I_G = 57{,}600\sqrt{\overline{0.0264}^2 + \overline{0.00025}^2} = 153$ amp.

The full-load copper loss $= 3 \times (153)^2 \times 30 = 2{,}115{,}000$ watts $= 2{,}115$ kw. $= \dfrac{2{,}115}{30{,}000 \times 0.88} \times 100 = 8$ per cent.

Taking the transformer iron losses as 1 per cent., the efficiency of the transmission is 91 per cent. The losses in the phase modifier should be subtracted.

437. High-voltage Direct-current System.—The transmission of power over long distances by direct current is limited by the difficulty of obtaining a sufficiently high voltage. In the Thury system the required line voltage is obtained by connecting a number of series-wound generators in series and the line current is automatically maintained constant. Series machines are built generating from 1,500 to 5,000 volts with a single commutator. A single generating unit may have two commutators and two units may be driven by one prime mover giving a direct voltage up to 20,000 volts per set. If five such machines are connected in series the voltage between lines will be 100,000 volts.

To maintain constant line current with variable load, the generator voltage must be varied; this is accomplished by means of an automatic regulator which moves the brushes and varies the resistance in a rheostat or diverter shunting part of the field winding. In some cases the speed of the prime mover is varied. The regulator may be set for any required current. In the systems at present in operation currents from 50 to 200 or 300 amp. are employed.

It is impossible to insulate rotating machines for the high voltages required for transmission and it is therefore necessary to mount the generators on insulated platforms and to couple them to the prime movers by insulating couplings.

The various generating units need not be included in a single station but may be located at any point along the proposed line. In this way a number of small generating stations may be linked up into one large system and since the current is constant they cannot be overloaded under any condition.

Fig. 492 shows the layout of a 60,000-volt, 150-amp. system. At periods of light load any generator may be taken out of service by disconnecting the regulator and moving the brushes over to the position of zero voltage and then closing the short-circuiting switch. To put it in service again it is brought up to speed, the short-circuiting switch is opened and the regulator set for the required current.

The substation equipment is very similar to that in the generating station and like it may be located in one or more stations. If it is necessary to operate the motors at constant speed a regulator is provided which maintains the speed constant by moving the brushes and shunting part of the motor field current.

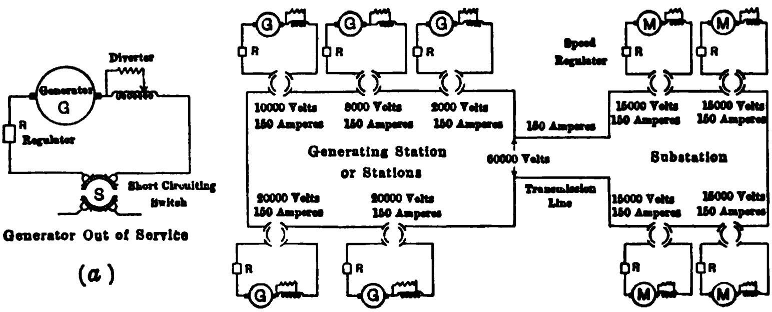


Fig. 492.—Thury or series system.

The motors must drive through insulating couplings and are put in operation by opening a short-circuiting switch and moving the brushes to the proper position. The regulator is then connected in and takes care of the speed.

In the majority of terminal stations the power which has been transmitted by direct current is converted into alternating current for local distribution, the series motors driving alternating-current generators.

438. Advantages and Disadvantages of the Thury or Series System.—*Advantages.*—1. Power factor is always unity as there is no reactance drop. Cables carrying direct currents can be laid in iron pipes if necessary.

2. Higher voltages may be used for the same line insulation. Direct voltages may be double the effective alternating voltages since there are no dielectric losses in the cables.

3. Two conductors only have to be insulated and the center point of the system may be grounded to reduce stresses.

4. A number of stations can be operated in series and a new station may be connected to the line at any point. The individual stations are entirely independent of local overloads or lack of demand since the current is maintained constant and regulation does not enter into the question.

5. Switching arrangements are very simple.

6. Cost of right-of-way will usually be very small since the line may be placed entirely underground making it possible to cross country where a right-of-way could not be obtained. In crossing large bodies of water high-voltage submarine cables may be used saving the expense of step-down and step-up transformers required in a similar case in the alternating-current system. To supply electric-railway substations the line could be placed underground along the right-of-way, a single conductor being tapped at each substation.

7. A single conductor with earth return may be used either as the ordinary method of operation or in case of emergency. All other grounds must then be removed and some of the apparatus insulated for the full-line voltage.

Disadvantages.—1. Insulated floors and couplings are required.

2. Units are of moderate size. A single unit delivering 250 amp. at 20,000 volts has a rating of only 5,000 kw.

3. Line loss is constant independent of the load.

4. Special regulating devices are required to keep the motor speed constant.

5. Motors have no overload capacity, since the current is constant.

CHAPTER XV

ELECTRICAL INSTRUMENTS

439. Electrical Instruments.—A complete study of electrical instruments is beyond the scope of this book but in the following pages the principles of a number of the more important types used in the measurement of electrical quantities are discussed.

440. Direct-current Voltmeters and Ammeters.—The majority of direct-current voltmeters and ammeters are of the movable-coil permanent-magnet type (Fig. 493). A light rectangular coil of wire *M*, wound on an aluminum frame, turns in the field of a permanent magnet *NS*. The field is made uniform by placing an iron cylinder *C* between the poles and this gives the instrument a uniform scale. The coil is supported on jewelled bearings

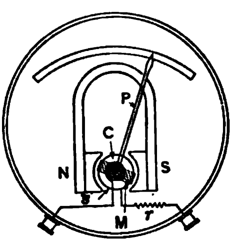


Fig. 493.—Weston permanent-magnet type of direct-current voltmeter or ammeter.

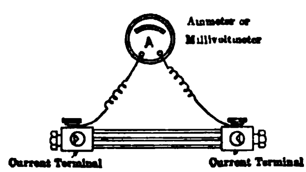


Fig. 494.—Ammeter shunt.

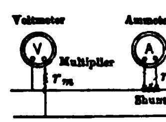


Fig. 495.

and its motion is controlled by two spiral springs *s*, *s*, above and below, coiled in opposite directions. The springs are used to lead the current in and out of the coil. The aluminum frame acts as a damper and makes the instrument dead beat.

A large resistance *r* must be inserted in series with the coil to limit the current in it to a suitable value. The resistance required varies from 50 to 150 ohms per volt and the current ranges from 7 to 20 milliamp. at full-scale deflection.

441. Voltmeter Multipliers.—If it is desired to change the range of a voltmeter an extra resistance r_m, Fig. 495, must be connected in series with the movable coil. If the resistance r_m

of the multiplier is exactly equal to the resistance of the coil plus any resistance r in series with it, then for a given line voltage the current is reduced to half and the scale reading to half. Such a multiplier doubles the range of the instrument and all readings must be multiplied by 2. Multipliers from 2 to 1 up to 20 to 1 are used with direct-current voltmeters.

442. Ammeter Shunts.—The ammeter is exactly similar to the voltmeter in construction (Fig. 493). A shunt of known resistance r_s, Fig. 494, is placed in the circuit and carries the current to be measured. A voltmeter or millivoltmeter is connected across the terminals and the pointer indicates the drop of voltage across the shunt. This drop is Ir_s and is proportional to the current since the resistance r_s is constant. By properly arranging the resistance of the shunt and the resistance r in series with the movable coil the instrument may be made to read directly in amperes.

If a shunt has a resistance $r_s = 0.001$ ohm and carries a current of 100 amp., the voltage drop is 0.1 volts = 100 millivolts. Instead of calibrating the scale in millivolts it may be calibrated directly in amperes.

If the ammeter is provided with a number of shunts it may be used to measure a large range of currents. For the smaller current ranges the shunt is usually placed inside the meter case.

The permanent magnet type is standard for direct-current instruments but cannot be used for alternating currents.

443. Thomson Inclined-coil Ammeter.—Fig. 496 shows a section of an inclined-coil ammeter or voltmeter. The current to be measured flows through the inclined coil and produces a magnetic field perpendicular to the plane of the coil. A soft-iron core is placed in the field and tends to turn until it becomes parallel to the direction of the flux. The motion is controlled by a spiral spring and the counterweight balances the moving parts. The motion of the pointer is damped by a light aluminum vane. In the ammeter the stationary coil consists of a few turns of large wire, while the voltmeter has a large number of turns of small wire. The scale is not uniform. Inclined-coil meters may be used for both alternating and direct currents but they are not so good as the permanent-magnet direct-current meters.

444. Weston Soft-iron-type Ammeters and Voltmeters.—This type of instrument is illustrated in Fig. 497. A is a fixed triangular piece of soft iron bent into the form of a cylinder and B

is a movable piece carried on a shaft. A magnetic field is produced by the coil C which carries the current to be measured or a current proportional to the voltage to be measured. The two soft-iron pieces become magnetized in the same direction and repel one another, producing a deflection of the pointer. Motion is controlled by a spiral spring and oscillations are damped out by a vane enclosed in an air chamber. The iron pieces are shaped to give a uniform scale.

Such instruments may be used to measure direct currents as well as alternating currents.

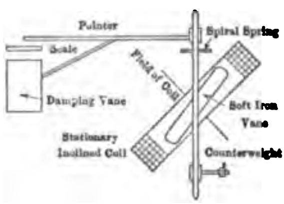


FIG. 496.—Thomson inclined-coil ammeter.

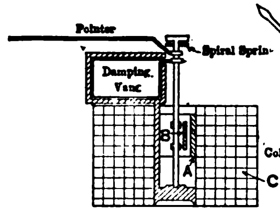


FIG. 497.—Weston soft-iron type ammeter or voltmeter.

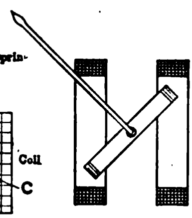


FIG. 498.—Electro dynamometer-type voltmeter.

445. Electrodynamometer-type Voltmeter.—The electrodynamometer-type voltmeter, Fig. 498, depends on the interaction between two coils carrying a current proportional to the voltage to be measured. The permanent magnet of Fig. 493 is replaced by a coil of wire in series with the moving coil. No iron is used in the instrument. This construction is not suitable for ammeters since the current must be carried by a moving contact between the coils. It can be used for voltmeters but other types are more satisfactory. Wattmeters are, however, designed on this principle.

446. Hot-wire Ammeters and Voltmeters.—The expansion of a wire, due to the heat produced by the passage of current through it, is utilized in this type of meter (Fig. 499). AB is a wire of platinum-silver alloy which carries the current to be measured. As it expands the tension on the fine phosphor-bronze wire CD is reduced and this enables the spring S to pull the silk fiber FE to the left. The silk fiber passes around a pulley on the shaft of the moving system and causes the pointer to move over the scale. The motion of the disc is damped by the aluminum disc which turns between the poles of the permanent magnet shown in Fig. 499(a).

For voltmeters a large resistance R is connected in series with the hot wire so that the current is proportional to the voltage to be measured. The zero can be adjusted by the screw at A.

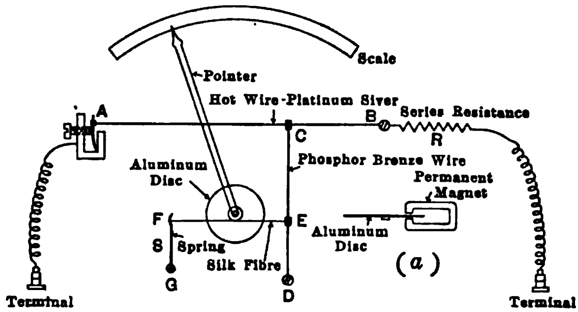


FIG. 499.—Hot-wire ammeter or voltmeter.

447. Dynamometer-type Wattmeter.—The dynamometer principle is satisfactory when applied to wattmeters, Fig. 500, because the two coils are not connected in series but carry different currents. The series coil of large wire carries the line current and takes the place of the permanent magnet in the direct-current voltmeter, Fig. 493. The voltage coil is exactly similar to the moving coil in the direct-current voltmeter and is supported and controlled in the same way.

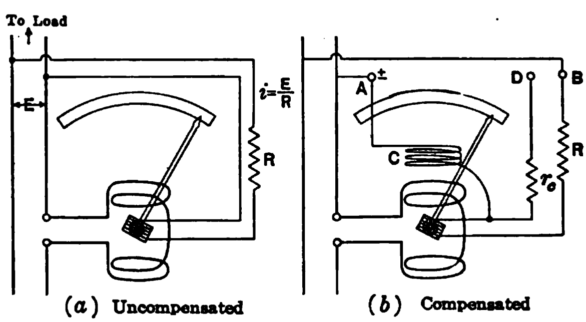


FIG. 500.—Dynamometer type wattmeter.

The torque exerted on the moving coil is proportional to the product of line current and line voltage, that is, to the power in the circuit.

If necessary a shunt may be used in parallel with the current coil and a multiplier in series with the voltage coil.

A wattmeter connected as in Fig. 500(a) has a slight zero error, because when no current is being supplied to the load, the current for the voltage coil passes through the series coil and the wattmeter indicates a small amount of power $= Ei = \frac{E^2}{R}$ where E is the line voltage and R is the resistance of the voltage coil. To correct this error a third coil C called the compensating coil Fig. 500(b) is added. It has the same number of turns as the series coil but is wound in the opposite direction and its m.m.f. neutralizes the m.m.f. of the small current $i = \frac{E}{R}$.

For ordinary measurements of power the voltage is applied between terminals A and B, but when the meter is to be calibrated, using separate voltage and current sources, no zero error is introduced and a third terminal D must be provided instead of A. In series with it is a resistance r_c equal to the resistance of the compensating coil.

Dynamometer-type wattmeters can be calibrated with direct current and used on alternating-current circuits.

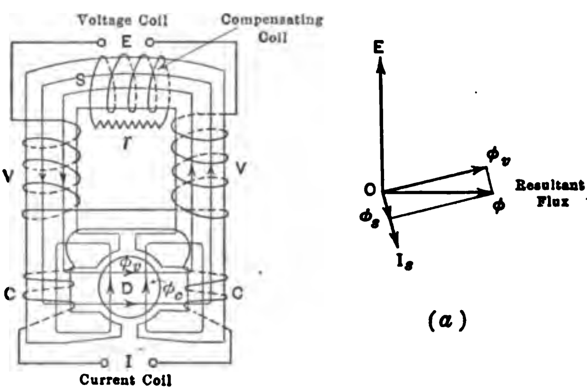


Fig. 501.—Induction-type wattmeter.

448. Induction-type Wattmeter.—An induction wattmeter is shown in Fig. 501. The voltage coils VV are connected in series across the line and the current coils are connected in series with the line. Both sets of coils are wound on the same laminated iron core.

The voltage coils on the two legs are wound in the same direction and their m.m.fs. add. The circuit is highly inductive and

the current through it is nearly 90 degrees behind the line voltage. The flux ϕ_v is in phase with the current and is therefore in quadrature behind the line voltage.

The current coils on the two legs are wound in opposite directions and both tend to produce a flux ϕ_c in the gap in mechanical quadrature with the flux ϕ_v. The flux ϕ_c is in phase with the line current and with non-inductive load it is 90 degrees ahead of the flux ϕ_v. A revolving field is thus produced in the space between the poles.

A light aluminum cup or disc D is pivoted in this field and tends to turn with it but is opposed by spiral springs. The torque is proportional to the product of the voltage and the line current, that is, to the power in the circuit.

The current in the circuit VV and the flux ϕ_v produced by it are never quite 90 degrees behind the line voltage E due to presence of resistance in the coil and to iron losses in the core. The error introduced would be negligible for power factors near unity but with inductive loads it would be serious. To make the potential flux lag exactly 90 degrees behind the line voltage a second coil S is wound on the upper part of the core and it is closed through an adjustable resistance r. The flux ϕ_v, Fig. 501(a), induces in the coil S an e.m.f. and current approximately 90 degrees behind it and this current produces a component of flux ϕ_s in phase with itself. ϕ_s combines with ϕ_v and produces a resultant potential flux ϕ exactly in quadrature behind E. ϕ_s may be adjusted by adjusting the resistance r in series with the coil S.

Polyphase wattmeters are made in the same way and consist of two single-phase elements exerting torque on a single disc.

Induction-type ammeters and voltmeters have also been constructed. All meters of this type are accurate only for the frequency for which they have been designed.

If the controlling springs are removed from a wattmeter and a registering mechanism is added the meter becomes a watt-hour meter and registers energy consumption.

449. Power-factor Meters.—Power-factor meters indicate the power factor of a circuit; they are constructed on the same principles as wattmeters, of both electrodynamometer and induction types and may be either single-phase or polyphase.

The circuits of a Weston single-phase power-factor meter are shown in Fig. 502. The current coil F is similar to that in the wattmeter, Fig. 500, but the moving system is made up of two

similar coils C_1 and C_2. The common terminal of the two coils is connected to one side of the line and the other terminals are connected through a resistance R and an inductance L to the other side of the line. The current in C_1 is therefore in phase with the line voltage and the current in C_2 is in quadrature behind the line voltage.

At non-inductive load the currents in C_1 and F are in phase and C_1 will be forced around until its plane is parallel to the plane of F. If the load current lags 90 degrees behind the voltage the current in C_2 is in phase with that in F and C_2 is turned into the plane of F. For intermediate power factors the moving element takes up intermediate positions.

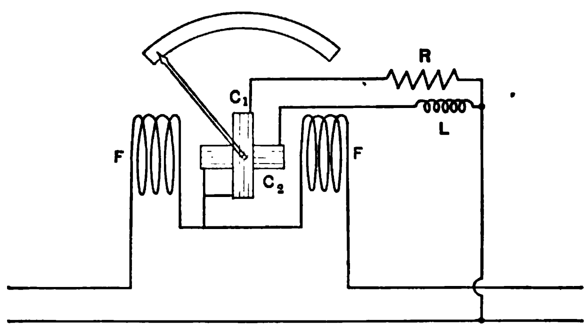


FIG. 502.—Weston single-phase power-factor motor.

This instrument can be used as a two-phase power-factor meter if C_1 and C_2 are each connected across one phase in series with a suitable resistance.

A three-phase meter has three coils on the moving element connected in star to the three lines.

450. Frequency Meters.—The vibrating reed frequency meter (Fig. 503) contains a number of steel strips of different lengths fixed at one end and free to vibrate at the other. An electromagnet is placed behind the strips and is excited from the circuit of which the frequency is required. The strip with a natural period corresponding to that of the magnetic field will be set in vibration. The ends of the reeds are painted white and when one is in vibration it shows as a white band. The periods of the reeds are adjusted during manufacture by attaching minute weights to the free ends.

451. The Weston Frequency Meter.—The elements of a Weston frequency meter are shown in Fig. 504. Two stationary coils A and B are fixed at right angles; a resistance R and inductance X are connected in series across the line; the coil A is connected in series with an inductance X_A across the resistance R and coil B is connected in series with a resistance R_B across the inductance X. The series inductance X_s is added to damp out higher harmonics.

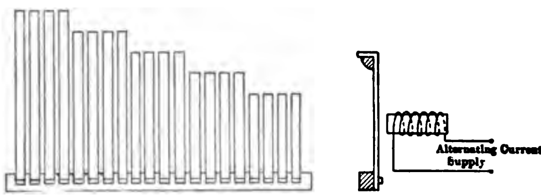


FIG. 503.—Vibrating reed frequency meter.

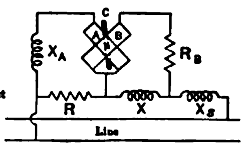


FIG. 504.—Weston frequency meter.

The moving element consists of a soft-iron core C mounted on a shaft without control of any kind and it takes up a position in the direction of the resultant of the two fields.

When the frequency increases, the current in A decreases and that in B increases and the core and pointer move with the resultant field.

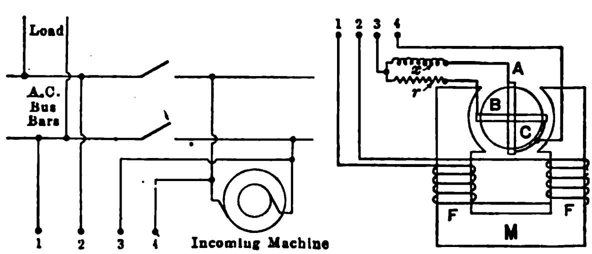


FIG. 505.—Synchroscope.

452. Synchroscope.—A synchroscope indicates: (1) whether the incoming machine is running too fast or too slow, and (2) the exact instant when synchronism is reached.

One form of synchroscope is shown in Fig. 505. It has a laminated bipolar magnetic circuit M excited by field coils FF, which are connected across the alternating-current busbars at 1 and 2 and produce an alternating field. The moving core C is also laminated and carries two windings A and B at right angles to

one another. Their common terminal is connected to one side of the incoming machine at 4. The other terminals of A and B are connected respectively through a reactance x and a resistance r to the other side of the machine at 3.

The current in F is in quadrature behind the line voltage, the current in A is in quadrature behind the machine voltage and the current in B is in phase with the machine voltage.

When the incoming machine is exactly in synchronism the coil A takes the position shown in the figure since the current in A is in phase with the current in F. When the machine is 90 degrees behind the position of synchronism the current in B is in phase with the current in F and the armature turns through 90 degrees and brings the coil B in line with the poles.

For intermediate phase relations the armature takes intermediate positions such that the revolving field produced by the armature winding is in line with the field poles when the current in F is maximum. The phase relation is indicated by a pointer on the dial of the synchroscope.

When the frequency of the incoming machine is lower than that of the line, the phase of the current in A continually falls behind that of F and the pointer rotates in the direction marked "Slow." When the incoming machine is running too fast, the rotation of the pointer is in the opposite direction marked "Fast."

When the machine is running exactly at synchronous speed and is exactly in phase, the pointer is vertical and stationary. The switch can then be closed and the speed and excitation adjusted until the machine takes its proper share of the load and operates at the proper power factor.

The synchroscope described is for a single-phase circuit. It can be used for a two-phase machine by connecting the two coils A and B to the two phases of the machine and the coils FF across one phase of the line.

For three-phase systems the armature carries a three-phase winding connected to the three phases of the machine.

453. Tirrill Regulator.—The Tirrill regulator is an automatic voltage regulator designed to maintain a steady voltage at the terminals of a direct-current generator irrespective of ordinary load fluctuations or changes in generator speed. It can also be made to compensate for line drop by increasing the generator voltage as the load increases.

The regulator controls the voltage by rapidly opening and clos-

ing a shunt circuit across the field rheostat of the generator. The rheostat is so adjusted that when in circuit it tends to reduce the voltage considerably below normal and when short-circuited the voltage tends to rise above normal. The relative lengths of time during which the short-circuit is closed or opened determines the average value of the field current and therefore the value of the terminal voltage.

The method of operation of the regulator is illustrated in Fig. 506. The regulator consists essentially of two magnets controlling two sets of contacts. The main control magnet has two independent windings, one, the potential winding, connected across

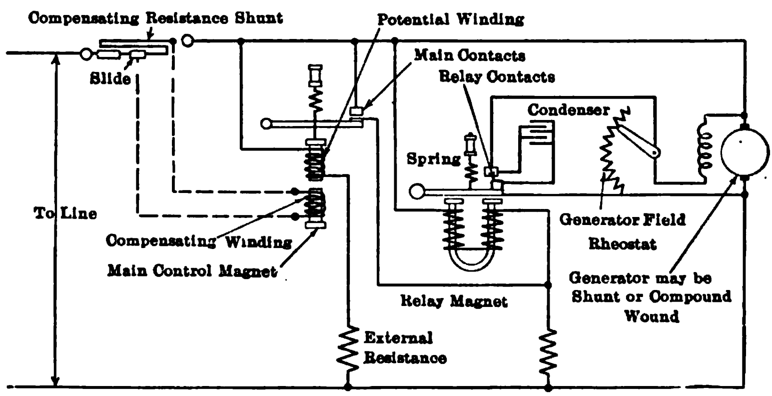


FIG. 506.—Automatic voltage regulator.

the generator terminals and the other across a shunt in the load circuit. The latter is the compensating winding and is used only when a rise of voltage with load is required. The relay magnet is differentially wound and controls the circuit shunting the field rheostat. The operation is as follows: When the short-circuit across the field rheostat is opened the voltage tends to fall below normal. The main control magnet is weakened and allows the spring to pull out the movable core until the main contacts are closed. This closes the second circuit of the differential relay and demagnetizes it. The relay spring then lifts the armature and closes the relay contacts. The field rheostat is short-circuited and the field current and terminal voltage tend to rise. The main control magnet is strengthened and opens the main contacts allowing the differential relay to open the short-circuit across the field rheostat. The terminal voltage falls again and

this cycle of operations is repeated at a very rapid rate maintaining a steady voltage at the generator terminals. When the compensating winding is not used the terminal voltage is maintained constant.

When it is necessary to compensate for line drop and maintain a constant voltage at the receiver end of the line, the compensating winding is connected across a shunt in the load circuit. The resistance of the shunt is adjusted to give the required compounding. The compensating winding opposes the action of the potential winding on the main control magnet so that as load increases a higher potential is necessary at the generator terminals in order to close the main contacts and open the shunt across the field rheostat. Thus, the generator voltage rises with load. The condenser connected across the relay contacts serves to reduce the sparking when the circuit is opened.

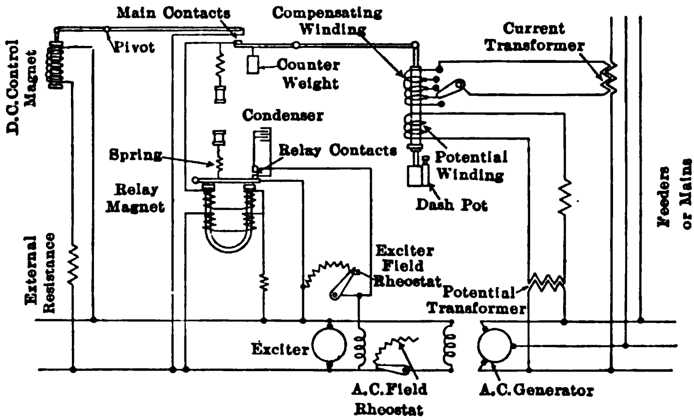


FIG. 507.—Automatic voltage regulator for alternating-current generators.

454. Automatic Voltage Regulator for Alternating-current Generators.—The regulator described in Art. 453 may be modified to regulate the voltage of alternators. The desired voltage is maintained by opening and closing a short-circuit across the exciter field rheostat.

The method of operation of the regulator can be understood from the diagram of connections shown in Fig. 507. The direct-current control magnet is connected to the exciter busbars and has a fixed core in the bottom and a movable core in the top attached to a pivoted lever, at the other end of which is a spring

which closes the main contacts. The alternating-current control magnet has a potential winding connected across one phase of the alternator and it may also have a compensating winding connected through a current transformer to one of the feeders. The core is movable and is connected to a pivoted lever controlled by a counterweight. The relay magnet is differentially wound and is connected as shown.

Operation.—The direct- and alternating-current control magnets are adjusted for the required voltage by means of the counterweight. The exciter field rheostat is then set to reduce the voltage about 65 per cent. below normal. This weakens both of the control magnets and the spring closes the main contacts and demagnetizes the relay magnet. The pivoted armature is released and the relay contacts are closed and thus short-circuit the exciter field rheostat and immediately raise the exciter voltage and the alternator voltage. When the alternator voltage reaches the value for which the regulator is adjusted, the main contacts open again, the relay magnet is again magnetized and the short-circuit on the exciter field rheostat is opened. This reduces the voltage as before and the cycle of operations is repeated at a very rapid rate and maintains a constant voltage at the terminals of the alternator if the compensating winding is not connected.

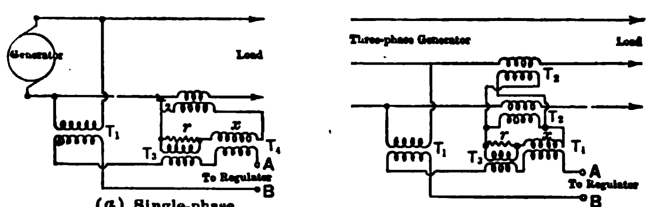


FIG. 508.—Line compensators.

When it is necessary to maintain a constant voltage at the receiver end of the line the compensating winding is connected as shown. As the load increases it brings the main contacts closer together and so increases the time of short-circuit of the field rheostat and thus increases the terminal voltage of the alternator. Using a current transformer and a dial switch any line drop up to 15 per cent. can be compensated for, but only at a given power factor. When the power factor of the load varies through a wide range a line compensator, Fig. 508(a), should be used in con-

junction with the potential coil and the compensating coil should be disconnected.

The line compensator forms a local circuit with the same voltage characteristics as the main line. The shunt transformer T_1 gives a secondary voltage proportional to the generator voltage. The current transformer T_2 produces through the circuit rx a current proportional to the load current. r is a resistance which consumes a voltage proportional to and in phase with the resistance drop in the line and this voltage is transferred to the compensator circuit by the potential transformer T_3. x is a reactance and consumes a voltage proportional to the reactance drop in the line. This voltage is transferred to the compensator circuit by the transformer T_4 which also forms the reactance. Thus there are in the compensator circuit three voltages proportional respectively to the generator voltage and the resistance and reactance drops. If the same proportions are maintained in each case, the voltage between the terminals AB will be proportional to the voltage at the end of the line, and, therefore, if the potential coil of the regulator is connected across AB it will maintain a constant voltage at the receiver end of the line. In the case of transmission lines of very high voltage a correction must be made for the capacity of the lines. With a three-phase alternator, as in Fig. 507, two current transformers must be used connected as shown in Fig. 508(b).

One automatic voltage regulator can be applied to control the voltage of a system operating two or more alternators in parallel.

The regulator may also be applied to the exciter of a synchronous-phase modifier to maintain a constant voltage at the receiver end of the line.

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